Mental Accounting, Access Motives, and Overinsurance

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Abstract

Consumers frequently overinsure modest risks. I argue that confining consumers' insurance motives to a single motive - risk aversion - is responsible for the difficulty to rationalize this behavior. People who perform mental accounting have an additional motive for buying insurance. They perceive a risk of having insufficient means to self-insure. This complements behavioral approaches to explain the profitability of warranties and the dislike of deductibles. It accounts for several empirical regularities that are difficult to reconcile within existing models. Finally, it suggests that the way in which an insurer pays benefits influences the value and the cost of insurance.

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Introduction

Insurance decisions have become an increasingly popular topic in behavioral economics since, on the one side, insurance markets are an integral part of modern economies, and, on the other side, these markets have produced plenty of evidence that documents departures from the benchmark of expected-utility theory.[1] Two types of behavior that have typically been described as instances of overinsurance are the avoidance of deductibles and the purchase of extended warranties.

There is a common view among economists that extended warranties are exploitative devices.[2] Due to our understanding of insurance as a device that allows consumption-smoothing across states, warranties cannot produce a significant surplus since people should be approximately risk-neutral with regard to the expenses that warranties insure. Consequently, the observed attractiveness of warranties to consumers and the resulting possibility for firms to reap significant profits from their sale has left economists puzzled.[3] Given the discrepancy between the predictions of standard models and observed behavior, several attempts have been made to explain this sort of behavior as an instance of mistaken decision-making. First, the overestimation of the claim probability has been proposed as a possible explanation, either because of probability weighting or because of an underestimation of future claim cost.[4] In addition, myopic loss-aversion has been identified as a possible reason for warranty purchase.[5] There is empirical evidence supporting both the view that customers overestimate the claim probability and the view that consumers' loss aversion plays an important role in the purchase of warranties.[6] Yet, there is also empirical evidence calling into question whether this can be the whole story. First, it has been observed that customers' propensity to buy a warranty is strongly related to the value of the product (Chen, Kalra, and Sun (2009) and OFT (2012)). Second, despite the high profitability of warranties, their sale is often confined to expensive products (OFT (2012)). Finally, there is evidence suggesting that warranty purchase varies with income - yet the sign of the variation differs across studies.[7]

[1] See e.g. Kunreuther, Pauly, and McMorrow (2013) for an overview.

[2] See e.g. Baker and Siegelman (2014) for a nice exposition of the basic argument.

[3] In a much noted article, Businessweek (2004) reports that "profits from warranties accounted for all of Circuit City's operating income and almost half of Best Buy's". Ten years later, Warranty Week (2014) notes in its 2014 Mid-Year Service Contract Report that "Consumers will pay nearly $ 40 billion this year for product protection plans, despite the best efforts of watchdogs who tell them not to".

[4] See e.g. Cutler and Zeckhauser (2004), and Michel (2014)

[5] See e.g. Rabin and Thaler (2001).

[6] See e.g. Jindahl (2014).

[7] Chen, Kalra, and Sun (2009) find a negative relationship for warranties covering electronic devices. Padmanabhan and Rao (1993) find a positive relationship for extended service contracts for cars. Chu and Chintagunta (2011) find a concave relationship between income and the duration of a purchased car warranty.

Neither the standard model of insurance nor the behavioral models of mistaken overinsurance are able to explain these empirical patterns.

At the same time, deductibles are an important part of insurance contracts in most insurance markets. This is based on the insight from insurance economics that a certain amount of risk sharing through deductibles helps to mitigate moral hazard. As long as consumers retain a "modest" amount of risk, the utility loss from incomplete coverage is negligible since consumers should be approximately risk neutral with regard to the stakes created by deductibles. In contrast to this prediction, consumers seem willing to accept significant premium increases in exchange for a full elimination or at least a decrease of the deductible prescribed by an insurance policy. Again, the most popular explanations for this discrepancy are probability weighting and/or loss aversion.[8] While these approaches succeed in predicting an aversion toward deductibles, they fail to explain the observed context sensitivity of this aversion.[9] Deductible avoidance has been documented in the context of flood insurance (Michel-Kerjan and Kousky (2010)), a market typically associated with probability underweighting, not overweighting. At the same time, Brown and Finkelstein (2007) find no evidence for deductible avoidance in the US market for private long-term care (LTC) insurance. The latter is particularly surprising as deductibles in the LTC insurance market are sizable enough that even standard expected-utility theory predicts more comprehensive insurance to be desirable.

I argue that a part of our models' inability to explain the observed attractiveness of warranties and unattractiveness of deductibles is a result of these models confining the value of insurance to its consumption-smoothing role. That is, part of economists' puzzle with regard to warranty demand and deductible avoidance is due to risk aversion being regarded as the sole motive for buying insurance.

Following an idea initially proposed by Nyman (2003) in the context of health insurance, I argue that insurance can be valuable as it helps to overcome budget constraints. Having experienced the loss of an asset (sickness, product failure), an individual may not possess the funds that are necessary to remedy the loss (medical expenditure, product replacement or repair). An insurance eliminates or at least alleviates this budget risk thereby granting the individual access to the remedy that financial constraints would otherwise inhibit. In this way, an insurance protects the

[8]See e.g. Johnson, Hershey, Meszaros, and Kunreuther (2000), Sydnor (2010), and Barseghyan, Molinari, O'Donoghue, and Teitelbaum (2013).

[9]See Barseghyan, Prince, and Teitelbaum (2011).

consumption value of the insured asset. This is not the case in the standard view of insurance, in which insurance is a device to mitigate the consumption variation due to the cost of the remedy. I argue that this *access motive* is also relevant for modest stakes such as the expenses insured by warranties or the expense to pay a deductible since there is ample evidence that people have a tendency to perform mental accounting.[10] With this minor modification, the model can explain why there is a significant gap between a customer's willingness-to-pay for the warranty and its actuarial value even if the customer is risk-neutral, does not misjudge the claim probability, and is not loss averse. This allows a seller to reap significant monopoly profits from selling warranties. In addition, it explains why a customer's valuation for a warranty is related to his valuation of the insured product. Finally, it explains why poorer customers as well as customers that buy the product on promotion are more likely to buy a warranty (Chen, Kalra, and Sun (2009)) and why warranties are more likely to be sold on expensive products (OFT (2012)). Allowing for a deductible, I show that the influence of income on insurance demand changes with the size of the deductible. This is because a poorer customer may envisage the possibility of not being able to pay the deductible, making the insurance policy effectively worthless. As this risk increases in the size of the deductible, customers can show a strong aversion towards deductibles. In addition, it can explain why a negative income effect has been observed for warranties insuring electronics that come with no deductible (Chen, Kalra, and Sun (2009)), while a positive income effect or a concave effect has been observed for extended service contracts for cars that typically involve a deductible (Padmanabhan and Rao (1993); Chu and Chintagunta (2011)). Investigating the attitudes towards deductibles that the model predicts, I find that deductible avoidance is strongly related to the value of the insured asset. That can explain the evidence on context-dependence of deductible avoidance that has been documented by Barseghyan, Prince, and Teitelbaum (2011). Finally, I show that the way an insurance pays benefits has a strong effect on both the value and the cost of insurance. In particular, there is a strict order in both value and cost of insurance dependent on whether insurance benefits are paid unconditionally, conditional on a deductible payment, or by reimbursement. This is because insurance benefits are paid in all loss states in the first case. In the second case, they are paid only in the states in which the insuree is able to pay the deductible. In the case of reimbursement, they are only paid in the states in which the insuree is able to advance the money to cover the complete loss. The different valuations dependent on payment style can rationalize why a strong deductible avoidance is observed for flood insurance

[10] See e.g. Thaler (1990), Heath and Soll (1996), Thaler (1999), and Hastings and Shapiro (2012).

(Michel-Kerjan and Kousky (2010)), where benefits are paid unconditionally. At the same time, it can explain why there is no evidence for deductible avoidance in the private market for LTC insurance in the US, in which benefits have traditionally been paid by reimbursement (Brown and Finkelstein (2007)).

I proceed as follows. In section 1, I use a simple framework to show why a standard model cannot account for the substantial profit margins in warranty markets and the strength of deductible avoidance that is typically observed. I show how assuming loss aversion or probability weighting can help to overcome this discrepancy. I continue by pointing out that these alternative approaches are still unable to accommodate several empirical patterns concerning warranty demand and deductible avoidance. In section 2, I introduce a simple modification that is applicable both to the standard model and behavioral models of insurance and can account for the large profit margins in the warranty markets as well as additional empirical patterns. Section 3 explains why deductible avoidance is context-dependent and discusses the role of deductibles in accounting for seemingly contradictory evidence on the impact of household income on warranty purchase. In section 4, I show how the payment details of an insurance contract affect its value and cost, as well as customers' attitudes toward deductibles. Section 5 discusses the role of mental accounting for producing the additional insurance motive that is proposed. Also, I outline how this adaptation complements previous approaches based on probability weighting and/or loss aversion. Finally, I discuss the relationship with the literature on background risk and risk taking. In section 6, I conclude. All proofs are deferred to the Appendix.

1 A Simple Model of Insurance

Suppose an individual possesses an asset that he values at V. That asset can take several forms: a consumption good - a plasma TV, a cell phone, or a car - or his good health, i.e. the absence of a disease. There is a probability $\pi \in (0, 1)$ that he looses this asset: the TV or car may malfunction, or he may be stricken by a disease. In all of these cases, there is a remedy available at a price $p < V$: repair or replacement of the consumption good, or a treatment for the disease that returns the individual to good health. The individual will then purchase the remedy in case of a loss. His utility without insurance is thus given by

$$\mathbb{E}u_0 = (1-\pi)u(V) + \pi u(V-p). \tag{1}$$

Suppose now, the individual is offered an insurance at a price w that completely covers the cost of the remedy p in case of a loss. His utility with insurance is then given by

$$\mathbb{E}u_I = u(V-w). \tag{2}$$

Thus, the individual will purchase the insurance if and only if

$$\mathbb{E}u_I - \mathbb{E}u_0 \geq 0 \Leftrightarrow w \leq \pi p + \text{risk premium}.$$

With a similar logic, the maximal willingness-to-pay to avoid a deductible $d \in (0,p)$ can be calculated by replacing p with d in equation (1) and subtracting it from the utility of full insurance given by (2). This gives a maximal willingness-to-pay of (πd+risk premium) to avoid a deductible of size d.

There are many instances in which it is argued that the risk premium is - or should be - negligible in these calculations. When the stakes are modest, people should be approximately risk neutral.[11] That is, if p is modest, as is argued in the case of e.g. extended warranties, then the maximal willingness-to-pay for this warranty should be approximately πp. In other settings, such as home, car, or health insurance, it is argued that at least the deductibles are low enough that people should be approximately risk-neutral with regard to the stakes imposed by deductibles. Hence, these people should show a maximal willingness-to-pay of approximately πd to avoid a deductible. In the following, I want to give a short account of the model's prediction that are at odds with empirical observations.

1. The model predicts that there are no (significant) gains from trade generated by extended warranties as measured by the difference between maximal willingness-to-pay and expected cost. Hence, a monopolist should not be able to reap a significant profit from their sale. Furthermore, competitive pressure in the market for warranties should have no significant effect on warranty prices. In contrast to this prediction, firms make significant profits from the

[11] See e.g. Rabin (2000), and Rabin and Thaler (2001).

sale of extended warranties and competition in warranty markets lowers prices (Businessweek (2004), OFT (2012)).

2. Controlling for the price of the product p, the model predicts a negative correlation between the willingness-to-pay for the product and the willingness-to-pay for the warranty. This is the result of the first being negatively related to break-down risk, and the second being positively related to break-down risk. Controlling for both price and break-down probability, there should be no correlation between the two. However, research has found a positive correlation between the value of the product and the propensity to buy a warranty (Chen, Kalra, and Sun (2009), OFT (2012), Chark and Muthukrishnan (2013)).

3. In contrast to the model's prediction, there is a correlation between warranty purchase and income. The sign of this correlation varies across studies (Padmanabhan and Rao (1993), Chen, Kalra, and Sun (2009), Chu and Chintagunta (2011)).

4. According to the model, consumers should be willing to accept a (higher) deductible d as long as they are compensated with a reduction in the premium w that is (slightly more favorable than) actuarially fair: πd. However, empirical results suggest that consumers avoid deductibles. They require a reduction of the premium that is much larger than πd in order to choose a policy that specifies a deductible of size d over a policy that does not require a deductible (Sydnor (2010)).

5. The willingness-to-pay to avoid a deductible is predicted to be a function of π and d alone. However, the existence and extent of deductible avoidance are found to be context-dependent (Barseghyan, Prince, and Teitelbaum (2011), Michel-Kerjan and Kousky (2010), Brown and Finkelstein (2007)).

These empirical findings are hence at odds with the standard model's theoretical prediction. The most prominent approaches to address these discrepancies typically focus on the first and fourth point. They posit that customers overweight the loss probability π, either because they perform probability weighting or they overestimate the probability π with which a loss occurs. Alternatively, people's loss aversion can lead them to overweight the payment p (or d respectively). These approaches are successful in predicting a positive wedge between the willingness-to-pay $\bar{w}$

for the warranty and the expected cost of coverage:

$$\bar{w} - \pi p = \omega(\pi)p - \pi p = (\omega(\pi) - \pi)p > 0, \tag{3}$$

$$\bar{w} - \pi p = \pi\lambda p - \pi p = (\lambda - 1)\pi p > 0. \tag{4}$$

with $\omega(\cdot)$ denoting a probability-weighting function, and λ denoting a parameter measuring the degree of loss aversion. It easy to see that such modifications can result in a firm with market power to be able to sell a warranty with positive profit. The same modifications can explain why consumers are found to demand a premium reduction larger than πd to accept a deductible. These behavioral approaches can thus explain the profitability of warranties and the avoidance of deductibles. However, given that these approaches only change the weighting of either π and/or p (d), they cannot account for the influence of other variables, such as asset value or income.

In the following, I want to suggest a simple way to accommodate all of these observations. It suggests that one reason for the discrepancy between observed behavior and the predictions of both standard and behavioral insurance models may lie in these models narrowing insurance motives down to consumption-smoothing motives. In this way, we fail to appreciate an additional value that consumers ascribe to insurance.

2 Budget Constraints

Contrary to the previous model, suppose that the consumer expects a chance to be unable to pay p in case of a loss. I discuss this central assumption and its relation to the concept of mental accounting in more detail in section 5. Suppose for simplicity, the probability that the consumer is unable to repurchase the product is given by $0 < \rho < 1$ and independent of a loss occurring. In this case, the utility from self-insuring to a risk-neutral individual[12] is given by

$$\mathbb{E}u_0 = (1 - \pi)V + \pi(1 - \rho)(V - p). \tag{5}$$

As the outside option becomes less attractive, this changes the consumer's valuation of insurance as measured by his maximal willingness-to-pay.

[12] I assume risk neutrality throughout the whole derivation in order to carve out the effects that are entirely due to the alternative insurance motive that is proposed here.

Proposition 1. *A risk-neutral individual has a maximum willingness-to-pay of* $\bar{w} = \pi(\rho V + (1 - \rho)p) = \pi p + \pi\rho(V - p)$ *for full insurance.*

If the individual perceives a chance to be unable to pay p in case of a loss, then the value of insurance exceeds its actuarially-fair value πp even if the individual is risk-neutral. An insurance gives the individual access to the loss remedy when his own funds do not suffice to purchase it on his own. Accordingly, this value has been termed *access value* in the context of health insurance (Nyman (2003)). I argue here that this value is of interest more generally. In particular, if people perform mental accounting, then they can perceive a risk of not being able to pay p even if p is rather modest. In addition, as I argue in the next section, they can perceive a chance of being unable to pay even a deductible. Note the following comparative statics that result from this proposition. First, $\bar{w}$ strictly increases in V as long as $\rho > 0$. This can explain a positive correlation between the propensity to buy a warranty and the value of the product, even after controlling for both product price and break-down risk. Second, $\bar{w}$ strictly increases in ρ as long as $p < V$. Hence, people with a higher budget risk are predicted to have a larger willingness-to-pay for warranties that fully replace a broken product. This is consistent with poorer people showing a stronger inclination to purchase such warranties. Since the budget risk ρ simply measures the probability with which a customer expects not to be able to repurchase the product in case of break-down, this comparative static is also consistent with the finding that a warranty purchase is more likely if the product was bought at a promotion price (Chen, Kalra, and Sun (2009)). Third, if $F(x)$ is the distribution associated with the customer's available budget, with $f(x)$ being the associated density, then $\frac{\partial \bar{w}}{\partial p} \geq \pi$ if and only if $f(p)(V - p) \geq F(p)$. We can expect this latter inequality to be fulfilled in markets in which (a) there is competition in the base good market, such that p is low, and (b) a high-value product is sold, such that V is high. If this inequality is fulfilled, then the willingness-to-pay is predicted to respond stronger to changes in p than predicted by the standard model. Such an overresponse has typically been interpreted as evidence of probability weighting and/or loss aversion.

In the following, I seek to derive the value of insurance if this insurance comes with a deductible of size d. This allows to investigate the attitudes towards deductibles that the adaptation predicts. Also, I want to point out how the existence of a deductible can explain the seemingly contradictory evidence on the effect of income on warranty purchase.

3 Partial Insurance

Consider the case in which the insuree has to pay a deductible $d < p$ when making a claim. A risk-neutral individual derives a utility

$$\mathbb{E}u_{I(d)} = (1-\pi)(V-w) + \pi\left[(1-\rho)(V-w-d) + \rho(-w)\right] \tag{6}$$

from such an insurance. Hence, he has a maximal willingness-to-pay of

$$\begin{aligned}\bar{w}(d) =& \pi\left[(p-d) + \rho(V-p) - \delta(V-d)\right] &(7)\\ =& \pi\left[(p-d) + \underbrace{(\rho-\delta)(V-p)}_{\text{access value}} \underbrace{-\delta(p-d)}_{\text{claim risk}}\right] &(8)\end{aligned}$$

where $\rho = F(p), \delta = F(d)$. Equation (8) indicates two reasons for people to avoid deductibles even in the absence of probability weighting or loss aversion. First, a deductible reduces the access value provided by insurance. With a probability $\delta = F(d)$, the individual's budget falls below d. In this case, even if he receives a benefit payment $p - d$ by the insurer, he cannot make the payment p that is necessary to remedy the loss of V. Second, very frequently, the benefit payment is conditional on deductible payment.[13] If that is the case, the insuree perceives a claim risk of $\delta = F(d)$ that he will not receive any insurance benefit despite incurring a loss. Due to these two effects, the model predicts a willingness-to-pay of $\pi d + \pi\delta(V - d) > \pi d$ to avoid a deductible. More generally, for any two deductibles d_h, d_l with $0 \leq d_l < d_h < p$, it is argued that for the typical sizes of deductibles observed, consumers should behave approximately risk-neutral. That is, the difference $\bar{w}(d_l) - \bar{w}(d_h)$ is predicted to be approximately $\pi(d_h - d_l)$. Yet, the observed willingness-to-pay for a lower deductible often far exceeds that value. The model can predict a strong aversion to higher deductibles and can thus complement previous approaches based on probability weighting and loss aversion.

Proposition 2. *For any two deductibles d_h, d_l with $0 \leq d_l < d_h < p$, the willingness-to-pay for*

[13] This is the case for health insurance or for car warranties among others. If the insuree is unable to pay his part d of the bill, he receives no service. And if he does not receive any service, the insurer does not need to settle any claim. A deductible can thus prevent an insuree from filing a claim in case of a loss. This *claim risk* depends on the way in which the insurance pays benefits. I will return to this point in the following section.

the lower deductible exceeds the value $\pi(d_h - d_l)$ *if and only if*

$$\frac{F(d_h) - F(d_l)}{F(d_h)} > \frac{d_h - d_l}{V - d_l}. \tag{9}$$

The model predicts an aversion to deductibles if the relative increase in budget risk due to the higher deductible exceeds the reduction in consumer surplus due to the higher deductible. Note that this predicts a stronger aversion towards deductibles for insurances covering assets of higher value V. This is consistent with evidence presented by Barseghyan, Prince, and Teitelbaum (2011) who find a stronger inclination to choose a lower deductible for home as compared to car insurance.

In addition, the presence of a deductible can explain the seemingly contradictory evidence on the effect of income on warranty purchase. While Chen, Kalra, and Sun (2009) find a negative effect, Padmanabhan and Rao (1993) find a positive effect. Chu and Chintagunta (2011) find a concave relationship between income and warranty purchase. Yet, there is a notable difference between these studies. Chen, Kalra, and Sun (2009) consider warranties for consumer electronics that typically offer full insurance through repair or replacement of a broken device. In contrast, the other two studies consider car warranties that typically prescribe a deductible. In section 2, it was already shown that the model predicts a negative effect of income on the willingness-to-pay for full insurance. Thus, the model can explain the negative effect found in Chen, Kalra, and Sun (2009). I want to show how the sign of this effect can switch from negative to positive if the insurance prescribes a deductible payment.

Let $F_i,\ i = H, L$ denote the budget risk of the poor (H) and the rich (L) group.[14] Then the willingness of the rich group $\bar{w}_H$ is higher than the willingness-to-pay of the poor group $\bar{w}_L$ if and only if

$$[F_H(p) - F_L(p)]\,(V - p) < [F_H(d) - F_L(d)]\,(V - d).$$

Since $d \leq p$, a sufficient condition is $F_H(p) - F_H(d) < F_L(p) - F_L(d)$. The difference $F(p) - F(d)$ is the joint probability with which the individual expects to be unable to bear the full remedy cost p, yet able to pay the deductible d. If this joint probability is lower for poorer customers than for richer customers, the richer group has a higher willingness-to-pay for the warranty.

Proposition 3. *Let* $F_\theta,\ \theta = L, H$ *be twice continuously-differentiable and let* F_L *first-order*

[14] I assume $F_i(0) = 0,\ i = H, L$ throughout.

stochastically dominate F_H. Suppose further, $F_H(x) - F_L(x) > 0$ for some $0 < x < V$, and denote by $x^* < V$ a maximum of $\phi(x) = (F_H(x) - F_L(x))(V - x)$.

Then there exist $p, d:\ 0 < d < p < V$ such that $\bar{w}_L(d) > \bar{w}_H(d)$.

Suppose, in addition, that x^* is the unique interior maximum of $\phi(x)$ and $\phi' > 0,\ \forall x < x^*$ and $\phi' < 0,\ \forall x > x^*$. Then, if $p \leq x^*$, $\bar{w}_H(d) > \bar{w}_L(d)\ \forall d < p$. In contrast, if $p > x^*$, then $\exists! d^* < p:\ \bar{w}_H(d) > \bar{w}_L(d),\ \forall d < d^*$ and $\bar{w}_H(d) \leq \bar{w}_L(d),\ \forall d \geq d^*$, with strict inequality for all $d \in (d^*, p)$.

Furthermore, if a $d^* < p$ exists, then $\frac{\partial d^*(V, F_h, F_l, p)}{\partial p} < 0$ and $\lim_{p \to V} d^* = 0$.

Intuitively, a deductible reduces the access value and produces a claim risk, as argued previously. While it is not clear a priori which group enjoys a larger access value when the insurance prescribes a deductible, it is clear that the poorer group perceives a larger claim risk than the richer group. As soon as the difference in claim risk dominates the difference in access value, the richer customer group ascribes a larger value to the insurance. Moreover, if p is large enough, then this domination occurs for very small deductibles already. This can explain the differing results on the relationship between income and willingness-to-pay for warranties. It explains a negative effect of income on warranty purchase when investigating markets for consumer electronics where typically there are no deductibles. At the same time, it explains evidence on a positive relationship for extended warranties in cars that often prescribe a deductible.[15]

Finally, allowing for nonlinear relationships, Chu and Chintagunta (2011) find evidence on a concave relationship between income and propensity for warranty purchase. The model is able to explain such a finding as well.

Suppose there are three groups with different budget risk: high (H), medium (M), and low (L). Let $F_i,\ i = H, M, L$ denote the budget risk of type i where F_L (F_M) first-order stochastically dominates F_M (F_H). Denote by $\phi_{HM} = (F_H(x) - F_M(x))(V - x)$, $\phi_{HL} = (F_H(x) - F_L(x))(V - x)$, and $\phi_{ML} = (F_M(x) - F_L(x))(V - x)$. Suppose further that for all these three functions, there exists a unique maximum $x_j^*,\ j = HM, HL, ML$ and $\phi_j'(x) > 0$ for all $x < x_j^*$ and $\phi_j'(x) < 0$ for all $x > x_j^*$. Let r_{HM}^* denote the minimum deductible d such that $\bar{w}_H(d) \geq \bar{w}_M(d),\ \forall d \leq d_{HM}^*$ and

[15] Some car warranties do not work with conditional benefit payment, but through reimbursing the insuree once he hands in proof of payment for p. In the following section, I discuss the impact of this different way of paying benefits. I seek to highlight here that under reimbursement, the model predicts the effect of income to always be positive.

$\bar{w}(d)_H < \bar{w}_M(d)$, $\forall d^*_{HM} < d < p$.[16] Define r^*_j, $j = HL, ML$ accordingly. This allows to state the following result.

Proposition 4. *Suppose that for any $x' > x$, if $\phi_{HL}(x) = \phi_{HL}(x')$, then $\phi_{ML}(x) \leq \phi_{ML}(x')$. Then, it holds that $d^*_{HM} \leq d^*_{HL} \leq d^*_{ML}$.*

The condition specified in Proposition 4 rules out that the largest differences between the distributions F_M and F_L occur at lower x than the largest differences between F_H and F_L. If the above ordering of d^*_j is possible, then the effect of income on the willingness-to-pay for a warranty is a simple function of the deductible d.

Corollary 1. *If the condition specified in Proposition 4 is met, then*
for any $d < d^*_{HM}$ the willingness-to-pay for a warranty is strictly decreasing in income: $\bar{w}_L < \bar{w}_M < \bar{w}_H$,
for any $d^*_{HM} < d < d^*_{ML}$, the willingness-to-pay for a warranty is concave in income: $\bar{w}_L < \bar{w}_M$ and $\bar{w}_H < \bar{w}_M$,
and for any $d^*_{ML} < d < p$, the willingness-to-pay for a warranty is strictly increasing in income: $\bar{w}_L > \bar{w}_M > \bar{w}_H$.

Propositions 3 and 4, as well as Corollary 1 show how the size of the deductible influences the effect of income on people's inclination to buy a warranty. This influence helps to explain why Chen, Kalra, and Sun (2009) find a negative effect of income in electronics markets, while Chu and Chintagunta (2011) find a concave relationship and Padmanabhan and Rao (1993) find a positive relationship for car warranties.[17]

4 Payment details matter

In the last section, the production of a claim risk was given as one reason why consumers dislike deductibles. This consequence of deductibles, as well as their impact on the access value, are a function of how the insurance pays out benefits. Consider the following three cases.

[16] Note that if $p < x^*_j$, then $r^*_j = p$.

[17] Chu and Chintagunta (2011) find a negative relationship between firm size and warranty demand in the market for computer servers. If one reinterprets the budget risk $F(x)$ as the probability with which a firm holds insufficient liquid resources to replace a broken server and assumes this risk to be larger for smaller firms, the model can accommodate this finding without assuming different degrees of risk aversion between smaller and larger firms.

In the case we have considered so far, the customer has to pay the deductible d when making a claim. That is, the insurer only pays the benefit $(p-d)$ if the claimant is able (and willing) to pay the deductible. The benefit payment is then *conditional* on the deductible payment. If that is the case, a customer perceiving a budget risk anticipates the possibility that he will be unable to pay the deductible, in which case the insurance pays no benefit.

In contrast, consider the case in which the insurance always pays out a benefit of $(p-d)$ to the insuree in case of a loss. That is, the insurer pays the benefit *unconditionally.* Flood insurance is a prominent example of this practice. In contrast to conditional payment, the insurer pays the benefit $(p-d)$ no matter whether the insuree is able to pay the deductible.

Finally, consider the practice of *reimbursement.* In this case, the insuree has to advance the full remedy cost p in case of a loss, after which he is reimbursed the amount $p-d$ by the insurer. In this case, the insuree anticipates to receive a benefit payment only if he is able to pay p.[18]

It is possible to show that both the customer's willingness-to-pay for insurance as well as the expected cost of insurance coverage differ across the three cases. Denote by $\bar{w}_c, \bar{w}_{uc}, \bar{w}_{ri}$ the maximal willingness-to-pay for the insurance and by f_c, f_{uc}, f_{ri} the actuarially fair price of the insurance. Then it is possible to make the following statement.

Proposition 5. *(i) The maximal willingness-to-pay for the three types of insurance is given by*

$$\bar{w}_c = \pi\left[(p-d) + \rho(V-p) - \delta(V-d)\right], \tag{10}$$

$$\bar{w}_{uc} = \pi\left[(p-d) + (\rho-\delta)(V-p)\right], \tag{11}$$

$$\bar{w}_{ri} = \pi\left[(1-\rho)(p-d)\right], \tag{12}$$

and, hence,

(ii) $\bar{w}_{uc} \geq \bar{w}_c \geq \bar{w}_{ri}$*, with strict inequality for all* $0 < d < p$*.*

[18] Here, I make the strong assumption that the insuree is unable to borrow the amount p that is necessary to make the advance payment. One reason could be borrowing constraints that are prohibitively high. A different reason could be that the insuree performs a very strong type of mental accounting. In the latter case, he does not consider the possibility to transfer funds between different mental accounts even temporarily. I focus on the polar case of prohibitive borrowing constraints here. The weaker the actual or perceived borrowing constraints are, the closer the case of reimbursement is to the case of conditional payment. In the opposite polar case of no borrowing costs and constraints, the case of reimbursement is equivalent to the case of conditional payment.

(iii) The actuarially fair price of the three types of insurance is given by

$$f_c = \pi(1-\delta)(p-d) \tag{13}$$

$$f_{uc} = \pi(p-d) \tag{14}$$

$$f_{ri} = \pi(1-\rho)(p-d), \tag{15}$$

and, hence,

(iv) $f_{uc} \geq f_c \geq f_{ri}$*, with strict inequality for all* $0 < d < p$.

Proposition 5,(i) and (ii) show that customers perceiving a budget risk ascribe different values to an insurance depending on its method to pay benefits. This has two reasons. First, an insurance provides an access value only in the case of unconditional or conditional payment. In these two cases, insurance helps an insuree to afford the remedy in case his own funds do not suffice (as long as they suffice to pay the deductible). In contrast, an insurance that pays by reimbursement only pays a benefit if the insuree's own funds suffice to pay for the remedy. Hence, such an insurance is of no help in case the insuree cannot afford the remedy. Thereby, the insurance fails to create an access value. In addition to the difference in access value, the three types of insurance differ in the probability in which the insuree can actually file a claim given that a loss has occurred. I call this a difference in *claim risk*. Under unconditional payment, the insuree faces no claim risk as he can always file a claim, and, hence, always receives the benefit $p - d$ whenever a loss occurs. In contrast, he can only file a claim under conditional payment if his funds suffice to pay for the deductible. Hence, there exists a positive chance $\delta = F(d)$ that he will receive no insurance benefits despite having incurred a loss. Even worse, under reimbursement, he can only file a claim if his funds suffice to fully advance the remedy cost p. In consequence, there exists a chance $\rho = F(p)$ that he is unable to do so and thus receives no insurance benefits in the loss event. In sum, the method of benefit payment changes the value of insurance as it influences both the existence of an access value and the existence and size of the claim risk.

Parts (iii) and (iv) of Proposition 5 indicate that, when customers perceive a budget risk, the cost of insurance provision depend on the method of benefit payment as well. This is due to the difference in claim risk. Since the method of payment can exclude some insurees from claiming benefits despite having incurred a loss, they also differ in the expected cost of coverage provision.

These results allow interesting welfare comparisons. Let $s_b = \bar{w}_b - f_b$ be the gains from trade created by an insurance of type $b = c, uc, ri$.

Corollary 2. *The gains from trade created by insurance are given by*

$$s_c = s_{uc} = \pi(\rho - \delta)(V - p) \geq 0 = s_{ri}, \tag{16}$$

with strict inequality for all $d < p$.

Corollary 2 shows that unconditional and conditional benefit payment are equivalent from a welfare perspective, while being superior to reimbursement. Note that this is due to our model confining the value of insurance to its access value: the value it provides by enabling the insuree to pay p when he is unable to do so on his own. The probability of having insufficient resources to do so when insured equals $\delta = F(d)$ under conditional and unconditional payment. It equals $F(p) = \rho$ under reimbursement, the same probability the customer would face when self-insuring. Hence, reimbursement does not provide any access value and thus creates no gains from trade.[19]

Beside the welfare implications, the different payment methods predict different attitudes toward deductibles.

Corollary 3. *For any two deductibles d_h, d_l with $0 \leq d_l < d_h < p$, the willingness-to-pay for the lower deductible*
(i) is given by $\bar{w}_{uc}(d_l) - \bar{w}_{uc}(d_h) = \pi\left[(d_h - d_l) + (F(d_h) - F_(d_l))(V - p)\right] > \pi(d_h - d_l)$ if benefits are paid unconditionally, and
(ii) is given by $\bar{w}_{ri}(d_l) - \bar{w}_{ri}(d_h) = \pi(1 - \rho)(d_h - d_l) < \pi(d_h - d_l)$ if benefits are paid by reimbursement.

Corollary 3 shows that there is always a stronger aversion towards higher deductibles as compared to the risk-neutral benchmark with no access motive when benefits are paid unconditionally. This is because the access value is strictly decreasing in d. Since the access value is a function of

[19] An interesting addition to this analysis would be to consider the classic consumption-smoothing motive in addition to the access motive that is modeled here. An unconditional benefit payment transfers money to the insuree in all loss states. In contrast, conditional benefit payments exclude insurance coverage in states in which the budget falls below d. Finally, reimbursement excludes insurance coverage in states in which the budget falls below p. Thus, a risk-averse individual would derive strictly more utility from unconditional benefits than from conditional benefits, and strictly more utility from conditional benefits than from reimbursement. We can conclude that considering both access and consumption-smoothing motive would sharpen the prediction. Unconditional benefit payments then provide strictly larger welfare than the less expensive form of conditional benefit payments. In addition, the welfare gain resulting from a switch from reimbursement to the more expensive form of conditional benefit payments would increase.

V, this inclination to buy lower deductibles is rising in V. At the same time, there is a weaker aversion towards higher deductibles when benefits are paid through reimbursement. This is because (a) the insurance provides no access value, and (b) imposes a positive claim risk that is independent of d. These predictions are interesting when compared to empirical evidence.

First, despite the general agreement that the uptake of flood insurance suffers from people underweighting the probability of such events, Michel-Kerjan and Kousky (2010) find only 3 percent of policyholders to have the largest possible, while almost 80 percent chose the lowest possible deductible. Flood insurance benefits are paid unconditionally.[20]

At the same time, the market for private long-term care insurance in the US suffers from low uptake. Note that in this market insurance benefits have traditionally been paid in the form of reimbursement.[21] The model predicts such a form of insurance to provide no access value which might be one factor contributing to the low demand. In addition, it is exactly in this market in which deductibles are quite sizable as compared to other markets that Brown and Finkelstein (2007) find no evidence of customers seeking lower deductibles. On the contrary, they find customers to choose high deductibles despite more comprehensive coverage being available. Such a low attractiveness of deductibles in the long-term care insurance market is in line with our model's prediction of a low inclination to avoid deductibles when benefits are paid by reimbursement.

5 Discussion

5.1 Budget Risk and Mental Accounting

The argument that an anticipated budget risk increases the willingness-to-pay for full insurance or for a lower deductible rests on the assumption that the decision-maker expects himself to be *unable* to pay the price p and/or the deductible d in case of a loss. A mere *unwillingness* to pay p does not increase $\bar{w}$, since the monetary valuation V of the product then falls below p.

It is important to ask why people can perceive a significant budget risk with respect to the expenses covered by warranties or imposed by deductibles? After all, these expenses are typically

[20] It is, however, important to note that the access motive alone does not predict limited uptake. This suggests an interesting role for performing the proposed adaptation on a model involving probability underweighting. While probability underweighting alone cannot explain the inclination to buy lower deductibles, the access model alone cannot explain limited uptake. A hybrid model could explain both observations.

[21] This is about to change, however, as more and more providers offer benefit payments in indemnity or disability form.

described as being "modest" in size. Here, the observed tendency for mental accounting plays a crucial role. People have been found to subdivide the entire available budget into different budget categories, considering money to be imperfectly fungible between those categories.[22] The relevant budget that is available for paying p or d is then substantially smaller than the whole budget of a household. Instead, these expenses will be compared to the implicit or explicit budget associated with the consumption category of the asset that is insured. This tendency for mental categorization of expenses then leads a decision-maker to perceive a significant risk of not being able to self-insure or to pay the deductible when being formally insured. The predictions concerning the connection between income and insurance, in particular warranty purchase, hold as long as there is a sufficiently strong positive correlation between the size of the relevant budget and the household's overall income.

5.2 Complementarity with previous behavioral approaches

I want to underline how the adaptation I propose in this paper strongly complements with previous approaches assuming distorted probability weights and/or loss aversion. With such modifications the willingness-to-pay for full insurance is given by

$$\tilde{w} = \omega(\pi\rho)\lambda V + \omega(\pi(1-\rho))\lambda p$$

where $\omega(\cdot)$ denotes a probability-weighting function and $\lambda > 1$ is a parameter measuring the degree of loss aversion.[23] It is easy to see that the impact of loss aversion is stronger when the individual perceives a budget risk, as the customer does anticipate a loss greater than the cost p with positive probability. Also, since the loss is greater than p, the impact of overweighting the loss probability is larger. This is further strengthened by the fact that the loss may take two different values. The probability-weighting function is typically assumed to be sub-additive, so $\omega(\pi\rho) + \omega(\pi(1-\rho)) > \omega(\pi)$. Thus, the fact that the loss may take two different values depending on the realization of the budget risk, further strengthens the role of probability-distortions in explaining overinsurance.

I conclude that the adaptation, that I propose here, nicely complements previous approaches to model consumer mistakes in insurance purchase. It adds explanations for empirical patterns

[22] See e.g. Thaler (1990), Heath and Soll (1996), Thaler (1999), and Hastings and Shapiro (2012).

[23] I make the conventional assumption that the payment of the insurance premium is not regarded as a loss.

that could not be accommodated before, while strengthening the impact of previously-identified consumer mistakes in that context.

5.3 Relation to the Literature on Background Risk and Risk Aversion

There is a large body of literature on how an independent background risk can change a person's inclination towards taking over a given risk.[24] This literature shows that when preferences exhibit decreasing absolute risk aversion, then an independent, uninsurable background risk makes a person more willing to take up insurance against a risk that he can insure against. The evidence on decreasing absolute risk aversion reported by e.g. Guiso and Paiella (2008) indicate that this is relevant idea. In addition, given that the common example of an uninsurable background risk is a person's income risk, it is straightforward to think about a relation with the adaptation proposed here.

The results presented here are independent of this literature on background risk. First, the above literature points out how one insurance motive, risk aversion, is affected by the presence background risk. Given that I consider an entirely different insurance motive, the access motive, the results on how background risk affects risk aversion do not apply here. This is all the more obvious as I consider the case of risk neutrality throughout the paper. In consequence, the results that are proposed here cannot be a consequence of the budget risk influencing the individual's risk preferences.

Second, the literature on background risk is less helpful for the questions I investigate. The main focus of this paper is to better understand insurance behavior when stakes are modest, i.e. warranty purchase and deductible avoidance. Since people should be approximately risk neutral toward stakes of such size, the literature on background risk and risk aversion has little to say in this regard.

All this being said, it does not mean that there is no interesting connection to this literature. In section 4, I show that a budget risk leads some loss states to be effectively excluded from coverage if benefits are paid conditionally or by reimbursement. This claim risk matters for a risk neutral individual. It matters even more for someone who is risk averse. Since the claim risk decreases in income, the value of insurance falls more dramatically for poorer consumers depending

[24] See e.g. Pratt and Zeckhauser (1987), Kimball (1990), Kimball (1993), Gollier and Pratt (1996), and Eeckhoudt, Gollier, and Schlesinger (1996).

on whether benefits are paid unconditionally, conditionally, or by reimbursement. If lower income groups exhibit, in addition, a stronger degree of risk aversion this further reinforces the decline in value. This suggests an important complementarity between the results on budget risk presented here and the results on the impact of background (income) risk on risk aversion. It points to a very promising avenue for further research.

6 Conclusion

In this paper, a simple adaptation is proposed that can help to account for various empirical observations that have been made in the context of insurance demand. Allowing consumers to perceive a risk of not being able to replace a broken product helps explain the observation of a positive correlation between product value and the value of a warranty. It strengthens the role of probability weighting and loss aversion in explaining firms' ability to reap significant profits from the sale of warranties. Finally, it helps to reconcile seemingly conflicting empirical evidence regarding the effect of household income on warranty demand.

The same adaptation is able to explain the context sensitivity of deductible avoidance. In addition, the resulting model predicts the value and cost of insurance to be strongly influenced by the the way benefits are paid. This can account both for the observation of deductible avoidance for flood insurance and the lack of similar evidence in the context of long-term care insurance. Finally, the prediction of the significance of payment methods suggests a straightforward way in which the model can be tested.

Given the fact that warranties are an important source of profit in many branches and the significant role that deductibles play in insurance markets, it is vital to reach a better understanding of consumer behavior with respect to the insurance of modest risks. Probability misperceptions and loss aversion have been identified as significant aspects of this behavior. I argue that a broader view on what constitute insurance motives may further our understanding as well.

The proposed adaptation suggests strong complementarities to both the standard approach and behavioral approaches to insurance. Risk aversion and differences in risk tastes have a stronger influence on insurance behavior if people perceive a claim risk. Probability weighting and loss aversion have a stronger impact if people perceive an access value. I conclude that this simple adaptation suggests several avenues for further investigation.

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Appendix

Proof of Proposition 1

A risk-neutral individual derives a utility

$$\mathbb{E}u_0 = (1-\pi)V + \pi(1-\rho)(V-p) \tag{17}$$

from self-insuring while deriving utility

$$\mathbb{E}u_I = u(V-w) \tag{18}$$

from buying insurance that fully pays p in case of a loss. The maximal willingness-to-pay $\bar{w}$ is then given by

$$\begin{aligned} &\mathbb{E}u_I = V - \bar{w} = \mathbb{E}u_0 = (1-\pi)V + \pi(1-\rho)(V-p) \\ \Leftrightarrow &\bar{w} = \pi(\rho V + (1-\rho)p). \end{aligned} \tag{19}$$

Proof of Proposition 2

The maximal willingness-to-pay for an insurance specifying a deductible d is given by

$$\bar{w}(d) = \pi\left[(p-d) + \rho(V-p) - \delta(V-d)\right]. \tag{20}$$

The maximal willingness-to-pay to replace a higher by a lower deductible is then given by

$$\bar{w}(d_l) - \bar{w}(d_h) = \pi \left[d_h - d_l - F(d_l)(V - d_l) + F(d_h)(V - d_h)\right] \tag{21}$$

$$= \pi \left[d_h - d_l + (F(d_h) - F(d_l))(V - d_l) - F(d_h)(d_h - d_l)\right]. \tag{22}$$

It follows that

$$\bar{w}(d_l) - \bar{w}(d_h) > \pi(d_h - d_l) \Leftrightarrow \frac{F(d_h) - F(d_l)}{F(d_h)} > \frac{d_h - d_l}{V - d_l}. \tag{23}$$

Proof of Proposition 3

Note that the difference between the willingness-to-pay of the high-risk and the low-risk group can be expressed as

$$\bar{w}_H - \bar{w}_L = \phi(p) - \phi(d). \tag{24}$$

If $F_\theta(0) = 0,\ \theta = H, L$, then $\phi(0) = \phi(V) = 0$. Since $F_H(x) - F_L(x) > 0$ for some $x,\ 0 < x < V$ by assumption, $\phi(x)$ must have an interior maximum $x^* \in (0, V)$. Since F_H and F_L are continuous, so is $\phi(x)$. Hence, there exist $p \in (x^*, V)$ such that $\phi(p) < \phi(x^*)$. And for any such p, there exists a value d, specifically $d = x^*$, such that $\phi(p) - \phi(d) < 0$ and hence $\bar{w}_H(d) < \bar{w}_L$.

Suppose, in addition, that $\phi(x) = (F_H(x) - F_L(x))(V - x)$ has a unique interior maximum x^* with $\phi'(x) > 0,\ \forall x < x^*$ and $\phi'(x) < 0,\ \forall x > x^*$. Then, for any $p < x^*$, $\phi(x) < \phi(p)$ for all $x < p$, and, hence, there exists no $0 < d < p$ such that $\bar{w}_H(d) < \bar{w}_L$. On the other hand, if $p > x^*$, then there exists a unique $d^* < x^*$ such that $\phi(d^*) = \phi(p)$. Since $\phi'(x) > 0,\ \forall x \in [d^*, x^*)$ and $\phi'(x) < 0,\ \forall x \in (x^*, p]$, we know that $\phi(x) > \phi(p),\ \forall x \in (d^*, p)$. Thus, for all $d \in (d^*, p)$, $\bar{w}_H < \bar{w}_L$. Since $\phi'(x) > 0,\ \forall x \in [0, d]$, we know that $\phi(x) < \phi(d^*),\ \forall x < d^*$. Hence, for all $d < d^*$, $\bar{w}_H > \bar{w}_L$.

Finally, suppose that there exists a $d^* < p$. Then $d^* < x^* < p$ must hold. Since d^* is implicitly defined by $\phi(p) = \phi(d^*)$, and we know that $\phi'(p) > 0$ and $\phi'(d^*) < 0$, we know that d^* declines p. With our assumptions on $\phi'(x)$, $\phi(x) = 0$ holds for $x = 0$ and $x = V$ only. So, as $\phi(p)$ converges to zero as p increases, so must $\phi(d^*)$. Hence, d^* converges to zero as well.

Proof of Proposition 4

Given the assumption on $\phi_j,\ j = HM, HL, ML$, we know from Proposition 3 that $d^*_{HM}, d^*_{HL}, d^*_{ML}$ are unique values. By definition, $\phi_{HL}(d^*_{HL}) = \phi_{HL}(p)$. By assumption, it then holds that $\phi_{ML}(d^*_{HL}) \leq \phi_{ML}(p)$. We can rule out that $x^*_{ML} < d^*_{ML} < p$, for that would imply $\phi_{ML}(d^*_{HL}) > \phi_{ML}(p)$, since $\phi'_{ML}(x) < 0,\ \forall x \in (x^*_{ML}, p)$. Since $\phi'(x) \geq 0,\ \forall x \leq x^*_{ML}$ and $\phi_{ML}(d^*_{HL}) \leq \phi_{ML}(p) \leq \phi_{ML}(x^*_{ML})$, we can conclude that there exists a unique $d \geq d^*_{HL}$ such that $\phi_{ML}(d) = \phi_{ML}(p)$. Yet, this unique d is exactly d^*_{ML}. Hence, $d^*_{ML} \geq d^*_{HL}$.

Finally, note that $\phi_{HM}(x) = \phi_{HL}(x) - \phi_{ML}(x)$. Thus, since at d^*_{HL}, $\phi_{HL}(p) - \phi_{HL}(d^*_{HL}) = 0$ and $\phi_{ML}(p) - \phi_{ML}(d^*_{HL}) \leq 0$, it follows that $\phi_{HM}(p) - \phi_{HM}(d^*_{HL}) \geq 0$. Since $\phi_{HM}(p) - \phi_{HM}(d) \leq 0$ for all $d \geq d^*_{HM}$, with strict inequality for all $d^*_{HM} < d < p$, we conclude that $d^*_{HM} \leq d^*_{HL}$.

Proof of Proposition 5

In all three cases the expected utility from self-insuring is given by

$$\mathbb{E}u_0 = (1-\pi)V + \pi(1-\rho)(V-p). \tag{25}$$

The expected utility from insuring when benefits are paid *unconditionally* is given by

$$\mathbb{E}u_{I(uc)} = (1-\pi)(V-w) + \pi\left[(1-\delta)(V-d-w) + \delta(p-d-w)\right]. \tag{26}$$

The expected utility from insuring when benefits are paid *conditionally* is given by

$$\mathbb{E}u_{I(c)} = (1-\pi)(V-w) + \pi\left[(1-\delta)(V-d-w) + \delta(-w)\right]. \tag{27}$$

Finally, the expected utility from insuring when benefits are paid by *reimbursement* is given by

$$\mathbb{E}u_{I(ri)} = (1-\pi)(V-w) + \pi\left[(1-\rho)(V-d-w) + \rho(-w)\right]. \tag{28}$$

The maximal willingness-to-pay for an insurance of a specific payment type can simply be derived by finding the level w at which the individual is indifferent between self-insuring and buying formal insurance:

$$\mathbb{E}u_{I(uc)} = \mathbb{E}u_0 \Leftrightarrow \bar{w}_{uc} = \pi\left[(p-d) + (\rho-\delta)(V-p)\right], \tag{29}$$

$$\mathbb{E}u_{I(c)} = \mathbb{E}u_0 \Leftrightarrow \bar{w}_{c} = \pi\left[(p-d) + \rho(V-p) - \delta(V-d)\right], \tag{30}$$

$$\mathbb{E}u_{I(ri)} = \mathbb{E}u_0 \Leftrightarrow \bar{w}_{ri} = \pi\left[(1-\rho)(p-d)\right]. \tag{31}$$

At the same time, an insurer needs to pay the benefit $(p-d)$ whenever a loss occurs under unconditional payment, when a loss occurs and the insuree is able to pay d under conditional payment, and when a loss occurs and the insuree is able to pay p under reimbursement. This straightforwardly gives the expected cost of coverage, i.e. the actuarially fair prices of insurance:

$$f_{uc} = \pi(p-d), \tag{32}$$

$$f_{c} = \pi(1-\delta)(p-d), \tag{33}$$

$$f_{ri} = \pi(1-\rho)(p-d). \tag{34}$$