A General Approach to Estimating Random Preference Models of Risk Attitudes

by

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December 2023

Abstract. The random preference (RP) model provides an integral framework for modeling within-individual heterogeneity in choice behavior, by attributing this heterogeneity to preference parameters in the underlying theory of risk attitudes instead of an additive error term that is external to the theory. However, most empirical studies in structural estimation of risk attitudes turn to additive error specifications because the RP likelihood function is computationally unattractive. We propose a general approach to estimating the RP model that facilitates empirical applications in this alternative modeling framework. Our estimation approach illustrates that the RP model is just as flexible as other stochastic choice models. By applying a kernel smoothing procedure, we can construct a versatile likelihood evaluator of the RP model that can accommodate any decision theory, types of lottery pairs, and parametric distribution of unobserved heterogeneity.

Keywords: behavioral noise, decision theory, experimental data, risk aversion, structural econometrics.

JEL codes: C51, D81, D90

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1. Introduction

Structural models of individual choice behavior typically assume that latent preference parameters are deterministic at the individual level and do not vary randomly *within an individual* over different decision tasks. To account for unexplained variation in individual choice behavior, the deterministic preference index function is subsequently combined with an additive error term that varies randomly over decision tasks. Recent theoretical developments have generated renewed interest in the random preference (RP) model of choice under risk (Becker, DeGroot and Marschak [1963]) by establishing its axiomatic foundations (Gul and Pesendorfer [2006]) and stochastic monotonicity with respect to the degree of risk aversion (Wilcox [2011]; Apesteguia, Ballester and Lu [2017]; Apesteguia and Ballester [2018]). The RP model excludes the additive error term and treats the preference parameters *per se* as stochastic variables that vary randomly within an individual over different decision tasks. From an empirical perspective, the RP model is known to be an unwieldy model to estimate except in a few special cases because the preference index function is not additively separable from the source of stochastic variation. One may apply a general purpose simulator based on the simple frequency logic to approximate the choice probabilities with relative ease, but the simulated likelihood function becomes a step function which does not lend itself to gradient-based optimization.[1]

We propose a general approach to estimating the RP model of choice under risk which can be applied to any empirical specification regardless of the underlying decision model and experimental design. The crux of our approach is the use of McFadden's [1989] perturbation strategy to construct a kernel-smoothed simulator of choice probabilities, which enables us to re-formulate the likelihood maximization problem in a way that the usual apparatus of maximum simulated likelihood (MSL) estimation can be applied to. McFadden's work was focused on the multinomial probit model, but we demonstrate that the same strategy can be fruitfully exploited in estimation of the RP model as well. The kernel-smoothed simulator replaces indicator functions in the simple frequency simulator with smoothing kernels such as logistic distribution functions. The

[1] Global search methods, such as differential evolution and particle swarm optimization algorithms (*e.g.*, Hole and Yoo [2017]), may help one maximize a step function but do not address problems with point identification of parameter values.

subsequent simulated likelihood function is a smooth function of preference parameters, which can be maximized by applying standard optimization techniques. Using smoothing kernels instead of indicator functions does not entail any constraints on the RP model itself or the data. This flexibility makes our approach applicable to a general class of RP models. The likelihood evaluator can be coded with relative ease and generalized to accommodate multidimensional risk preference parameters, as well as unobserved interpersonal heterogeneity, in a tractable fashion.

Existing approaches to estimating the RP model focus on restricted cases that enable one to derive analytic choice probabilities. These cases are based either on Expected Utility Theory (EUT) with a one-parameter utility function, thereby precluding non-EUT models as well as EUT with more flexible utility functions; or on restrictive types of decision tasks with lotteries over a universal set of at most four outcomes, thereby precluding experimental designs with more variation in outcomes. The only existing study that estimates a RP model with more than one random risk preference parameter is Wilcox [2008; §4.5].[2] It is also one of two existing studies that estimate RP models of non-EUT preferences.[3] He considers Rank-Dependent Utility Theory (RDU) which extends EUT by adding a probability weighting function (PWF) that complements the utility function (Quiggin [1982]). Wilcox places a *partial* RP structure on RDU by adopting a flexible utility function with two RP parameters, while maintaining a non-random PWF for each individual. His statistical approach is specifically designed for this partial RP-RDU framework. Additionally, it necessitates the use of a bivariate gamma distribution, and of data from an experiment that features the restrictive types of decision tasks mentioned above. Our general approach can accommodate any

[2] The same case study is also reported in Wilcox [2011; pp.101-102]. Ironically, the main aim of this case study is to explain why it is difficult to generalize RP models to multi-parameter and non-EUT settings.

[3] Loomes, Moffatt, and Sugden [2002] estimate a Rank-Dependent Utility model akin to that in Wilcox [2008]. However, their RP model features only a single RP parameter within the utility function and employs a deterministic probability weighting function. Apesteguia, Ballester, and Gutierrez-Daza [2023] and Jagelka [2023] estimate RP adaptations of the discounted expected utility model. This includes one RP parameter measuring risk aversion under EUT and another measuring delay aversion under exponential discounting. As the model's risk preference specification adheres to a one-parameter EUT framework, it avoids the complexities in specifying non-EUT models with multiple RP parameters, a challenge highlighted by Wilcox [2008] and which we will discuss further in Section 3.

decision model, any number of random parameters, any distributional assumption, and any set of decision tasks that allows one to identify the model specification of interest.[4]

McFadden's perturbation strategy has only been used in the estimation of multinomial probit. In fact, it has not even seen wider use in this area of application, except as an inspiration for the normal-error component logit-mixture model (Walker, Ben-Akiva and Bolduc [2007]), presumably because it is far more general than required for the task at hand. In multinomial probit models, random variations in individual choice behavior are attributed to alternative-specific error terms which follow a multivariate normal distribution. Leveraging this assumption of multivariate normality, the GHK simulator offers a practical method for constructing simulated choice probabilities which are inherently smooth with respect to model parameters.[5] This has made the GHK simulator the standard tool for estimating multinomial probit models. By contrast, McFadden's kernel-smoothed simulator can be applied to any distribution—be it normal or non-normal—but necessitates a subjective choice by the user regarding the degree of kernel smoothing. The RP model, compared to the multinomial probit, is better positioned to benefit from this tradeoff between generality and automated implementation. Depending on which preference parameters enter the embedded decision models and the logical constraints that they must satisfy, different RP model specifications may require the use of different distributional assumptions.

To illustrate the feasibility and practical application of kernel smoothing for the estimation of RP models, we use data from two existing experiments to estimate EUT and RDU preferences. We also examine extensions to a hybrid choice model, which incorporates a trembling mechanism to permit violations of stochastic dominance, and a model of population heterogeneity that addresses random taste variations both between and within individuals. The small number of restricted cases of EUT and RDU, which yield analytic RP choice probabilities, provide particularly interesting test

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[4] In Online Appendix A, we complement our general estimation approach by a simple, convenient approach to estimating one-parameter EUT models that satisfies the single-crossing condition for a given data set. In this setup the RP-EUT model has a dual representation as a standard binary response model. The duality allows one to use standard commands in software packages (*e.g.* -logit- and -xtlogit, re- in *Stata*) in the estimation of the one-parameter RP-EUT model.

[5] A Cholesky factorization of multivariate normal errors lead to a linear combination of independent univariate normal errors. The GHK simulator exploits this property to approximate multinomial probit probabilities by using a series of univariate normal distribution functions. Train [2009; §5.6.3] provides an accessible summary along with references to independent contributions by Geweke, Hajivassiliou, and Keane.

beds for our general approach albeit our general approach might be seen as an overkill for such cases. Since these cases can be estimated by the standard method of maximum likelihood (ML) as well as our general approach, we can directly compare the two sets of the results to gauge approximation noise introduced by the use of the perturbation strategy in authentic empirical settings.

In our empirical case studies, we also aim to offer practical guidance on the selection of the smoothing factor that determines the extent of kernel smoothing. One immediate complication is that the effect of a given value of the smoothing factor depends on the overall scale of utility, which does not have a natural unit of measurement. We find it useful to scale the index function of the RP model by the utility of the best available outcome in a given application, which enables us to think of the value of the smoothing factor in terms of percentage of the maximum utility level. A follow-up issue is to actually find an appropriate value of the smoothing factor. Our empirical results suggest that a value close to 1% of the maximum utility level provides a good default configuration. With this configuration, the kernel-smoothed MSL estimates are practically indistinguishable from the standard ML estimates where both sets of the results are available. This similarly suggests that with an appropriate setting the kernel-smoothed simulator can indeed provide reliable approximation to the underlying RP likelihood function.

We stress that the RP model *does not* refer to the usual random coefficient model that has been widely used in applied microeconomics. The two models serve quite different purposes, and one can combine the RP model with the fixed coefficient model (*e.g.*, Apesteguia and Ballester [2018]) or with the random coefficient model (*e.g.*, Wilcox [2008][2011]). The RP model uses a statistical distribution to describe variation in preference parameters *within* an individual over decision tasks, whereas the random coefficient model uses a statistical distribution to describe variation in preference parameters *between* individuals.[6] Our general estimation approach can be applied to both fixed and random coefficient versions of the RP model.

[6] The use of the term *random coefficient* to describe this approach to modeling interpersonal heterogeneity is well-established in all branches of applied microeconomics: See, for example, Revelt and Train [1998], Cohen and Einav [2007], Blass, Lach and Manski [2010], and Harrison, Lau and Yoo [2020].

2. One-dimensional Random Preference Models

We begin by introducing our general estimation approach to RP models in a simple setting with a single risk preference parameter. Expected Utility Theory (EUT) with a constant absolute risk aversion (CARA) or a constant relative risk aversion (CRRA) utility function, as well as one-parameter formulations of Yaari's [1987] Dual Theory, fall into this class of decision models. Even in this simple setting the likelihood function of the RP model does not have an analytic solution unless specific constraints are imposed on the decision tasks in the experimental design. Moreover, the standard frequency-based simulator of the likelihood function generates a step function that is not amenable to numerical optimization.[7] To tackle these computational challenges, we apply the kernel-smoothed frequency simulator that McFadden [1989] developed to address similar issues in the estimation of multinomial probit models.

A. Perturbation to EUT with CRRA Utility

Consider data from an experiment where subjects choose between two lotteries, A and B, in each decision task. Assume for now that every subject has the same "urn of *random* risk preference parameters," a metaphor to be clarified shortly. We use $n \in \{1, 2, \cdots, N\}$ to index subjects, and $t \in \{1, 2, \cdots, T\}$ to index decision tasks. Lottery $L \in \{A, B\}$ in subject n's decision task t is a probability distribution over K prizes which pays prize m_{Lknt} with probability p_{Lknt}, where $k \in \{1, 2, \cdots, K\}$. All prizes and probabilities are known to the subjects before they make their decisions. Finally, let m_{max} denote the maximum possible prize in all decision tasks.

We use EUT with CRRA utility to illustrate our kernel smoothing approach to estimate one-dimensional RP models. The utility of prize M is

$$U(M|\omega) = (M^{(1-\omega)} - 1)/(1 - \omega), \tag{1}$$

[7] Simulation is often required when additive error models are combined with random coefficients. As Revelt and Train [1998] explain the frequency-based simulator is not necessarily a step function in those cases: When the additive error term follows a parametric distribution which produces a smooth link function (*e.g.*, logit), the simulated likelihood function is a smooth function of the parameters to be estimated.

where $\omega \in (-\infty, \infty)$ is the coefficient of relative risk aversion. Without loss of generality, we assume that prizes are scaled such that $m_{max} > 1$, hence $U(m_{max} | \omega) > 0$ for all ω. The expected utility of lottery $L \in \{A, B\}$ is given by

$$EU_{Lnt}(\omega) = \sum_k p_{Lknt} \times U(m_{Lknt} | \omega) \qquad (2)$$

and an EU-maximizing subject will choose lottery B if $EU_{Bnt}(\omega) > EU_{Ant}(\omega)$ and lottery A if the sign of this inequality is reversed.

Structural econometric models usually account for unexplained variation in individual choice behavior by including idiosyncratic error terms which are distributed independently of the subject's preferences. By contrast, the RP model accommodates stochastic choice behavior by treating the relative risk aversion parameter as a random variable, ω_{nt}, that varies across subjects and, more importantly, *over decision tasks within a subject*. Suppose that the random risk parameter ω_{nt} is logistically distributed with mean μ_ω and scale σ_ω, and let $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ denote the density function. One can interpret the density function as an "urn" that contains different values of ω_{nt} (Wilcox [2008; p.213]), and μ_ω and σ_ω as the mean and dispersion of the urn's contents. In each task, the subject makes a new draw from the urn, with replacement, and the outcome of that draw determines her degree of relative risk aversion in that task. The probability that subject n chooses lottery B in task t is then

$$L_{nt}(\mu_\omega, \sigma_\omega) = \int \mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0] f(\omega_{nt} | \mu_\omega, \sigma_\omega) d\omega_{nt} \qquad (3)$$

where $\mathbf{I}[.]$ denotes an indicator function and $\Delta EU_{nt}(\omega_{nt})$ refers to a scaled EU difference between the two lotteries

$$\Delta EU_{nt}(\omega_{nt}) = (EU_{Bnt}(\omega_{nt}) - EU_{Ant}(\omega_{nt})) \,/\, U(m_{max} | \omega_{nt}). \qquad (4)$$

The choice probability in equation (3) is invariant to any increasing transformation of the EU difference in the numerator of equation (4).

The lack of separability between the index function $\Delta EU_{nt}(.)$ and the stochastic component ω_{nt} in the RP model makes it difficult to obtain analytic choice probabilities without further assumptions, such as the single-crossing property that we will discuss in Section 2.C. Given some candidate values of μ_ω and σ_ω, one may think of applying a standard frequency simulator to perform Monte Carlo integration and compute a simulated analogue to equation (3) as follows

$$S_{nt}(\mu_\omega, \sigma_\omega) = (1/R) \sum_r \mathbf{I}[\Delta EU_{nt}(\omega_{ntr}) > 0] \qquad (5)$$

by using R pseudo-random draws from $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$, where ω_{ntr} refers to the rth draw of ω_{nt} and $r \in \{1, 2, \cdots, R\}$. However, this approach is not amenable to maximum simulated likelihood (MSL) estimation of the unknown parameters μ_ω and σ_ω. Given a finite number of pseudo-random draws, the simulated choice probability in equation (5) may be equal to 0, and more importantly it is a step function which implies that different candidate values of μ_ω and σ_ω may return the same value of $S_{nt}(\mu_\omega, \sigma_\omega)$. The former issue implies that the sample log-likelihood value may be undefined, and the latter issue precludes the use of gradient-based maximization algorithms to compute μ_ω and σ_ω.[8]

We use the perturbation strategy by McFadden [1989; p.1001] to construct a kernel-smoothed frequency simulator that lends itself more easily to numerical maximization. The key idea is to perturb the inequality inside the indicator function by adding a *contaminating* disturbance term so $\mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0]$ is replaced with $\mathbf{I}[\Delta EU_{nt}(\omega_{nt}) + \varkappa \times v_{nt} > 0]$, where $\varkappa$ is a smoothing factor to be selected by the researcher prior to estimation and v_{nt} is a standard logistic variate. The disturbance term is *contaminating* since it does not form part of the assumed stochastic choice process; it represents an intentional specification error that is added to generate a perturbed model which is easier to simulate than the assumed model. Given a suitably small value of $\varkappa$, the perturbed model can approximate the assumed model to a desired degree of accuracy.[9] Potential bias due to the contaminating disturbance disappears asymptotically if $\varkappa$ is seen as an element of a sequence that decreases at a sufficiently fast rate as the sample size grows. This result supports the use of usual asymptotic inferential procedures along with the perturbation strategy.[10]

Using the law of iterative expectations, a perturbed version of the RP model in equation (3) can be written as

$$L_{nt}(\mu_\omega, \sigma_\omega) = \int \Lambda(\Delta EU_{nt}(\omega_{nt}) \;/\; \varkappa) f(\omega_{nt} | \mu_\omega, \sigma_\omega) d\omega_{nt} \tag{6}$$

[8] Thus, one cannot use standard algorithms, such as Newton-Raphson (NR) and Broyden-Fletcher-Goldfarb-Shanno (BFGS), to maximize the simulated log-likelihood function. The use of gradient-free algorithms may help locate a maximum, but will not fix the failure of identification.

[9] McFadden and Train [2000; p.451] provide a theorem which formally illustrates this intuition.

[10] The asymptotic bias disappears if $\varkappa$ tends to zero at a rate faster than the square root of the sample size. This result is reminiscent of a similar result supporting simulation-assisted estimation, which states that simulation bias vanishes asymptotically if the number of simulated draws grows at a sufficiently fast rate relative to the sample size. Both types of results establish the sense in which the use of perturbation or simulation does not interfere with asymptotic inferences, though they do not help one select the value of $\varkappa$ or the number of simulated draws in empirical applications where the sample size is fixed.

where $\Lambda(z) = \exp(z) / (1 + \exp(z))$ is the standard logistic distribution function that results from integrating out the contaminating disturbance. The kernel-smoothed frequency simulator of the RP model is a simulated analogue to the perturbed model in equation (6):

$$S_{nt}(\mu_\omega, \sigma_\omega) = (1/R) \textstyle\sum_r \Lambda(\Delta EU_{nt}(\omega_{ntr}) / \varkappa) \tag{7}$$

where ω_{ntr} is the r^{th} draw of ω_{nt} from $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ as defined earlier. From a computational angle, the logistic distribution function Λ in equation (7) can be seen as a smoothing kernel for the indicator function in equation (5) rather than an expectation over the contaminating disturbance, hence the name of the simulator. Since the kernel-smoothed simulator is a finite sum of logistic distribution functions, it returns algebraically positive probabilities and is twice continuously differentiable in ω_{ntr}. We can thus construct a sample likelihood function, which can be maximized by using conventional gradient-based algorithms to obtain MSL estimates of the unknown parameters μ_ω and σ_ω that characterize the RP urn.

In the literature on semi-parametric estimation of discrete choice models, the smoothing factor $\varkappa$ is often set at a value proportional to $1/(N \times T)^{0.2}$ to smooth the maximum score estimator (*e.g.,* Horowitz [1992] and Yan and Yoo [2019]), where $N \times T$ is the total number of choice observations. In our empirical applications we collate results from multiple estimation runs with $\varkappa = \# /(N \times T)^{0.2}$ using multiplicative factors $\# \in \{0.01, 0.05, 0.1, 0.25, 0.5, 1, 2\}$. We find that values of $\varkappa$ close to 0.01 work best in terms of convergence, as well as approximation to maximum likelihood (ML) estimates for analytic likelihood functions in those special cases where such ML estimates are available. Scaling the EU difference by the maximum utility in all decision tasks is equivalent to normalizing maximum utility to unity. Our results thus suggest that setting the smoothing factor to a value close to 1% of maximum utility provides a useful default configuration in implementations of the kernel smoothing approach.

B. General Approach to Estimating One-Dimensional RP Models

We can generalize the estimation procedure to other decision models with a single random preference parameter. Let $\Delta V_{nt}(\alpha_{nt})$ denote an index that represents the subject's relative valuation of two options as a function of the parameter α_{nt}, where the exact form of the index may vary across theory and data. Suppose that $\Delta V_{nt}(\alpha_{nt}) > 0$ corresponds to the choice of option B, and let $\zeta(\alpha_{nt} | \boldsymbol{\theta})$

denote the density of α_{nt} which is characterized by the distributional parameters in $\boldsymbol{\theta}$. The probability that subject n chooses option B in task t, $L_{nt}(\boldsymbol{\theta}) = \int \mathbf{I}[\Delta V_{nt}(\alpha_{nt}) > 0]\zeta(\alpha_{nt} | \boldsymbol{\theta})d\alpha_{nt}$, is simulated by

$$S_{nt}(\boldsymbol{\theta}) = (1/R) \textstyle\sum_r \Lambda(\Delta V_{nt}(\alpha_{ntr}) / \varkappa) \tag{8}$$

where α_{ntr} refers to the r^{th} draw of $\boldsymbol{\alpha}_{nt}$ from $\zeta(\alpha_{nt} | \boldsymbol{\theta})$.

Given the simulated choice probability, the likelihood of subject n's choice in task t is

$$h_{nt}(\boldsymbol{\theta}) = S_{nt}(\boldsymbol{\theta})^{ynt} \times [1 - S_{nt}(\boldsymbol{\theta})]^{(1 - ynt)} \tag{9}$$

where y_{nt} is a binary indicator that is equal to 1 if the observed choice is option B and equal to 0 if it is option A. The joint likelihood of all T choices by subject n is

$$H_n(\boldsymbol{\theta}) = \textstyle\prod_t h_{nt}(\boldsymbol{\theta}) \tag{10}$$

where $t \in \{1, 2, \cdots, T\}$.[11] The MSL estimates of $\boldsymbol{\theta}$ can be computed by maximizing the sample log-likelihood function, $H(\boldsymbol{\theta}) = \sum_n \ln(H_n(\boldsymbol{\theta}))$, where $n \in \{1, 2, \cdots, N\}$.

From a programming perspective, the likelihood evaluator for the RP model can be coded by adapting the likelihood evaluator for the pooled logit model. The sample log-likelihood function, $H(\boldsymbol{\theta}) = \sum_n \ln(H_n(\boldsymbol{\theta}))$, has the same algebraic structure as the pooled logit likelihood, except that the simulated choice probability with R draws in equation (8) replaces the standard logit probability. Train [2009; §9] provides an accessible guide to drawing from alternative density functions. Our implementation makes draws based on a Halton sequence (Train [2009; §9.3.3]) as follows.

1. Prior to estimation, make N × T × R draws from a Halton sequence and allocate R distinct draws to each of the N × T choice observations. These draws are held constant throughout model estimation. Each Halton draw is a number in the unit interval (0, 1). Suppose that the data set is stored in the usual long form structure with N × T rows, where each row refers to a distinct choice observation subscripted by "nt" in our notation, and the columns refer to different variables (*e.g.*, the choice indicator and lottery characteristics). This step is then equivalent to adding R new variables to the data set, where each new variable stores a set of Halton draws allocated to N × T observations.

[11] To accommodate panel correlations across repeated choice observations on the same subject, one can combine this RP model of *intrapersonal* heterogeneity with a random coefficient model of *interpersonal heterogeneity*. We present this extension in Section 4.

2. At each iteration of model estimation, convert the Halton draws into draws of α_{nt} from $\zeta(\alpha_{nt}|\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is set to their most recent estimates. This conversion is achieved by inverting each Halton draw using the inverse cumulative distribution function associated with $\zeta(\alpha_{nt}|\boldsymbol{\theta})$. To avoid draws from the far right or left tail of $\zeta(\alpha_{nt}|\boldsymbol{\theta})$ that can induce numerical problems, we truncate the Halton draws on (0.005, 0.995) prior to applying the inversion.[12] In the context of our EUT example in equation (5), this step entails applying the inverse logistic distribution function separately to each truncated Halton draw. From a data management perspective, this step is akin to generating R new variables which are non-linear transformations of the R variables generated in Step 1.

3. At each iteration of model estimation, re-evaluate the kernel-smoothed simulator in equation (8) at the draws of α_{nt} obtained in Step 2. Finally, use the results to re-evaluate the likelihood function in equation (10).

Once the likelihood evaluator has been coded, model estimation can proceed by applying a numerical optimization technique to update the estimates of $\boldsymbol{\theta}$ between iterations. We use the BFGS technique which is known to work well in maximizing simulated likelihood functions.

C. Application: EUT with CRRA Utility

To estimate the EUT model with CRRA utility, we use data from Andersen, Harrison, Lau and Rutström [2014]. The data set includes 413 subjects from the general adult population in Denmark, and each subject in the experiment was asked to make choices from 40 distinct pairs of lotteries A and B. Each lottery pair can be written as $A = \{(m_{A1}, (1 - p_2)), (m_{A2}, p_2)\}$ and $B = \{(m_{B1}, (1 - p_2)), (m_{B2}, p_2)\}$ where $m_{B1} < m_{A1} < m_{A2} < m_{B2}$. For each set of prizes $[m_{A1}, m_{A2}, m_{B1}, m_{B2}]$, the probability p_2 varied from 0.1 to 1 in increments of 0.1. For now, we exclude lottery pairs with dominant choices (*i.e.*, with $p_2 = 1$) which do not contribute to the identification of risk preferences

[12] The truncated draw is a weighted average of the two endpoints of the truncated interval, where the weight is the un-truncated draw. Let d_{nt} denote a Halton draw allocated to observation nt; a truncated draw is then equal to $0.995d_{nt} + 0.005(1 - d_{nt})$. In the context of estimating Generalized Multinomial Logit models, Keane, Fiebig, Louviere and Wasi [2010; §3] recommend truncation on (0.023, 0.977). Our experience suggests that a more conservative truncation on (0.005, 0.995) suffices for RP model specifications.

in the RP model.[13] There were four prize sets in the design, Set_1 = [1600, 2000, 100, 3850], Set_2 = [750, 1125, 250, 2000], Set_3 = [875, 1000, 75, 2000] and Set_4 = [1000, 2250, 50, 4500], where the amounts are in Danish kroner.[14] At the end of the experiment, one of the subject's 40 choices was randomly selected for payment, and each subject had a 10% chance of receiving the payment.

The lottery pairs follow the same algebraic structure as popular multiple price lists, which have been designed to make the EUT model with CRRA utility conform to the single-crossing property. Thus, in each decision task without dominant choices (*i.e.*, $p_{2nt} < 1$), there exists a unique and pre-determined value of w_{nt} that solves $\Delta EU_{nt}(w_{nt}) = 0$, where $\Delta EU_{nt}(.)$ refers to the scaled EU difference in equation (4).[15] In relation to the RP model, the single-crossing property implies that the subject chooses lottery A ($\Delta EU_{nt}(\omega_{nt}) < 0$) if the person is more risk-averse than the indifference point, *i.e.* $\omega_{nt} > w_{nt}$, and lottery B ($\Delta EU_{nt}(\omega_{nt}) > 0$) if $\omega_{nt} < w_{nt}$. Since $\Pr(\Delta EU_{nt}(\omega_{nt}) > 0)$ is now equal to $\Pr(\omega_{nt} < w_{nt})$, there is an analytic expression for equation (3) that specifies the probability of subject n choosing lottery B in task t, namely $L_{nt}(\mu_\omega, \sigma_\omega) = \Lambda((w_{nt} - \mu_\omega)/\sigma_\omega)$.[16]

By using this analytic expression to construct a sample log-likelihood function, one can apply the standard ML procedure to estimate μ_ω and σ_ω without relying on kernel-smoothed simulation. These *analytic* ML estimates provide a useful benchmark for the empirical performance of our approach. The analytic choice probability, $\Lambda((w_{nt} - \mu_\omega)/\sigma_\omega)$, is what our kernel-smoothed simulator is intended to approximate. Given a small approximation error, the kernel-smoothed MSL estimates of μ_ω and σ_ω should be numerically similar to the corresponding analytic ML estimates.

Figure 1 shows that we can obtain MSL estimates that are practically indistinguishable from the analytic ML estimates if the smoothing factor $\varkappa$ is close to 1% of the utility of the highest prize in all decision tasks ($\varkappa \approx 0.010$).[17] The four RP urns in Figure 1 display logistic density functions, $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$, that are estimated by the analytic ML approach along with three kernel-smoothed MSL

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[13] Loomes, Moffatt and Sugden [2002; p.104] estimate a hybrid model that combines the RP model with a stochastic choice process known as trembles, which can accommodate violations of dominance. We use our kernel smoothing approach to estimate this hybrid choice model in Section 4.1.

[14] At the time of the experiment, the exchange rate was close to 5 kroner per US dollar.

[15] We report the indifference point for each lottery pair in Online Appendix A.

[16] The standard logistic distribution function $\Lambda(.)$ in this context represents the distributional assumption on ω_{nt}, and is not a smoothing kernel.

[17] Detailed results are reported in Online Appendix B, Table B1.

estimates based on $\varkappa \in \{0.037, 0.015, 0.007\}$ and R = 100 Halton draws.[18] The four sets of estimates for μ_ω and σ_ω have *p*-values < 0.001. Further tightening or loosening of the smoothing factor tends to provide poorer approximation to the analytic ML estimates than the results reported in Figure 1. The logistic RP urn estimated by the analytic ML approach has a mean (μ_ω) of 0.535 and a scale (σ_ω) of 0.575. By comparison, the kernel-smoothed MSL estimates are $\mu_\omega = 0.528$ and $\sigma_\omega = 0.565$ for $\varkappa = 0.007$.[19] If the mean is seen as a core risk aversion parameter and the scale as a behavioral noise parameter, an interpretation adopted by Apesteguia and Ballester [2018], then the average agent can be classified as risk-averse. Random fluctuations in risk preferences, however, implies that the agent occasionally makes choices that one expects of risk seeking decision makers. Both ML and MSL imply that this inconsistency occurs in about 28% of choice occasions, which is based on the formula $\Pr(\omega_{nt} < 0) = \Lambda(-\mu_\omega/\sigma_\omega)$.[20]

3. Multidimensional Random Preference Models

We next extend our kernel-smoothed estimation approach to RP models with more than one random risk preference parameter. These models naturally arise when one is interested in studying EUT with a multi-parameter utility function (Gul and Pesendorfer [2006]) or non-EUT preferences that allow for multiple sources of risk aversion. Popular examples of the latter include RDU that attribute risk aversion to utility curvature and probability weighting, and Cumulative Prospect Theory

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[18] These three values are based on $\varkappa = \#/(N \times T)^{0.2}$ where $\# \in \{0.25, 0.10, 0.05\}$ and $N \times T = 14{,}868$. Bhat [2001] finds that 100 Halton draws provide approximately the same level of accuracy as 2,000 pseudo-random draws in a Monte Carlo study of discrete choice models. The difference is attributed to the deliberate construction of Halton draws to provide good coverage of the parametric space.

[19] The two other sets of the MSL estimates are $\mu_\omega = 0.523$ and $\sigma_\omega = 0.546$ for $\varkappa = 0.015$; and $\mu_\omega = 0.506$ and $\sigma_\omega = 0.463$ for $\varkappa = 0.037$.

[20] Figure B1 in Online Appendix B displays ML and MSL estimates of μ_ω and σ_ω from a Monte Carlo experiment with 1,000 simulated data sets. The data generating process (DGP) for the Monte Carlo experiment assumes that the decision maker's RP urn is equivalent to the analytic ML estimates, *i.e.* $f(\omega_{nt} \mid \mu_\omega = 0.535, \sigma_\omega = 0.575)$, and we generate 1,000 simulated data sets of the same size (14,868 choice observations) as the original sample. We use the original set of lottery pairs in each simulated data set, but replace actual choices made by each subject with simulated choices based on draws of ω_{nt} from the DGP. Two sets of estimates for μ_ω and σ_ω are then computed per simulated data set by analytic ML and kernel-smoothed MSL with $\varkappa = 0.007$. The ML and MSL estimates are almost perfectly correlated, which suggests that the similar results in the empirical application are not a coincidence. The Monte Carlo experiment also suggests that the use of kernel smoothing *per se* does not have detrimental effects on the finite sample behavior of asymptotically justified inferential procedures.

(Tversky and Kahneman [1992]) which includes a loss aversion parameter along with reference-dependent utility curvature and probability weighting parameters.

A. Current Progress in Estimating Non-EUT Preferences

Before we present our general approach to estimating multi-dimensional RP models, it is useful to review Wilcox's approach to estimating a two-dimensional RP model since it illustrates the econometric challenges that have precluded wider use of RP models. He focuses on an experimental design with four distinct prizes, $m_1 < m_2 < m_3 < m_4$. In a given decision task the subject chooses between two lotteries over the same three-element subset of the four prizes but with different probability distributions. Let $M_{-1} = \{m_2, m_3, m_4\}$, $M_{-2} = \{m_1, m_3, m_4\}$, $M_{-3} = \{m_1, m_2, m_4\}$ and $M_{-4} = \{m_1, m_2, m_3\}$ denote the four prize sets in the experiment, and let p_{Lknt} denote the probability of prize m_{Lknt} in lottery L of task t to subject n.

The analytic ML approach by Wilcox is focused on RDU preferences that combine a non-parametric utility specification over the four prizes with a one-parameter PWF proposed by Prelec [1998]. The utility level of each prize, $U(m_k) = u_k$, is treated as a distinct parameter. We follow the original normalization by setting $u_1 = 0$ and $u_2 = 1$ for identification, and retaining u_3 and u_4 as free parameters with $1 < u_3 < u_4 < \infty$ to ensure monotonicity. The PWF is specified by

$$\pi(P\,|\,\varphi) = \exp\{-(-\ln(P))^{\varphi}\} \tag{11}$$

where $\pi(0\,|\,\varphi) \equiv 0$, and $\varphi \in (0, \infty)$ is a parameter that determines whether the shape of the PWF is regular S ($\varphi > 1$) or inverse-S ($0 < \varphi < 1$). Subject n's evaluation of lottery L in task t is then given by

$$\begin{aligned} RDU_{Lnt}(u_3, u_4, \varphi) &= \textstyle\sum_k (\pi(P_{Lknt}\,|\,\varphi) - \pi(P_{L(k+1)nt}\,|\,\varphi)) \times u_{knt} \\ &= \textstyle\sum_k d_{Lknt} \times u_{knt} \end{aligned} \tag{12}$$

where $P_{Lknt} = \sum_{j\geq k} p_{Ljnt}$ is the cumulative probability of receiving m_k or a higher prize in lottery L, with $P_{L5nt} \equiv 0$. In the special case where $\varphi = 1$, the PWF is linear and RDU is equivalent to EUT.

Wilcox estimates a *partial* RP-RDU model that places a RP structure on the utility function, but not on the PWF. The two utility levels $\{u_3, u_4\}$ are specified as RP parameters $\{u_{3nt}, u_{4nt}\}$ that vary randomly within a subject across decision tasks, whereas the PWF parameter φ is non-random for analytic simplicity and does not carry an nt subscript. We specify the scaled difference in the subject's evaluation of the two lotteries as[21]

$$\Delta RDU_{nt}(u_{3nt}, u_{4nt}, \varphi) = (RDU_{Bnt}(u_{3nt}, u_{4nt}, \varphi) - RDU_{Ant}(u_{3nt}, u_{4nt}, \varphi)) / u_{4nt} \tag{13}$$

and denote the difference in decision weights between the two lotteries by

$$\Delta d_{knt} = (d_{Bknt} - d_{Aknt}). \tag{14}$$

Since each decision task involves three distinct prizes, the probability that subject n chooses lottery B in task t can be enumerated for the four distinct prize sets as

$$\begin{aligned} \Pr(\Delta RDU_{nt}(u_{3nt}, u_{4nt}, \varphi) > 0) &= \Pr((u_{4nt} - u_{3nt})/(u_{3nt} - 1) > \Delta d_{2nt}/\Delta d_{4nt}) && \text{for } M_{-1} \\ &= \Pr((u_{4nt} - u_{3nt})/(u_{3nt}) > \Delta d_{1nt}/\Delta d_{4nt}) && \text{for } M_{-2} \\ &= \Pr((u_{4nt} - 1) > \Delta d_{1nt}/\Delta d_{4nt}) && \text{for } M_{-3} \\ &= \Pr((u_{3nt} - 1) > \Delta d_{1nt}/\Delta d_{3nt}) && \text{for } M_{-4} \end{aligned} \tag{15}$$

where each set-specific inequality collates all RP components of the model on the left-hand side and all non-random components on the right-hand side.

There is no joint distribution of u_{3nt} and u_{4nt} that translates all four types of choice probabilities in (15) into likelihood functions that can be used in ML estimation. When viewed in isolation, finding tractable marginal distributions of $(u_{4nt} - 1)$ for M_{-3} and $(u_{3nt} - 1)$ for M_{-4} is straightforward as one may choose from any distribution with support on $(0, \infty)$. Loomes, Moffatt and Sugden [2002] estimate the partial RP-RDU model without difficulty because all lotteries in their decision tasks contained the same set of three prizes. Complications arise in a four-prize setting because any joint distribution of $(u_{3nt} - 1)$ and $(u_{4nt} - 1)$ gives an intractable marginal distribution of the ratio $(u_{4nt} - u_{3nt})/(u_{3nt} - 1)$ for M_{-1}, as well as $(u_{4nt} - u_{3nt})/(u_{3nt})$ for M_{-2}.

The solution by Wilcox excludes all observations on M_{-2} from consideration. This constraint on the sample allows one to use McKay's [1934] bivariate gamma distribution that provides tractable marginal distributions for the remaining three prize sets: $(u_{3nt} - 1)$ and $(u_{4nt} - 1)$ follow univariate gamma distributions, whereas $(u_{4nt} - u_{3nt})/(u_{3nt} - 1)$ follows a "beta-prime" distribution that is related to an *F*-distribution. Let $\Pr(X \leq x) = F(x; a_1, a_2)$ denote the distribution function of an *F*-distributed random variate X, where a_1 and a_2 are numerator and denominator degrees of freedom, respectively.

[21] Wilcox did not scale the raw RDU difference by the highest attainable utility, u_4, but we introduce the scaled difference in equation (13) in anticipation of the kernel smoothing procedure.

Similarly, let $\Pr(X \leq x) = G(x; a_3, a_4)$ denote the distribution function of a gamma-distributed random variable X with shape parameter a_3 and scale parameter a_4. Under the bivariate gamma distribution of $(u_{3nt} - 1)$ and $(u_{4nt} - 1)$, an analytic solution to equation (15) is given by

$$\begin{aligned} L_{nt}(\alpha_3, \alpha_4, \gamma, \varphi) &= 1 - F(\alpha_3/\alpha_4 \times \Delta d_{1nt}/\Delta d_{4nt}; 2\alpha_4, 2\alpha_3) && \text{for } M_{-1} \\ &= 1 - G(\Delta d_{1nt}/\Delta d_{4nt}; \alpha_3 + \alpha_4, \gamma) && \text{for } M_{-3} \\ &= 1 - G(\Delta d_{1nt}/\Delta d_{3nt}; \alpha_3, \gamma) && \text{for } M_{-4} \end{aligned} \tag{16}$$

where $L_{nt}(\alpha_3, \alpha_4, \gamma, \varphi) = \Pr(\Delta RDU_{nt}(u_{3nt,}\ u_{4nt}, \varphi) > 0)$ is the probability that subject n chooses lottery B in decision task t. One can thus use equation (16) to construct the sample likelihood function and compute ML estimates of the non-random probability weighting parameter φ and the distributional parameters α_3, α_4 and γ that describe the RP urn for u_{3nt} and u_{4nt}.[22]

The restrictions imposed by this solution on the prize sets and the distributional assumptions also apply to the RP-EUT model with a non-parametric utility specification. Hence, despite the prominence of RP-EUT models with non-parametric utility in theoretic formulations such as Gul and Pesendorfer [2006], there is until now no available approach to estimate this structural model in a more general data environment and with alternative classes of RP urns.

B. General Approach to Estimating Multi-Dimensional RP Models

We can easily adapt the kernel-smoothed simulator of the one-dimensional RP-EUT model in equation (7) to the partial RP-RDU model with two RP parameters. Let $g(v_{3nt}, v_{4nt} | \alpha_3, \alpha_4, \gamma)$ denote the bivariate gamma density of the two RP parameters in the partial RP-RDU model, where $v_{3nt} \equiv (u_{3nt} - 1)$ and $v_{4nt} \equiv (u_{4nt} - 1)$. The probability that subject n chooses lottery B in task t is

$$L_{nt}(\alpha_3, \alpha_4, \gamma, \varphi) = \int\int \mathbf{I}[\Delta RDU_{nt}(u_{3nt}, u_{4nt}, \varphi) > 0] g(v_{3nt}, v_{4nt} | \alpha_3, \alpha_4, \gamma) dv_{3nt} dv_{4nt} \tag{17}$$

where $\Delta RDU_{nt}(u_{3nt,}\ u_{4nt}, \varphi)$ is the scaled RDU difference in equation (13), $u_{3nt} = (v_{3nt} + 1)$ and $u_{4nt} = (v_{4nt} + 1)$. Let v_{3ntr} and v_{4ntr} denote the r^{th} draw of v_{3nt} and v_{4nt} from the bivariate density function, where $r \in \{1, 2, \cdots, R\}$. The required kernel-smoothed simulator can then be constructed as

$$S_{nt}(\alpha_1, \alpha_2, \gamma, \varphi) = (1/R) \textstyle\sum_r \Lambda(\Delta RDU_{nt}(u_{3ntr}, u_{4ntr}, \varphi) / \varkappa) \tag{18}$$

[22] The sample likelihood function has a similar algebraic form as the usual pooled logit model. The only difference is that the standard logit probability of choice B (choice A) is replaced with the RP probability of choice B (choice A). The RP probability of choice A is equal to $1 - L_{nt}(\alpha_3, \alpha_4, \varkappa, \varphi)$.

where $\varkappa$ is a smoothing factor selected prior to estimation, and the standard logistic distribution function $\Lambda(.)$ is the smoothing kernel. The MSL estimates of α_3, α_4, γ and φ can be computed by using $S_{nt}(\alpha_3, \alpha_4, \gamma, \varphi)$ instead of the analytic choice probabilities in equation (16) when one specifies the sample likelihood function for the three prize sets M_{-1}, M_{-3} and M_{-4}.

Of course, with the kernel smoothing approach it is not necessary to restrict the estimation sample to those three prize sets unless one is interested in obtaining MSL estimates that are directly comparable to Wilcox's analytic ML estimates. The choice probability in (17) and the simulator in (18) can accommodate M_{-2} as easily as the three other prize sets. We can also replace the bivariate gamma distribution with other bivariate distributions since the algebraic structure of our simulator remains the same regardless of which distribution u_{3ntr} and u_{4ntr} are drawn from.

The kernel-smoothed simulator can be also adapted to estimate the full RP-RDU model with RP structures on both the utility function and the PWF. Suppose that we augment the partial RP-RDU model by specifying a log-normally distributed random parameter φ_{nt} in addition to the two random utility parameters. The choice probability in equation (17) is then changed to

$$L_{nt}(\alpha_3, \alpha_4, \gamma, \mu_\varphi, \sigma_\varphi) = \int\int\int \mathbf{I}[\Delta RDU_{nt}(u_{3nt}, u_{4nt}, \varphi_{nt}) > 0] g(v_{3nt}, v_{4nt} | \alpha_3, \alpha_4, \gamma) \times \xi(\ln(\varphi_{nt}) | m_\varphi, s_\varphi) dv_{3nt} dv_{4nt} d\varphi_{nt} \quad (19)$$

where $\xi(\ln(\varphi_{nt}) | m_\varphi, s_\varphi)$ is a normal density function for $\ln(\varphi_{nt})$. The full RP-RDU model with three random parameters can be simulated by

$$S_{nt}(\alpha_1, \alpha_2, \gamma, \mu_\varphi, \sigma_\varphi) = (1/R) \textstyle\sum_r \Lambda(\Delta RDU_{nt}(u_{3ntr}, u_{4ntr}, \varphi_{ntr}) / \varkappa) \quad (20)$$

where φ_{ntr} is the r^{th} draw of φ_{nt} from the log-normal density, and otherwise equation (20) is identical to (18) for the partial RP-RDU model.

More generally, our estimation approach can be adapted to any preference index $\Delta V_{nt}(\boldsymbol{\alpha}_{nt})$ that represents the subject's relative valuation of two lotteries as a function of a multidimensional vector of RP parameters $\boldsymbol{\alpha}_{nt}$, where the exact form of the index may vary across theory and data. Suppose that $\Delta V_{nt}(\boldsymbol{\alpha}_{nt}) > 0$ corresponds to the choice of lottery B, and let $\zeta(\boldsymbol{\alpha}_{nt} | \boldsymbol{\theta})$ denote the joint density of $\boldsymbol{\alpha}_{nt}$ which is characterized by the distributional parameters in $\boldsymbol{\theta}$. The probability that subject n chooses lottery B in task t is simulated by

$$S_{nt}(\boldsymbol{\theta}) = (1/R) \textstyle\sum_r \Lambda(\Delta V_{nt}(\boldsymbol{\alpha}_{ntr}) / \varkappa), \quad (21)$$

where $\boldsymbol{\alpha}_{ntr}$ is the rth draw of $\boldsymbol{\alpha}_{nt}$ from $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta})$. The MSL estimates of $\boldsymbol{\theta}$ can be computed by maximizing the sample log-likelihood function $H(\boldsymbol{\theta}) = \sum_n \ln(H_n(\boldsymbol{\theta}))$, where the subject-level joint likelihood function $H_n(\boldsymbol{\theta})$ is identical to equation (10) except that the underlying preference index is now a function of multidimensional $\boldsymbol{\alpha}_{ntr}$ instead of unidimensional α_{ntr}. The likelihood evaluator can be programmed in substantively the same three steps as the unidimensional RP model. With emphasis on aspects related to the multi-dimensionality of $\boldsymbol{\alpha}_{ntr}$, these steps are:

1. Prior to estimation, make $N \times T \times R$ draws from each of k_{RP} distinct Halton sequences, where k_{RP} is equal to the dimension of $\boldsymbol{\alpha}_{ntr}$. For example, $k_{RP} = 2$ for the partial RP-RDU model in equation (17) and $k_{RP} = 3$ for the full model in equation (19). Allocate $R \times k_{RP}$ distinct draws to each of the $N \times T$ choice observations.

2. At each iteration of model estimation, convert the Halton draws into draws of $\boldsymbol{\alpha}_{nt}$ from $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ are set to their most recent estimates. For example, consider the partial RP-RDU model in equation (17) and let $G^{-1}(x; \alpha, \gamma)$ denote the inverse distribution function of a *univariate* gamma variable with the location parameter α and the scale parameter γ. The utility parameter u_{3nt} can be simulated by applying $G^{-1}(x; \alpha_3, \gamma)$ to draws from one of the two Halton sequences, and adding the inverted Halton draws v_{3ntr} to unity: $u_{3ntr} = 1 + v_{3ntr}$. Similarly, the utility parameter u_{4nt} can be simulated by applying $G^{-1}(x; \alpha_4, \gamma)$ to draws from the other Halton sequence, and adding the inverted Halton draws v^*_{4ntr} to u_{3ntr}: $u_{4ntr} = 1 + (v_{3ntr} + v^*_{4ntr})$. The sum in the parentheses makes up draw v_{4ntr} which display correlation with v_{3ntr} that is consistent with the bivariate gamma density of v_{3nt} and v_{4nt}.

3. At each iteration of model estimation, re-evaluate the kernel-smoothed simulator in equation (21) at the draws of $\boldsymbol{\alpha}_{nt}$ obtained in Step 2, and use the results to re-evaluate the likelihood function.

In our empirical illustration, we use R = 100 sets of Halton draws per choice observation to simulate the sample log-likelihood function, and find MSL estimates by applying the BFGS technique for numerical maximization.

C. Application: RDU with Non-Parametric Utility

To estimate RDU with non-parametric utility, we use data from Harrison and Rutström [2008; §2.6]. The experiment was conducted with a sample of 63 students at the University of Central Florida, who made choices from 60 pairs of lotteries, A and B. The two lotteries in each pair had the same set of three prizes [m_1, m_2, m_3] but different probability distributions: A = {(m_1, p_{A1})), (m_2, p_{A2}), (m_3, p_{A3})} and B = {(m_1, p_{B1})), (m_2, p_{B2}), (m_3, p_{B3})}. Each lottery pair can thus be seen as two points in the Marschak-Machina (MM) triangle. Each probability p_{Lknt} took a value of 0, 0.13, 0.25, 0.37, 0.5, 0.62, 0.75 or 0.87. The three probabilities in each lottery summed to one, and none of the lottery pairs had a stochastically dominated choice. The three prizes m_{1nt}, m_{2nt} and m_{3nt} were selected from one of four prize sets denominated in US dollars, M_{-1} = [5, 10, 15], M_{-2} = [0, 10, 15], M_{-3} = [0, 5, 15] and M_{-4} = [0, 5, 10]. At the end of the experiment, three of the subject's 60 choices were randomly selected for payment.[23]

The experiment replicates the data environment in Wilcox [2008; §4.5][2011]. We thus have an opportunity to compare our kernel-smoothed MSL estimates with analytic ML estimates for two random preference parameters. Once the prize set M_{-2} is excluded from the sample, we can use Wilcox's solution to the partial RP-RDU model in equation (16) and derive the sample likelihood function analytically. We can also use the same bivariate gamma density distribution and the same subset of data when we construct our kernel-smoothed simulator in equation (18). This will allow us to directly compare the two estimation approaches.

Figure 2 illustrates that our MSL estimates are almost identical to the analytic ML estimates of the partial RP-RDU model when we set the smoothing factor $\varkappa$ to 0.01. The results thus reinforce our earlier observation based on EUT that $\varkappa \approx 0.01$ is an appropriate default configuration for kernel smoothing. The structural model includes two RP parameters for the utility function, $u_{3nt} \in (1, \infty)$ and $u_{4nt} \in (u_{3nt}, \infty)$, along with a non-random parameter $\varphi \in (0, \infty)$ for the PWF. The two RP parameters measure the utility of \$10 and \$15 after normalizing utility of \$0 ($u_{1nt} = 0$) and \$5 ($u_{2nt} = 1$). The upper

[23] Harrison and Rutström [2008] paid for three of the subject's 60 choices to ensure comparability of rewards with other experiments in which subjects made 40 or 20 choices over pairwise lotteries, and where two decision tasks or one decision task, respectively, was selected at random for payment. This so-called Random Lottery Incentive Method is common in these types of experiments, and it assumes that subjects isolate each pairwise lottery choice within the series from each other. One can alternatively pay subjects for all their decisions, but it opens up for hedging opportunities and portfolio effects, as well as wealth effects depending on information and payment procedures.

panel in Figure 2 displays the marginal gamma density of u_{3nt}, which is characterized by the shape parameter α_3 and scale parameter γ; and the lower panel displays the same density of u_{4nt}, which is characterized by the shape parameter $(\alpha_3 + \alpha_4)$ and scale parameter γ. The four pairs of RP urns in the two panels are estimated by applying analytic ML estimation along with kernel-smoothed MSL estimation at $\varkappa \in \{0.020, 0.010, 0.002\}$ and R = 100 Halton draws.[24] All four sets of estimates for α_3, α_4, γ and φ have *p*-values < 0.001.

The analytic ML estimates of the three parameters characterizing the bivariate gamma RP urns are equal to $\{\alpha_3, \alpha_4, \gamma\} = \{0.702, 0.830, 0.791\}$. Multiplying the shape and scale parameters of a gamma distribution produces its mean, and dividing the mean by the square root of the shape parameter produces its standard deviation. We thus obtain $E[u_{3nt}] = 1.556$ with $SD[u_{3nt}] = 0.663$, and $E[u_{4nt}] = 2.212$ with $SD[u_{4nt}] = 0.979$, where E[.] and SD[.] denote the mean and standard deviation of the RP parameter in the argument. Given the normalized utility levels of $u_{1nt} = 0$ and $u_{2nt} = 1$, the utility function is linear when $u_{3nt} = 2$ and $u_{4nt} = 3$. Should $E[u_{3nt}]$ and $E[u_{4nt}]$ be viewed as the core preference parameters, the results would suggest that the decision maker has a concave utility function. Also, the RP urns suggest that the possible realizations of utility functions generally are concave despite random fluctuations, with $Pr(u_{3nt} < 2) = 0.824$, $Pr(u_{4nt} < 3) = 0.826$, and $Pr(u_{3nt} < 2, u_{4nt} < 3) = 0.642$. The non-random parameter φ in the PWF is equal to 1.011, with a 95% confidence interval of (0.949, 1.072), and we cannot reject the null hypothesis of a linear PWF at the 5% significance level.

The MSL estimates with a smoothing factor of $\varkappa = 0.010$ generate RP urns which lead to substantively the same inferences as the analytic ML estimates. The parameters characterizing the bivariate gamma distribution are estimated at $\{\alpha_3, \alpha_4, \gamma\} = \{0.795, 0.953, 0.663\}$, implying $E[u_{3nt}] = 1.527$ with $SD[u_{3nt}] = 0.591$, and $E[u_{4nt}] = 2.158$ with $SD[u_{4nt}] = 0.876$. The utility function is again generally concave with $Pr(u_{3nt} < 2) = 0.841$, $Pr(u_{4nt} < 3) = 0.850$ and $Pr(u_{3nt} < 2, u_{4nt} < 3) = 0.667$. Moreover, we cannot reject the null hypothesis that the PWF is linear since the non-random φ

[24] The smoothing factor values are obtained by setting $\varkappa = \#/(N \times T)^{0.2}$ where $\# \in \{0.10, 0.05, 0.01\}$ and $N \times T = 2{,}985$.

parameter is equal to 1.002 with a 95% confidence interval of (0.942, 1.062). The MSL estimates at the other two values of $\varkappa$ lead to similar conclusions.[25]

4. Two Other Types of Heterogeneity

Our general estimation approach can be also applied to other variants of the RP model that incorporate different aspects of individual heterogeneity in choice behavior. We consider two applications. The first application is a finite mixture model that combines the RP model with another stochastic choice process known as trembles (Loomes, Moffatt and Sugden [2002]). If the embedded decision model satisfies stochastic dominance, the RP model is inconsistent with any instance of stochastically dominated choices. The finite mixture model helps explain such instances without discarding the RP modeling framework entirely. The second application accounts for preference heterogeneity across individuals by specifying a model that allows each subject to have a personal RP urn. This model has a hierarchical structure that uses the random coefficient model to represent *interpersonal* heterogeneity and the RP model to represent *intrapersonal* heterogeneity. Once our general estimation approach has been applied to evaluate RP choice probabilities, the sample log-likelihood function can be simulated in the same manner as mixed logit models (Train [2009; §6]) which are widely used in applied microeconomics.

A. Mixture of Random Preferences and Trembles

Subjects in experiments are occasionally presented with decision tasks where one choice dominates the other. The experiment by Andersen, Harrison, Lau and Rutström [2014], for example, provides a simple example, in which 4 out of 40 tasks present choices between two degenerate lotteries where option B offers a higher amount than option A. So far we have excluded decision tasks with dominant choices to focus squarely on estimating the RP model. We now consider a hybrid model that combines random preferences with trembles (Loomes, Moffatt and Sugden [2002]; Wilcox [2008; §2.1]), which allows one to include dominated choice tasks in the sample.

[25] Detailed results are reported in Online Appendix B, Table B3.

Suppose that lottery B dominates lottery A. Since the embedded decision criterion always favors lottery B regardless of the preference parameter values that the subject happens to draw from the RP urn, the RP model predicts that the subject always chooses lottery B. If the subject's observed choice is indeed B, this data point contributes ln(1) = 0 to the sample log-likelihood function, adding no information to the estimation of the RP model. If the observed choice instead is A, this data point precludes ML or MSL estimation since its log-likelihood contribution is ln(0), thereby rendering the value of the sample log-likelihood function undefined.

To accommodate dominated choices without discarding the RP framework entirely, one must combine the RP model with a secondary stochastic model that allows for violations of stochastic dominance. Loomes, Moffatt and Sugden [2002; p.114] introduce a hybrid model where a simple process known as trembles takes this secondary role. Suppose that the RP component of the hybrid model is based on a general preference index $\Delta V_{nt}(\boldsymbol{\alpha}_{nt})$, where $\boldsymbol{\alpha}_{nt}$ is a vector of RP parameters with density function $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta})$. The tremble mechanism suggests that the subject metaphorically has a trembling hand that can lead to unintentional choices. In the hybrid model, the probability that subject n chooses lottery B in task t is equal to a weighted average of choice probabilities under the two components

$$L_{nt}(\boldsymbol{\theta}, \tau) = (1 - \tau) \int I[\Delta V_{nt}(\boldsymbol{\alpha}_{nt}) > 0]\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta})d\boldsymbol{\alpha}_{nt} + \tau\, 0.5 \tag{22}$$

where $\tau \in (0, 1)$ is the probability of trembling. The RP integral in (22) is identical to equation (3) except that we have generalized the notation to accommodate multiple RP parameters. With no trembling ($\tau = 0$), the probability of selecting either lottery is predicted by the RP integral. When $\tau = 1$, the RP integral is redundant and the subject is equally likely to choose either lottery.

The hybrid model has an analytic expression only if the RP integral has one, and it inherits all computational challenges from the RP model otherwise. It is, however, straightforward to apply our kernel smoothing procedure to simulate the hybrid model in equation (22) as

$$S_{nt}(\boldsymbol{\theta}, \tau) = (1 - \tau)\ (1/R) \sum_r \Lambda(\Delta V_{nt}(\boldsymbol{\alpha}_{ntr}) / \varkappa) + \tau\, 0.5 \tag{23}$$

where $\varkappa$ is a pre-selected smoothing factor, and $\boldsymbol{\alpha}_{ntr}$ is the r^{th} draw of $\boldsymbol{\alpha}_{nt}$ from $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta})$. The MSL estimates of the parameters $\boldsymbol{\theta}$ that describe the subject's RP urn and tremble parameter τ can be obtained by maximizing a sample log-likelihood function which uses equation (23) instead of $S_{nt}(\boldsymbol{\theta})$ in equation (9).

To study the empirical performance of our kernel smoothing procedure in this context, we return to the RP-EUT application in Section 2.C and estimate the hybrid EUT model with trembles. We use the same data from Andersen, Harrison, Lau and Rutström [2014], but now add observations from the dominated choice tasks which were excluded from the RP-EUT model. Since the one-dimensional RP-EUT model for the non-dominant tasks has an analytic log-likelihood, the hybrid EUT model for the full sample of 16,520 choice observations also has an analytic log-likelihood function. We can thus compare our kernel-smoothed MSL results to the analytic ML benchmark.

Figure 3 displays estimated logistic density functions of the risk preference parameter $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ using the analytic ML approach and the kernel-smoothed MSL approach with $\varkappa \in \{0.036, 0.014, 0.007\}$. The estimated RP urn with the analytic ML approach is $f(\omega_{nt} | 0.523, 0.337)$ and the estimated tremble parameter is 0.162. We obtain practically the same results with the MSL approach for $\varkappa = 0.007$ and $\varkappa = 0.014$, whereas the estimated RP urn for $\varkappa = 0.036$ is somewhat different from the analytic benchmark.[26] In both ML and MSL cases, compared to the RP-EUT estimates in Figure 1, we thus find practically the same mean but less dispersion for the RP urn since stochastic choice behavior is now attributed to both the dispersion of the RP urn and trembling.

B. Random Preferences with Interpersonal Heterogeneity

The RP models considered so far assume that all subjects draw their RP parameters from the same urn. Loomes, Moffatt and Sugden [2002] and Wilcox [2008][2011] show that it is possible to specify a more flexible model that allows each subject to have a personal RP urn by combining the RP model of *intrapersonal* heterogeneity with the random coefficient model of *interpersonal heterogeneity*. Their statistical models, however, are specifically designed for partial RP-RDU specifications that lead to analytic choice probabilities conditional on specific draws of the random coefficients.

We can extend our estimation approach based on kernel smoothing and apply it to any class of RP models with interpersonal preference heterogeneity. Consider first the one-dimensional RP-EUT model in equation (3), but now suppose that the logistic density function is characterized by subject-specific mean and scale parameters $\mu_{\omega n}$ and $\sigma_{\omega n}$. That is, while the risk aversion parameter ω_{nt}

[26] When $\varkappa = 0.007$, the RP urn is $f(\omega_{nt} | 0.524, 0.341)$ and τ is 0.158; when $\varkappa = 0.014$, the RP urn is $f(\omega_{nt} | 0.522, 0.332)$ and τ is 0.156; and when $\varkappa = 0.037$, the RP urn is $f(\omega_{nt} | 0.517, 0.281)$ and τ is 0.144.

is logistically distributed *within* each subject and across the decision tasks, the mean and dispersion of this logistic distribution varies from subject to subject. The random coefficient model accommodates interpersonal heterogeneity by assuming that the subject-specific parameters $\mu_{\omega n}$ and $\sigma_{\omega n}$ are randomly distributed *between* different subjects in the population. To complete the random coefficient specification, one must put further structure on this population distribution. Suppose that the mean and log-scale parameters are jointly normally distributed *between* subjects, $[\mu_{\omega n}, \ln(\sigma_{\omega n})]' \sim MVN(\mathbf{b}_{EUT}, \mathbf{V}_{EUT})$. Each subject n thus has a distinct pair of values $\mu_{\omega n}$ and $\ln(\sigma_{\omega n})$, and the distribution of these bivariate parameters has between-subject mean $\mathbf{b}_{EUT}$ and covariance matrix $\mathbf{V}_{EUT}$. Model estimation is now concerned with these between-subject parameters which constitute the primitive parameters of the assumed statistical structure.

More generally, we can consider a preference index function $\Delta V_{nt}(\boldsymbol{\alpha}_{nt})$ of a RP vector $\boldsymbol{\alpha}_{nt}$. Let $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta}_n)$ denote the assumed joint density of $\boldsymbol{\alpha}_{nt}$ as a function of parameters $\boldsymbol{\theta}_n$, which are now assumed to vary between different subjects $n \in \{1, 2, \cdots, N\}$. Suppose that $\boldsymbol{\theta}_n$ is distributed as per the joint density function $\mathscr{F}(\boldsymbol{\theta}_n|\boldsymbol{\Psi})$, where hyper-parameters $\boldsymbol{\Psi}$ characterize the between-subject distribution of $\boldsymbol{\theta}_n$. The statistical model thus has a hierarchical structure. At the upper level, each subject's RP urn is seen as a draw from $\mathscr{F}(\boldsymbol{\theta}_n|\boldsymbol{\Psi})$ in the same way as interpersonal heterogeneity is captured in the random coefficient model. At the lower level, the subject is assumed to make a new draw of RP parameters from her personal RP urn $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta}_n)$ in each decision task t, in the same way as stochastic choice behavior is captured in the RP model.

The likelihood function of T choice observations on subject n, say $J_n(\boldsymbol{\Psi})$, can be derived by integrating out $\boldsymbol{\theta}_n$ from the corresponding likelihood function under the RP model. That is, $J_n(\boldsymbol{\Psi}) = E(H_n(\boldsymbol{\theta}_n)) = \int H_n(\boldsymbol{\theta}_n)\mathscr{F}(\boldsymbol{\theta}_n|\boldsymbol{\Psi})d\boldsymbol{\theta}_n$, where $H_n(\boldsymbol{\theta}_n)$ is the RP likelihood function conditional on particular $\boldsymbol{\theta}_n$ from the population distribution. Once $H_n(\boldsymbol{\theta}_n)$ has been simulated using the kernel smoothing approach as in equation (9), $H_n(\boldsymbol{\theta}_n)$ becomes a smooth function of $\boldsymbol{\theta}_n$, which enables us to approximate this expectation by applying the standard frequency simulator. Simulation of $J_n(\boldsymbol{\Psi})$ can proceed in a hierarchical fashion by sequentially generating draws from $\mathscr{F}(\boldsymbol{\theta}_n|\boldsymbol{\Psi})$ and $\zeta(\boldsymbol{\alpha}_{nt}|\boldsymbol{\theta}_n)$ at each iteration as follows:

1. Prior to estimation, make N × T × R draws from each of k_{RP} distinct Halton sequences, where k_{RP} is the dimension of $\boldsymbol{\alpha}_{nt}$. Allocate R × k_{RP} distinct draws to each of the N × T choice observations, and call these choice-level draws.

2. Prior to estimation, make N × Q draws from each of k_{RC} distinct Halton sequences, where k_{RC} is the dimension of $\boldsymbol{\theta}_n$. Allocate Q × k_{RC} distinct draws to each of the N subjects, and call these subject-level draws. From a data management perspective, the choice-level allocation in step 1 is equivalent to generating R × k_{RP} new variables whose values vary from row to row in the data set. The subject-level allocation in step 2 is equivalent to generating R × k_{RC} new variables whose values are repeated within blocks of T data rows for the same subject but vary between those blocks.

3. At each iteration of model estimation, convert the subject-level Halton draws into draws of $\boldsymbol{\theta}_n$ from $\mathscr{F}(\boldsymbol{\theta}_n | \boldsymbol{\Psi})$, where $\boldsymbol{\Psi}$ is set to their most recent estimates, and let $\boldsymbol{\theta}_{nq}$ denote the q^{th} draw of $\boldsymbol{\theta}_n$. Like the RP examples earlier, this conversion is achieved by applying an inverse cumulative distribution function associated with $\mathscr{F}(\boldsymbol{\theta}_n | \boldsymbol{\Psi})$ to the Halton draws.

4. At each iteration of model estimation, convert the choice-level Halton draws into draws of $\boldsymbol{\alpha}_{nt}$ from $\zeta(\boldsymbol{\alpha}_{nt} | \boldsymbol{\theta}_{nq})$ by applying an inverse cumulative distribution function associated with $\zeta(\boldsymbol{\alpha}_{nt} | \boldsymbol{\theta}_{nq})$, where $\boldsymbol{\theta}_{nq}$ has been generated in step 3. Let $\boldsymbol{\alpha}_{ntrq}$ denote the r^{th} draw of $\boldsymbol{\alpha}_{nt}$; this draw is subscripted by both r and q since the underlying RP urn is conditioned on $\boldsymbol{\theta}_{nq}$.

5. At each iteration of model estimation, re-evaluate the RP choice probabilities in equation (21) at each draw $\boldsymbol{\alpha}_{ntrq}$ from step 4 to calculate $S_{nt}(\boldsymbol{\theta}_{nq})$, and use the results to re-evaluate the subject-level RP likelihood in equation (9), $H_n(\boldsymbol{\theta}_{nq})$.

6. At each iteration of model estimation, re-evaluate a simulated analogue to the subject-level likelihood $J_n(\boldsymbol{\Psi})$ by averaging $H_n(\boldsymbol{\theta}_{nq})$ across Q draws of $\boldsymbol{\theta}_{nq}$: $J_n(\boldsymbol{\Psi}) = (1/Q) \sum_q H_n(\boldsymbol{\theta}_{nq})$. The sum of $\ln(J_n(\boldsymbol{\Psi}))$ across N subjects gives us the sample log-likelihood function, which is maximized to obtain the MSL estimates of $\boldsymbol{\Psi}$.

Model estimation can proceed by using any numerical optimization technique to update the estimates of $\boldsymbol{\Psi}$ between iterations. Our own implementation uses R = Q = 100 Halton draws to simulate each level of the hierarchical structure, and the BFGS technique for numerical maximization of the sample log-likelihood function. For numerical stability we again truncate each Halton draw on (0.005, 0.995) prior to applying the inverse distribution functions.

We now turn to applying this approach to estimate a RP-RDU model with interpersonal heterogeneity. We use the same data from Andersen, Harrison, Lau and Rutström [2014] without the dominated choice tasks to focus on the RP model, although our approach also can be adapted to estimate a hybrid model with trembles. Given the algebraic structure of the decision tasks, where each lottery pair can be written as $A = \{(m_{A1}, (1 - p_2)), (m_{A2}, p_2)\}$ and $B = \{(m_{B1}, (1 - p_2)), (m_{B2}, p_2)\}$ and $m_{B1} < m_{A1} < m_{A2} < m_{B2}$, we can specify subject n's evaluation of lottery $L \in \{A, B\}$ as

$$RDU_{Lnt}(\omega_{nt}, \varphi_{nt}) = (1-\pi(p_{2nt} | \varphi_{nt}))U(m_{L1nt} | \omega_{nt}) + \pi(p_{2nt} | \varphi_{nt})U(m_{L2nt} | \omega_{nt}) \tag{24}$$

where $U(. | \omega_{nt})$ is the CRRA utility function in equation (1) and $\pi(. | \varphi_{nt})$ is the PWF in equation (11). The preference index function $\Delta V_{nt}(\boldsymbol{\alpha}_{nt})$ then refers to the scaled RDU difference

$$\Delta RDU_{Lnt}(\omega_{nt}, \varphi_{nt}) = (RDU_{Bnt}(\omega_{nt}, \varphi_{nt}) - RDU_{Ant}(\omega_{nt}, \varphi_{nt})) \;/\; U(m_{max} | \omega_{nt}) \tag{25}$$

where m_{max} is the highest prize in all decision tasks. We assume that the utility urn has logistic density $f(\omega_{nt} | \mu_{\omega n}, \sigma_{\omega n})$ and the PWF urn has log-normal density $\xi(\ln(\varphi_{nt}) | m_{\varphi n}, s_{\varphi n})$. We model the subject-specific parameters $\mu_{\omega n}$, $\sigma_{\omega n}$, $m_{\varphi n}$ and $s_{\varphi n}$ as independent random coefficients. The between-subject distributions of $\mu_{\omega n}$ and $m_{\varphi n}$ are specified as normals since these mean parameters do not have theoretical bounds, and the distributions of scale parameters $\sigma_{\omega n}$ and $s_{\varphi n}$ as folded normals to satisfy positivity constraints. This specification gives a total of 8 hyper-parameters to be estimated in $\boldsymbol{\Psi}$, namely one location parameter and one scale parameter for each of the four between-subject distributions. For normally distributed $\mu_{\omega n}$ and $m_{\varphi n}$, the location (scale) parameter can be interpreted as the averages (standard deviations) of $\mu_{\omega n}$ and $m_{\varphi n}$ in the subject population.

Figure 4 reports the estimated between-subject distributions of $\mu_{\omega n}$ and $m_{\varphi n}$ at $\varkappa \in \{0.037, 0.015, 0.007\}$. If the within-subject mean of each personal RP urn is seen as the core risk preference parameter, the results in Figure 4 can be interpreted as population distributions of risk preferences under the RDU model. We will focus on the configuration with $\varkappa = 0.007$ since the other two configurations produce similar results. The utility parameter, $\mu_{\omega n}$, is normally distributed with population mean $E[\mu_{\omega n}] = 0.558$ and standard deviation $SD[\mu_{\omega n}] = 0.765$, and both coefficients are significantly different from 0 (p-values < 0.001). The results imply that the average decision maker, along with 76.7% of the decision makers, has a concave utility function. The log shape parameter in the PWF, $m_{\varphi n}$, is normally distributed with $E[m_{\varphi n}] = 0.038$ (p-value = 0.665) and $SD[m_{\varphi n}] = 1.164$ (p-value < 0.001) and we cannot reject the hypothesis that the population is equally divided between those with inverse-S and S shaped PWFs (p-value = 0.665). Nevertheless, given the estimated between-subject mean and standard deviation of $m_{\varphi n}$, the implied population mean of the shape parameter φ_{nt} is equal to 2.044, which is significantly greater than unity (p-value < 0.001) and suggests that the average decision maker has an S-shaped PWF.

The results can be compared to alternative model estimates reported by Harrison, Lau and Yoo [2020] for the same data set.[27] They combine RDU with Fechnerian and Contextual Utility models (Wilcox [2008][2011]), two alternative stochastic choice models that attribute unexplained variations in choice behavior to additive error terms. Using random coefficient specifications that adopt a normal distribution of the utility curvature parameter ω_n and a log-normal distribution of the PWF parameter φ_n, they estimate the population mean of the former at 0.553 (Fechner) and 0.605 (Contextual) and the latter at 2.401 (Fechner) and 3.293 (Contextual).[28] In comparison, we estimate the former and the latter at 0.558 and 2.044 in the RP framework.

5. Conclusion

The RP model provides an integral framework for modeling within-individual heterogeneity in choice behavior, by attributing this heterogeneity to preference parameters in the underlying theory of risk attitudes instead of an additive error term that is external to the theory. However, most empirical studies in structural estimation of risk attitudes turn to additive error specifications because the RP likelihood function is computationally unattractive. We propose a general approach to estimating the RP model that facilitates empirical applications in this alternative modeling framework. Our

[27] Since neither the RP-RDU model nor the random coefficient model has analytic expressions, we do not have an analytic ML benchmark that can be compared to the MSL results. As usual with random coefficient choice models, the required computer run time is non-trivial, making it impractical to implement a Monte Carlo experiment.

[28] The two random coefficients are only subscripted by n because the Fechner and Contextual Utility models assume that the preference parameters are constant within an individual. Harrison, Lau and Yoo [2020] estimate these random coefficient RDU specifications for two different waves of a longitudinal experiment. We cite estimates for the first wave of the experiment, which coincides with the data that we use in our empirical illustration. They generalize the model specifications further by adding statistical correction for endogenous sample selection and panel attrition. We cite their pre-correction estimates because we do not make these corrections.

estimation approach illustrates that the RP model is just as flexible as other stochastic choice models. By applying a kernel smoothing procedure, we can construct a versatile likelihood evaluator of the RP model that can accommodate any decision theory, types of lottery pairs, and parametric distribution of unobserved heterogeneity.

Our approach helps advance the boundaries in structural estimation of choice under risk by making the RP model accessible to decision theories with multiple parameters. The empirical applications presented in this paper do not exhaust the analytic opportunities that our versatile approach opens up. One may also apply our approach to other non-EUT models such as Prospect Theory; to decision tasks with more than two options by using multinomial or rank-ordered logit link functions as smoothing kernels; and to other domains of preferences such as non-exponential discounting functions with multi-dimensional time preference parameters.

Figure 1: RP-EUT with CRRA Utility

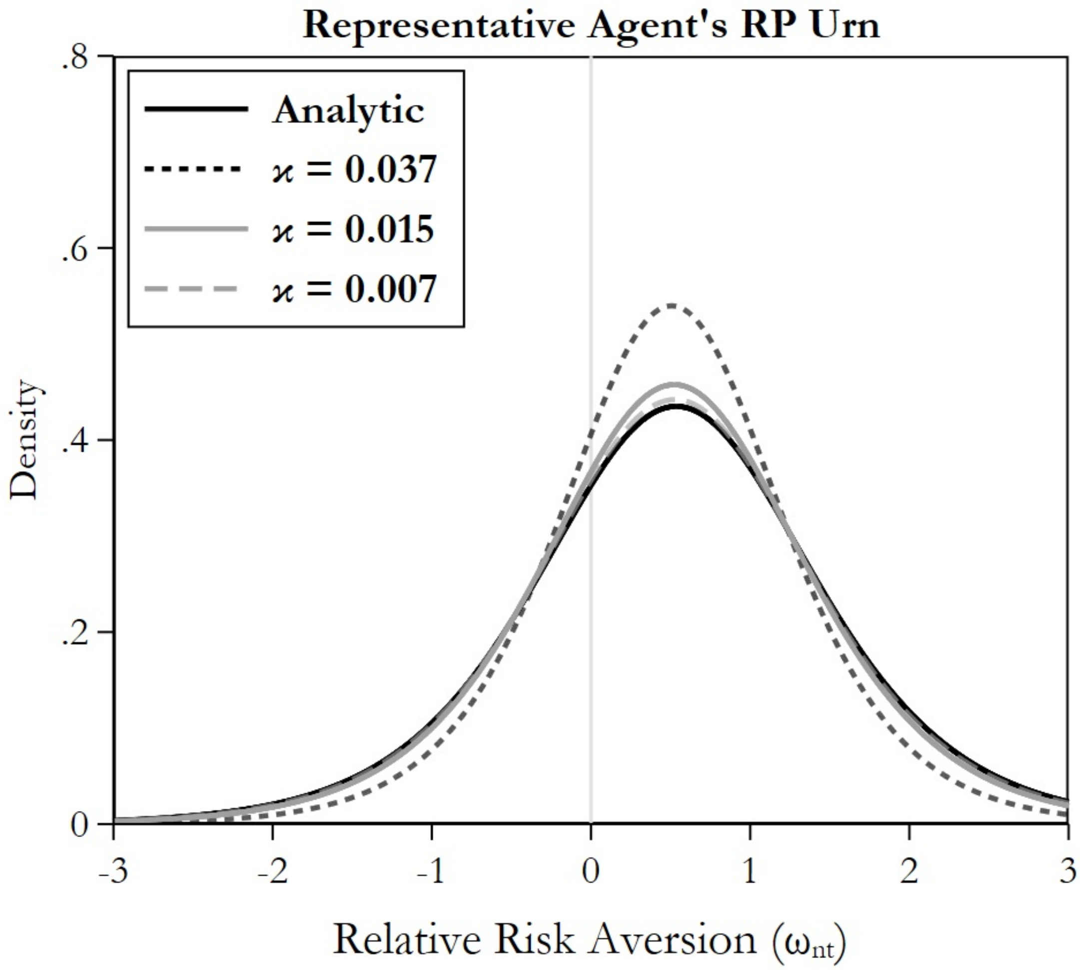


Figure 2: Partial RP-RDU with NP Utility

Representative Agent's RP Urns

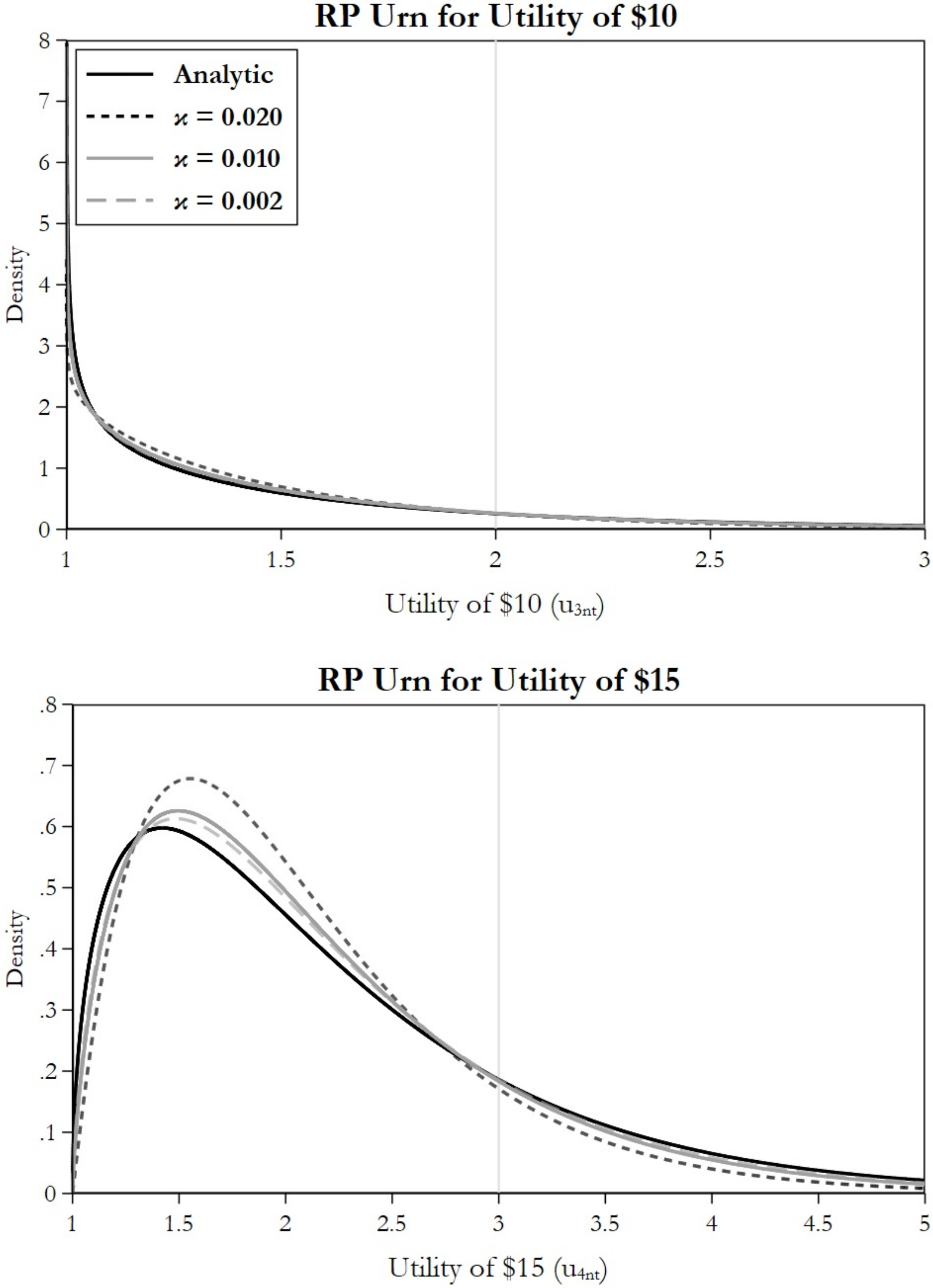


Figure 3: RP-Tremble-EUT with CRRA Utility

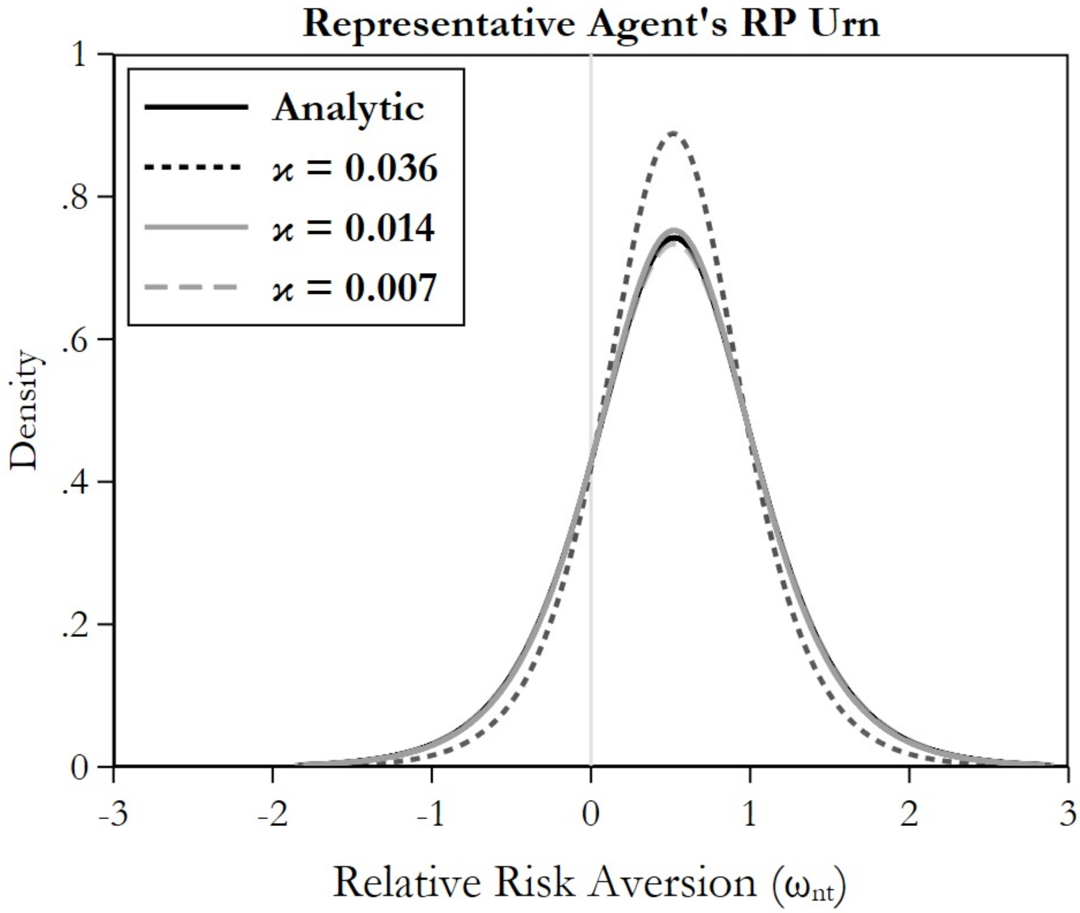
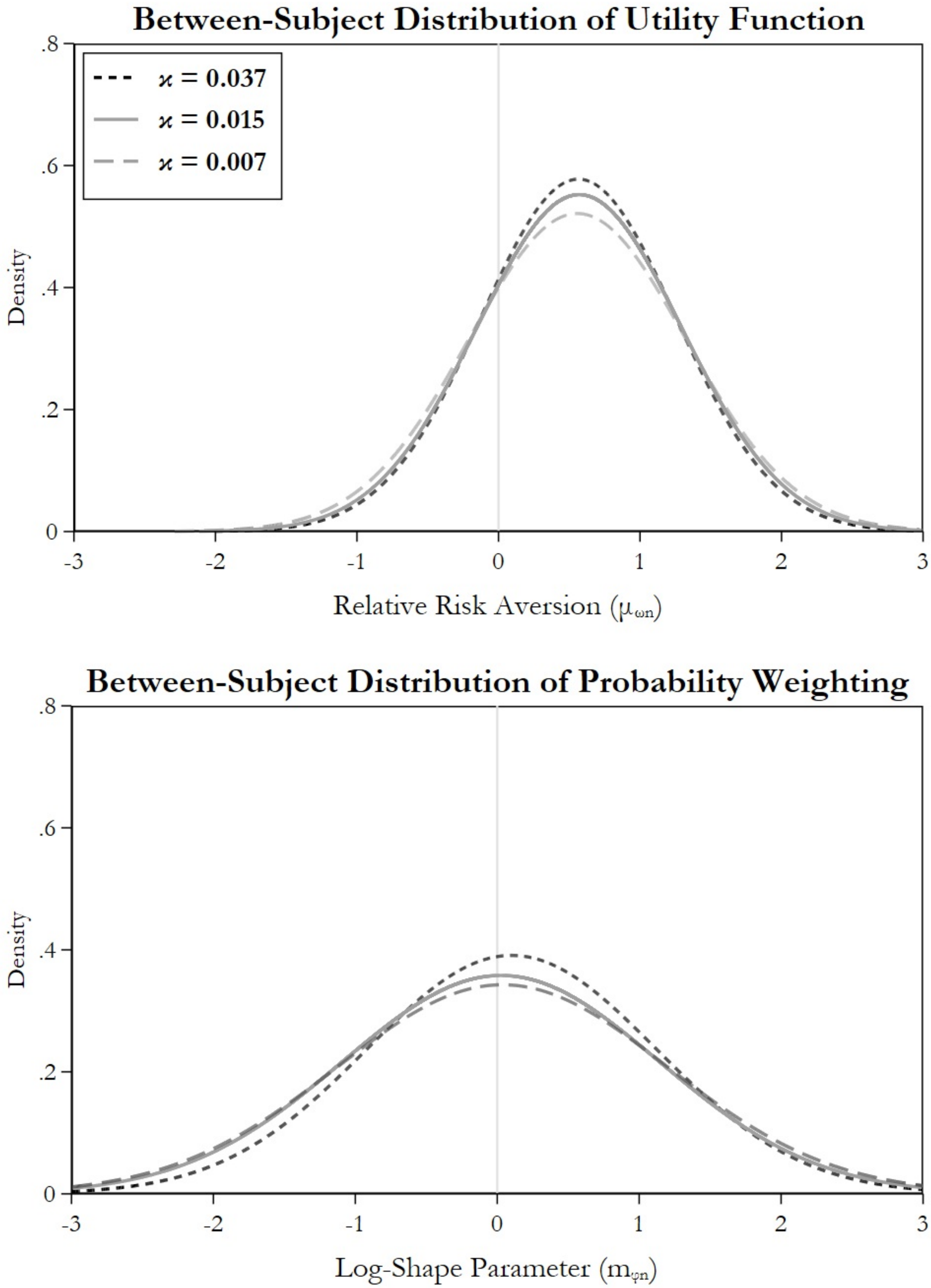


Figure 4: Random Coefficient RP-RDU with CRRA Utility



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Online Appendix A: Dual Estimation of RP-EUT with CRRA Utility

In Section 2.C of the main text, we estimate the RP-EUT model with CRRA utility using data from Andersen, Harrison, Lau and Rutström [2014]. Since the binary choice tasks in this experiment are based on the same logic as the multiple price lists in Holt and Laury [2002], the scaled EU difference in equation (4) has a single crossing point (*e.g.*, Gans and Smart [1996], Athey [2001] and Quah and Strulovici [2012]). That is, in each decision task without a dominated choice, there exists a unique indifference point w_{nt} that solves $\Delta EU_{nt}(w_{nt}) = 0$ such that the subject chooses lottery B (*i.e.*, $\Delta EU_{nt}(\omega_{nt}) > 0$) if her risk aversion parameter $\omega_{nt} < w_{nt}$ and lottery A (*i.e.*, $\Delta EU_{nt}(\omega_{nt}) < 0$) if $\omega_{nt} > w_{nt}$.[29] Table A1 reports the indifference point for each non-dominated decision task in the data set.

In this appendix, we introduce a convenient approach to estimating the RP-EUT model with CRRA utility that takes advantage of the single crossing property. Viewing ω_{nt} as a latent dependent variable and w_{nt} as a known threshold allows us to recast the RP-EUT model as a linear index model with an additive error term, just like a standard discrete choice model. One can thus use logit and probit regression commands for the dual standard model to estimate the primal RP-EUT model. Discrete choice models for panel data, such as random effects (RE) logit and mixed logit, provide accessible avenues to incorporate interpersonal heterogeneity by making the distribution of the random preference parameter individual-specific. We are not aware of any existing study that applies standard regression commands in structural estimation of risk attitudes, be it in the stochastic framework of the RP model or the additive error model.

The dual pooled logit model below explains how one may obtain the analytic ML estimates of the logistic RP urn that are reported in Figure 1. We expect that the dual RE logit and mixed logit models will be useful toolkits for studies on socio-economic determinants of risk preferences that consider an individual's risk attitude as a one-dimensional trait (*e.g.*, Dohmen, Falk, Huffman and Sunde [2010], Filippin and Crosetto [2016], Guiso, Sapienza and Zingales [2018] and Hryshko, Luengo-Prado and Sørensen [2011]). With our dual approach, the RP-EUT model with CRRA utility becomes an attractive alternative to reduced-form regression models which are widely used. Both types of models can be estimated using standard regression commands in software packages, but the RP model has a more solid theoretical foundation that allows one to distinguish interpersonal preference heterogeneity from behavioral noise. Large household surveys, such as the Panel Study of Income Dynamics in the USA, Socio-Economic Panel in Germany and the UK Household Longitudinal Study, include binary choice tasks under risk for which one-parameter formulations of EUT display the single crossing property, thereby increasing the appeal of our dual approach.[30]

A. Homogeneous Risk Aversion and Noise

Consider a data environment where the scaled EU difference displays the single crossing property, and let w_{nt} be the indifference point where the difference in expected utility is equal to zero.

[29] We stress that single crossing is a *joint* property of theory *and* data. For example, EUT with CRRA utility has a *unique* indifference point in each binary choice task following the logic in multiple price lists, as reported in Table A1. The *same* decision model has *two* indifference points in a third-order risk apportionment task in the style of Deck and Schlesinger [2010], namely one at $\omega_{nt} = 0$ and the other at $\omega_{nt} = -1$, thereby violating the single crossing property.

[30] Combining the RP model with the single crossing property also plays an important analytic role in Barseghyan, Molinari, and Thirkettle [2021], who develop a semi-nonparametric estimator of risk aversion in market settings where the analyst does not fully observe the decision maker's choice sets and relevant product attributes.

The indifference point w_{nt} is a function of the prizes and probabilities in each pair of lotteries and is therefore an observed characteristic of the decision task. Suppose that $\omega_{nt} > w_{nt}$ implies $\Delta EU_{nt}(\omega_{nt}) < 0$, and $\omega_{nt} < w_{nt}$ implies $\Delta EU_{nt}(\omega_{nt}) > 0$. For example, subject n chooses lottery A (B) in task t if she is more (less) risk averse than w_{nt}. Let y_{nt} denote a choice indicator that is equal to 1 if her choice is B and equal to 0 if it is A.

The RP-EUT model in equation (3) specifies the distribution of ω_{nt} as logistic with mean μ_ω and scale σ_ω. We assume for now that every subject has the same "RP urn" and refer to μ_ω and σ_ω as the risk aversion parameter and the noise parameter, respectively. We can exploit the single crossing property of ω_{nt} and show that an analytic solution to equation (3) is given by

$$L_{nt}(\mu_\omega, \sigma_\omega) = \int \mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0] f(\omega_{nt} | \mu_\omega, \sigma_\omega) d\omega_{nt} = \Lambda((w_{nt} - \mu_\omega)/\sigma_\omega), \tag{A1}$$

where $\Lambda(.)$ is the standard logistic distribution function. We can also show that this specification is equivalent to the *pooled* logit model

$$L_{nt}(\beta_0, \beta_1) = \Lambda(\beta_0 + \beta_1 w_{nt}), \tag{A2}$$

which can be estimated by running a logit regression of the choice indicator y_{nt} on the independent variable w_{nt} over all subjects n and decision tasks t. The steps involved in moving from the RP-EUT model in equation (A1) to the pooled logit model in equation (A2) are straightforward, at least with hindsight, but the dual link has not been identified in the empirical literature on choice under risk.

The probability that subject n chooses lottery B in task t, $L_{nt}(\mu_\omega, \sigma_\omega)$, is equal to $\Pr(\Delta EU_{nt}(\omega_{nt}) > 0)$. Given the single crossing condition, $\Pr(\Delta EU_{nt}(\omega_{nt}) > 0)$ is equal to the probability that subject n in task t is less risk averse than the indifference point w_{nt}, $\Pr(\omega_{nt} < w_{nt})$. It is useful to write out the "core risk aversion plus random fluctuations" interpretation of ω_{nt} explicitly

$$\omega_{nt} = \mu_\omega + \sigma_\omega \times e_{nt}, \tag{A3}$$

where e_{nt} is a standard logistic random variable. It follows that $\Pr(\omega_{nt} < w_{nt}) = \Pr(\mu_\omega + \sigma_\omega \times e_{nt} < w_{nt}) = \Pr(e_{nt} < (w_{nt} - \mu_\omega)/\sigma_\omega)$. Hence, the probability that subject n chooses lottery B in task t is the cumulative probability that the random variable e_{nt} is smaller than the standardized difference between the indifference point and risk aversion parameter $(w_{nt} - \mu_\omega)/\sigma_\omega$. One can evaluate these choice probabilities using the standard logistic distribution function $\Lambda(.)$ in (A1).

The RP-EUT model in (A1) can be indirectly estimated as the pooled logit model in (A2), since the latter is equivalent to the former with $\beta_0 = -\mu_\omega/\sigma_\omega$ and $\beta_1 = 1/\sigma_\omega$. One can thus obtain maximum likelihood estimates (MLEs) of β_0 and β_1 in the pooled logit model, and use the results to infer the risk aversion parameter $\mu_\omega = -\beta_0/\beta_1$ and noise parameter $\sigma_\omega = 1/\beta_1$. The invariance property of MLE implies that the transformed parameter estimates from the pooled logit model are equivalent to the direct ML estimates of μ_ω and σ_ω, which can be obtained by a user-written likelihood evaluator. As usual, standard errors of the transformed parameters can be obtained by the delta method.

B. Heterogeneous Risk Aversion and Homogeneous Noise

Since the RP-EUT model in equation (A1) is dual to the pooled logit model in equation (A2), the two models share the same fundamental limitations. First, neither model accounts for panel correlation across repeated choice observations on the same subject. Each choice is modeled as an independent observation, even though it forms part of a set of choices by the same subject. Second, neither model accounts for unobserved heterogeneity in choice behavior across subjects. In the RP model, this translates into the assumption that every subject has the same urn of random risk aversion parameters.

The random effects (RE) logit model for panel data addresses these two limitations of the pooled logit model, and it is dual to an EUT model that captures between-subject heterogeneity by replacing the fixed coefficient μ_ω with a random coefficient $\mu_{\omega n}$. Suppose that the random risk aversion parameter ω_{nt} is logistically distributed *within* subject n with density $f(\omega_{nt} | \mu_{\omega n}, \sigma_\omega)$. That is, if one compares the RP urns of two subjects, the contents of the two urns have different means but the same standard deviation. Suppose further that the risk aversion parameter $\mu_{\omega n}$ is normally distributed *between* different subjects, $\mu_{\omega n} \sim N(E[\mu_{\omega n}], SD[\mu_{\omega n}]^2)$, *i.e.* each subject n in the population has her own value of $\mu_{\omega n}$, and the between-subject mean and standard deviation of those values in the population are equal to $E[\mu_{\omega n}]$ and $SD[\mu_{\omega n}]$, respectively. One can interpret $N(E[\mu_{\omega n}], SD[\mu_{\omega n}]^2)$ as the population distribution of risk attitudes in this model.

Conditional on a particular $\mu_{\omega n}$ from the population distribution, the probability that subject n chooses lottery B in task t is specified as

$$\begin{aligned} L_{nt}(\mu_{\omega n}, \sigma_\omega) &= \int \mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0] f(\omega_{nt} | \mu_{\omega n}, \sigma_\omega) d\omega_{nt} \\ &= \Lambda((w_{nt} - \mu_{\omega n})/\sigma_\omega) \text{ with } \mu_{\omega n} \sim N(E[\mu_{\omega n}], SD[\mu_{\omega n}]^2), \end{aligned} \tag{A4}$$

where $E[\mu_{\omega n}]$, $SD[\mu_{\omega n}]$ and σ_ω are parameters to be estimated.[31] This model is equivalent to the RE logit model that augments the pooled logit model in equation (A2) with a normally distributed error component υ_n. Conditional on a particular υ_n from the population distribution, the probability that subject n chooses lottery B in task t in the RE logit model is

$$L_{nt}(\beta_0, \beta_1, \upsilon_n) = \Lambda(\beta_0 + \beta_1 w_{nt} + \upsilon_n) \text{ with } \upsilon_n \sim N(0, \sigma_0^2), \tag{A5}$$

where β_0, β_1 and σ_0 are parameters to be estimated. This model has a homogeneous slope coefficient β_1 and a heterogeneous intercept, $\alpha_n = (\beta_0 + \upsilon_n)$, which is normally distributed between subjects, $\alpha_n \sim N(\beta_0, \sigma_0^2)$. The error component υ_n captures between-subject heterogeneity around the mean intercept β_0, and the standard deviation σ_0 measures the extent of that heterogeneity.

When the RE logit model in equation (A5) is interpreted as a random intercept model, the dual link to the EUT model in equation (A4) with between-subject heterogeneity in risk aversion becomes apparent. The slope coefficient β_1 is equivalent to $1/\sigma_\omega$, and the random intercept $\alpha_n = (\beta_0 + \upsilon_n)$ is equivalent to $-\mu_{\omega n}/\sigma_\omega$. Thus, one can use the RE logit estimates of β_0, β_1 and σ_0 to compute $\sigma_\omega = 1/\beta_1$, $E[\mu_\omega] = -\beta_0/\beta_1$ and $SD[\mu_\omega] = \sigma_0/\beta_1$, and apply the delta method to obtain standard errors of the transformed parameters.

It is useful to clarify the meaning of "random" since the term has been used to describe two different kinds of random variations. The term "random" in the RP model refers to the random variable ω_{nt} with density $f(\omega_{nt} | \mu_{\omega n}, \sigma_\omega)$ that describes the seemingly unstable risk attitude of subject n over decision tasks. It captures *within-subject* variation that vanishes as the noise parameter σ_ω tends to zero. The term "random" in the random coefficient and RE models refers to the use of random

[31] As Train [2009; §6] and Wooldridge [2010; p.613] explain, the sample log-likelihood function can be specified in terms of $E[\mu_{\omega n}]$, $SD[\mu_{\omega n}]$ and σ_μ by "integrating out" $\mu_{\omega n}$.

variables $\mu_{\omega n}$ and α_n to describe interpersonal heterogeneity in the underlying population. It captures *between-subject* variation which is present even if every subject makes deterministic choices according to the non-stochastic version of EUT. The model specification in equation (A4) includes both kinds of random variations.

C. Heterogeneous Risk Aversion and Noise

The RP-EUT model in equation (A4) allows for between-subject heterogeneity in the risk aversion parameter, $\mu_{\omega n}$, but not in the noise parameter, σ_ω. We can capture between-subject heterogeneity in the noise parameter by introducing a second random coefficient, $\sigma_{\omega n}$, that replaces the fixed coefficient σ_ω. Assume that the random risk aversion parameter ω_{nt} is logistically distributed *within* subject n with density $f(\omega_{nt} | \mu_{\omega n}, \sigma_{\omega n})$. The subject now has an individual-specific risk aversion parameter, $\mu_{\omega n}$, *and* an individual-specific noise parameter, $\sigma_{\omega n}$. Assume further that the risk aversion and log-noise parameters, $\mu_{\omega n}$ and $\ln(\sigma_{\omega n})$, are jointly normally distributed *between* subjects, $[\mu_{\omega n}, \ln(\sigma_{\omega n})]' \sim MVN(\mathbf{b}_{EUT}, \mathbf{V}_{EUT})$. Each subject n thus has a distinct pair of values $\mu_{\omega n}$ and $\ln(\sigma_{\omega n})$, and the distribution of these bivariate parameters has between-subject mean $\mathbf{b}_{EUT}$ and covariance matrix $\mathbf{V}_{EUT}$. Conditional on particular $\mu_{\omega n}$ and $\sigma_{\omega n}$ from the population distribution, the probability that subject n chooses lottery B in task t is

$$\begin{aligned} L_{nt}(\mu_{\omega n}, \sigma_{\omega n}) &= \int \mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0] f(\omega_{nt} | \mu_{\omega n}, \sigma_{\omega n}) d\omega_{nt} \\ &= \Lambda((w_{nt} - \mu_{\omega n})/\sigma_{\omega n}) \text{ with } [\mu_{\omega n}, \ln(\sigma_{\omega n})]' \sim MVN(\mathbf{b}_{EUT}, \mathbf{V}_{EUT}), \end{aligned} \tag{A6}$$

where the mean vector $\mathbf{b}_{EUT}$ and covariance matrix $\mathbf{V}_{EUT}$ are parameters to be estimated.

The more general RP-EUT model in (A6) is dual to the mixed logit model in the willingness-to-pay (WTP) space (Train and Weeks [2005]). Although less known compared to pooled logit and RE logit models, the WTP space model is a standard econometric model that one can readily estimate in popular statistical packages.[32] In the WTP space model, conditional on particular α_n and λ_n from the population distribution, the probability that subject n chooses lottery B in task t is

$$\begin{aligned} L_{nt}(\alpha_n, \lambda_n) &= \exp((\alpha_n + w_{nt}) \times \lambda_n) \,/\, [\exp(0) + \exp((\alpha_n + w_{nt}) \times \lambda_n)] \\ &= \Lambda((\alpha_n + w_{nt}) \times \lambda_n) \text{ with } [\alpha_n, \ln(\lambda_n)]' \sim MVN(\mathbf{b}_{WTP}, \mathbf{V}_{WTP}) \end{aligned} \tag{A7}$$

where the mean vector $\mathbf{b}_{WTP}$ and the covariance matrix $\mathbf{V}_{WTP}$ are parameters to be estimated. The first equality in equation (A7) is algebraically redundant but conveys useful operating advice. Since the WTP space model is primarily intended for multinomial choice applications, the available programs evaluate a separate index function for each alternative in a choice set. In our dual approach, one can set the index functions for lottery A to 0 and lottery B to $(\alpha_n + w_{nt}) \times \lambda_n$.

To express the dual link between the RP-EUT model in equation (A6) and the WTP space model in equation (A7) more explicitly, the former may be arranged as

$$L_{nt}(\mu_{\omega n}, \sigma_{\omega n}) = \Lambda((-\mu_{\omega n} + w_{nt}) \times (1/\sigma_{\omega n})) \text{ with } [\mu_{\omega n}, \ln(\sigma_{\omega n})]' \sim MVN(\mathbf{b}_{EUT}, \mathbf{V}_{EUT}). \tag{A6'}$$

[32] The GMNL model of Fiebig, Keane, Louviere and Wasi [2010] is also a dual model since it algebraically nests the WTP space model (Greene and Hensher [2010; p.416]). Estimation programs for the GMNL model are also widely available in different software packages.

Comparing the primal representation in equation (A6') to the dual representation in equation (A7) shows that $\alpha_n = -\mu_{\omega n}$ and $\lambda_n = 1/\sigma_{\omega n}$. Moreover, taking the natural log of both sides in $\lambda_n = 1/\sigma_{\omega n}$ shows that $\ln(\lambda_n) = -\ln(\sigma_{\omega n})$. Since $[-\mu_{\omega n}, -\ln(\sigma_{\omega n})] = [\alpha_n, \ln(\lambda_n)]$, it follows that $\mathbf{b}_{EUT} = -\mathbf{b}_{WTP}$ and $\mathbf{V}_{EUT} = \mathbf{V}_{WTP}$. We can thus multiply the estimates of $\mathbf{b}_{WTP}$ by -1 to compute the estimates of $\mathbf{b}_{EUT}$, and use $\mathbf{V}_{WTP}$ directly in place of $\mathbf{V}_{EUT}$.

This dual link is computationally very convenient. The use of random coefficients to capture interpersonal preference heterogeneity has become a well-established practice in environmental economics, health economics, marketing science and transportation research because standard estimation programs for the family of mixed logit models enable one to estimate the structural models of interest in those fields. By contrast, the structural models of risk preferences, seemingly, do not fit in with the mixed logit structure and the empirical analysis of those models typically relies on fixed coefficient specifications. The dual link that we have identified provides convenient means to apply recent advances in discrete choice methods to behavioral research.

We stress that this dual link *does not* make any use of the subject's WTP for a lottery under EUT.[33] Nor does it assume that the indifferent point w_{nt} in (A7) is measured in terms of WTP: w_{nt} still refers to the indifference point measured in terms of the coefficient of RRA. The potentially confusing term *WTP space* has no behavioral content in relation to the subject's risk attitude; it is simply an inherited label which describes the standard discrete choice model that we use in dual estimation, much as *pooled* and *RE* in our earlier examples.

To discuss why the WTP space model has been dubbed as it is, we must address its origin in the non-market valuation literature. Typically non-market valuation studies specify the consumption utility in a particular choice situation as a linear combination of available product attributes. Suppose now that the attribute taking place of w_{nt} in equation (A7) is equal to the price of product A minus the price of product B. Then, if the consumer has a constant marginal utility of money that is proportional to the precision parameter λ_n, the random intercept α_n can be seen as the consumer's WTP for acquiring product B rather than product A. Train and Weeks [2005] have named this type of model specification the "WTP space model" to distinguish it from the "preference space model" which would substitute $(\beta_{0n} + \beta_{1n} w_{nt})$ for $(\alpha_n + w_{nt}) \times \lambda_n$.[34] The slope parameter $\beta_{1n} = 1/\lambda_n$ in the preference space measures the consumer's marginal utility of money given the linear utility function, and the intercept $\beta_{0n} = 1/\alpha_n$ expresses the consumer's preference for product B relative to A in terms of *utils* rather than monetary units. Train and Weeks [2005], Scarpa, Thiene and Train [2008], and Oviedo and Yoo [2017] provide further comparisons of parameters in the two different spaces. This way of interpreting the WTP space model parameters is peculiar to the non-market valuation studies, and is irrelevant to our usage of the WTP space model which is entirely motivated by its algebraic structure rather than behavioral content.

D. Empirical Illustration of Dual Estimation: Baseline Models

We illustrate dual estimation of the RP-EUT model with CRRA utility using data from the field experiment reported in Andersen, Harrison, Lau and Rutström [2014]. The sample excludes

[33] We thank an anonymous reviewer for alerting us to the importance of clarifying this potential point of confusion.

[34] The WTP in this sense is sometimes called implicit prices or equivalent prices (Hausman and Ruud [1987]) instead.

lottery pairs with dominated choices which do not contribute to the identification of risk preferences in the RP framework.

Panel A in Table A2 reports the estimation results for the RP-EUT model in equation (A1), which assumes that every subject carries the same urn of RP parameters with logistic density $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$. We estimate the dual pooled logit model in equation (A2) by using the *logit* command in *Stata* to regress the choice indicator y_{nt} on the indifference point w_{nt}. The estimated intercept (β_0) and coefficient on w_{nt} (β_1) are equal to −0.932 and 1.740, respectively. Unless stated otherwise, all estimated coefficients and transformed parameters have *p*-values < 0.001. We can use the estimated coefficients to derive the relative risk aversion parameter $\mu_\omega = -\beta_0/\beta_1 = 0.535$ and noise parameter $\sigma_\omega = 1/\beta_1 = 0.575$. We thus find that the representative agent generally is risk averse with significant variation in choice behavior. These estimates of μ_ω and σ_ω are identical to the analytic ML estimates reported in Figure 1.

Panel B in Table A2 reports the results for the RP-EUT model in (A4) which accommodates between-subject heterogeneity in risk aversion. The RP urn of subject n is $f(\omega_{nt} | \mu_{\omega n}, \sigma_\omega)$ where the risk aversion parameter $\mu_{\omega n}$ is now subject-specific and assumed to be normally distributed between subjects, with population mean $E[\mu_{\omega n}]$ and standard deviation $SD[\mu_{\omega n}]$. Since this model is dual to the RE logit model, we regress y_{nt} on w_{nt} using the *xtlogit, re* command in *Stata*, which applies a Gauss-Hermite quadrature to integrate out the normal error component υ_n in equation (A5). The random intercept is normally distributed with an estimated population mean (β_0) of −1.327 and a standard deviation (σ_0) of 1.778, while the estimated coefficient β_1 on w_{nt} is equal to 2.411. The transformed parameters $E[\mu_{\omega n}] = -\beta_0/\beta_1$, $SD[\mu_{\omega n}] = \sigma_0/\beta_1$ and $\sigma_\omega = 1/\beta_1$ are equal to 0.550, 0.737 and 0.415, respectively. Hence, the coefficient of RRA is estimated to be 0.550 on average, with significant between- and within-subject variation, and we find that 77.2% of the population are risk averse.[35]

Panel C in Table A2 reports the results for the RP-EUT model in equation (A6) which accommodates between-subject heterogeneity in both risk aversion and noise. The RP urn of subject n is $f(\omega_{nt} | \mu_{\omega n}, \sigma_{\omega n})$, where both risk aversion ($\mu_{\omega n}$) and noise ($\sigma_{\omega n}$) parameters are subject-specific. The model assumes that $\mu_{\omega n}$ and $\ln(\sigma_{\omega n})$ are jointly normally distributed between subjects. We estimate the dual mixed logit model in WTP space and use the *mixlogitwtp* command in *Stata* by Hole [2015], which applies simulation to integrate out the joint normal random coefficients α_n and $\ln(\lambda_n)$ in (A7). Our simulated integrals are based on 100 Halton draws per subject. The estimated intercept α_n is normally distributed between subjects with a mean (β_0) of −0.519 and a standard deviation (σ_0) of 0.846. The log-precision parameter $\ln(\lambda_n)$ is normally distributed between subjects with a mean (τ) of 1.304 and a standard deviation (σ_τ) of 0.997. We can use these estimates to evaluate the between-subject distribution of the risk aversion parameter $\mu_{\omega n}$ and log-noise parameter $\ln(\sigma_{\omega n})$ in the population. The results suggest that $E[\mu_{\omega n}] = -\beta_0 = 0.519$ and $SD[\mu_{\omega n}] = \sigma_0 = 0.846$, while $E[\ln \sigma_{\omega n}] = -\tau = -1.304$ and $SD[\ln \sigma_{\omega n}] = \sigma_0 = 0.997$.[36] Hence, the estimated population mean of relative risk aversion is equal to 0.519, and we find that 73% of the population are risk averse.[37]

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[35] Since the between-subject distribution of $\mu_{\omega n}$ is normal, the population share of risk averse subjects is equal to $\Pr(\mu_{\omega n} > 0) = 1-\Phi(-E[\mu_{\omega n}]/SD[\mu_{\omega n}])$, where $\Phi(.)$ is the standard normal distribution function. The estimated share is significantly greater than 0.50 or 50% (one-sided *p*-value < 0.001).

[36] Using the analytic properties of the log-normal distribution, we can also evaluate the population moments of $\sigma_{\omega n}$. We find that $E[\sigma_{\omega n}] = 0.446$, $SD[\sigma_{\omega n}] = 0.582$, and $\varrho(\mu_{\omega n}, \sigma_{\omega n}) = -0.042$ (*p*-value = 0.570).

[37] The between-subject correlation coefficient for α_n and $\ln(\lambda_n)$, $\varrho_{\tau 0}$, is equal to −0.055 with *p*-value = 0.564. Since the correlation coefficient for $\mu_{\omega n}$ and $\ln(\sigma_{\omega n})$, $\varrho[\mu_{\omega n}, \ln \sigma_{\omega n}]$, is identical to $\varrho_{\tau 0}$, we do not find that

E. Empirical Illustration: Incorporating Observed Heterogeneity

We can extend the dual estimation approach to incorporate observed heterogeneity in risk preferences by adding relevant independent variables to the dual logit models in the usual manner. To facilitate discussion, let female_n denote a binary indicator variable that is equal to one if subject n is female and zero otherwise. Suppose now that we add this variable to the dual pooled logit model as follows

$$L_{nt}(b_0, b_1, \beta_1) = \Lambda(b_0 + b_1\text{female}_n + \beta_1 w_{nt}), \tag{A8}$$

which can be seen as a specification that replaces a constant intercept β_0 in equation (A2) with a demographic intercept $(b_0 + b_1\text{female}_n)$. Just as the constant specification in equation (A2) is dual to the RP-EUT model with the risk aversion parameter $\mu_\omega = -\beta_0/\beta_1$ and the noise parameter $\sigma_\omega = 1/\beta_1$, the demographic specification (A8) is dual to the logistic EUT model with the risk aversion parameter $(m_0 + m_1\text{female}_n) = (-b_0/\beta_1 - b_1/\beta_1)$ and the noise parameter $\sigma_\omega = 1/\beta_1$. This insight extends directly to the RE logit in equation (A5) and the mixed logit model in the WTP space in equation (A7); in the latter model, we can also replace a constant intercept τ in the log-precision parameter, $\ln(\lambda_n) \sim N(\tau, \sigma_\tau^2)$, with a demographic intercept, thereby inducing observed heteroskedasticity in the log-noise parameter of the primal model.

Table A3 reports the demographic extension of each dual logit model in Table A2. To make the link between the two tables clearer, we use β_0:base and β_0:female to denote b_0 and b_1, and likewise we use μ_ω:base and μ_ω:female to denote m_0 and m_1. Other parameter labels with base and female suffixes can be similarly associated with the baseline intercept and the demographic slope coefficient. As with the preceding subsection, we use the *logit and xtlogit, re* commands in *Stata* to estimate the pooled and RE logit models. Hole's [2016] *mixlogitwtp* command does not allow one to include observed heterogeneity in the log-precision parameter of the WTP space model. We therefore use the *gmnl* command by Gu, Knox and Hole [2013] and estimate the WTP space model as a special case of the GMNL-II model (Fiebig, Keane, Louviere, and Wasi [2010]), an approach inspired by Greene and Hensher [2010; p.416].[38]

The pooled logit model reported in the top panel of Table A3 suggests that while both men and women are risk averse, women tend to be more risk averse than men. The coefficient of relative risk aversion is equal to 0.416 for men (μ_ω:base) and 0.664 (μ_ω:base + μ_ω:female) for women; the female-male difference of 0.248 (μ_ω:female) is significantly greater than zero (*p*-value < 0.001). We draw qualitatively similar conclusions from the RE logit model in the middle panel and the WTP space model in the bottom panel, which account for unobserved between-subject heterogeneity in risk aversion on top of the observed heterogeneity. The WTP space model also includes an extra parameter $E[\ln \sigma_{\omega n}]$:female, which captures whether women's risk preferences tend to show greater or smaller random fluctuations than men's. We do not find evidence of such demographic heteroskedasticity: The point estimate of 0.176 is small in magnitude compared to the overall intercept ($E[\ln \sigma_{\omega n}]$:base = −1.139) and is not significantly different from zero at the 5% significance level (*p*-value = 0.076).

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more risk averse subjects have more or less variation in risk preferences.

[38] We thank Arne Risa Hole for alerting us to this reference.

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Table A1: Indifference Points for EUT Model with CRRA Utility in Experiment by Andersen, Harrison, Lau and Rutström [2014]

Lottery A	Lottery B	Indifference Point
10% of 2000 and 90% of 1600	10% of 3850 and 90% of 100	w = −1.713
20% of 2000 and 80% of 1600	20% of 3850 and 80% of 100	w = −0.947
30% of 2000 and 70% of 1600	30% of 3850 and 70% of 100	w = −0.487
40% of 2000 and 60% of 1600	40% of 3850 and 60% of 100	w = −0.143
50% of 2000 and 50% of 1600	50% of 3850 and 50% of 100	w = 0.146
60% of 2000 and 40% of 1600	60% of 3850 and 40% of 100	w = 0.412
70% of 2000 and 30% of 1600	70% of 3850 and 30% of 100	w = 0.676
80% of 2000 and 20% of 1600	80% of 3850 and 20% of 100	w = 0.971
90% of 2000 and 10% of 1600	90% of 3850 and 10% of 100	w = 1.368
10% of 1125 and 90% of 750	10% of 2000 and 90% of 250	w = −1.454
20% of 1125 and 80% of 750	20% of 2000 and 80% of 250	w = −0.720
30% of 1125 and 70% of 750	30% of 2000 and 70% of 250	w = −0.247
40% of 1125 and 60% of 750	40% of 2000 and 60% of 250	w = 0.131
50% of 1125 and 50% of 750	50% of 2000 and 50% of 250	w = 0.471
60% of 1125 and 40% of 750	60% of 2000 and 40% of 250	w = 0.805
70% of 1125 and 30% of 750	70% of 2000 and 30% of 250	w = 1.161
80% of 1125 and 20% of 750	80% of 2000 and 20% of 250	w = 1.585
90% of 1125 and 10% of 750	90% of 2000 and 10% of 250	w = 2.206
10% of 1000 and 90% of 875	10% of 2000 and 90% of 75	w = −1.839
20% of 1000 and 80% of 875	20% of 2000 and 80% of 75	w = −1.013
30% of 1000 and 70% of 875	30% of 2000 and 70% of 75	w = −0.516
40% of 1000 and 60% of 875	40% of 2000 and 60% of 75	w = −0.144
50% of 1000 and 50% of 875	50% of 2000 and 50% of 75	w = 0.169
60% of 1000 and 40% of 875	60% of 2000 and 40% of 75	w = 0.457
70% of 1000 and 30% of 875	70% of 2000 and 30% of 75	w = 0.747
80% of 1000 and 20% of 875	80% of 2000 and 20% of 75	w = 1.070
90% of 1000 and 10% of 875	90% of 2000 and 10% of 75	w = 1.511
10% of 2250 and 90% of 1000	10% of 4500 and 90% of 50	w = −0.701
20% of 2250 and 80% of 1000	20% of 4500 and 80% of 50	w = −0.264
30% of 2250 and 70% of 1000	30% of 4500 and 70% of 50	w = 0.007
40% of 2250 and 60% of 1000	40% of 4500 and 60% of 50	w = 0.218
50% of 2250 and 50% of 1000	50% of 4500 and 50% of 50	w = 0.403
60% of 2250 and 40% of 1000	60% of 4500 and 40% of 50	w = 0.578
70% of 2250 and 30% of 1000	70% of 4500 and 30% of 50	w = 0.760
80% of 2250 and 20% of 1000	80% of 4500 and 20% of 50	w = 0.971
90% of 2250 and 10% of 1000	90% of 4500 and 10% of 50	w = 1.267

Table A2: RP-EUT with CRRA Utility - Dual Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
			A. Pooled Logit *(Log-likelihood = −7501.476)*		
β_0	−0.932	0.069	<0.001	−1.068	−0.795
β_1	1.740	0.088	<0.001	1.567	1.913
$\mu_\omega = -\beta_0/\beta_1$	0.535	0.033	<0.001	0.470	0.601
$\sigma_\omega = 1/\beta_1$	0.575	0.029	<0.001	0.518	0.632
			B. Random Effects Logit *(Log-likelihood = −6143.673)*		
β_0	−1.327	0.114	<0.001	−1.550	−1.104
β_1	2.411	0.133	<0.001	2.150	2.672
σ_0	1.778	0.143	<0.001	1.498	2.058
$E[\mu_{\omega n}] = -\beta_0/\beta_1$	0.550	0.038	<0.001	0.475	0.625
$SD[\mu_{\omega n}] = \sigma_0/\beta_1$	0.737	0.056	<0.001	0.628	0.847
$\sigma_\omega = 1/\beta_1$	0.415	0.023	<0.001	0.370	0.460
			C. Mixed Logit in WTP Space *(Log-likelihood = −5374.130)*		
β_0	−0.519	0.022	<0.001	−0.562	−0.475
τ	1.304	0.074	<0.001	1.158	1.450
σ_0	0.846	0.026	<0.001	0.794	0.898
σ_τ	0.997	0.070	<0.001	0.859	1.135
$\varrho_{0\tau}$	−0.055	0.095	0.564	−0.242	0.132
$E[\mu_{\omega n}] = -\beta_0$	0.519	0.022	<0.001	0.475	0.562
$E[\ln \sigma_{\omega n}] = -\tau$	−1.304	0.074	<0.001	−1.450	−1.158
$SD[\mu_{\omega n}] = \sigma_0$	0.846	0.026	<0.001	0.794	0.898
$SD[\ln \sigma_{\omega n}] = \sigma_\tau$	0.997	0.070	<0.001	0.859	1.135
$\varrho[\mu_{\omega n}, \ln \sigma_{\omega n}] = \varrho_{0\tau}$	−0.055	0.095	0.564	−0.242	0.132

Notes: All models have been estimated using the Andersen, Harrison, Lau and Rutström [2014] data set. Standard errors have been adjusted for clustering at the subject level, except in panel C. The *mixlogitwtp* (version 1.1.0) command in *Stata* does not support clustered standard errors. The coefficient $\varrho_{0\tau}$ is the correlation coefficient between the random intercept and the random log-precision parameter.

Table A3: RP-EUT with CRRA Utility and Observed Heterogeneity - Dual Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
A. Pooled Logit with Observed Heterogeneity in Risk Aversion (Log-likelihood = −7442.767)					
β_0:base	−0.730	0.089	<0.001	−0.904	−0.556
β_0:female	−0.436	0.113	<0.001	−0.658	−0.213
β_1	1.756	0.089	<0.001	1.582	1.930
μ_ω:base	0.416	0.046	<0.001	0.326	0.505
μ_ω:female	0.248	0.065	<0.001	0.120	0.376
$\sigma_\omega = 1/\beta_1$	0.569	0.029	<0.001	0.513	0.626
B. Random Effects Logit with Observed Heterogeneity in Risk Aversion (Log-likelihood = −6137.296)					
β_0:base	−1.015	0.140	<0.001	−1.290	−0.740
β_0:female	−0.645	0.182	<0.001	−1.001	−0.289
β_1	2.411	0.133	<0.001	2.150	2.672
σ_0	1.746	0.140	<0.001	1.471	2.021
$E[\mu_{\omega n}]$:base	0.421	0.053	<0.001	0.317	0.525
$E[\mu_{\omega n}]$:female	0.268	0.075	<0.001	0.120	0.415
$SD[\mu_{\omega n}] = \sigma_0/\beta_1$	0.724	0.055	<0.001	0.616	0.832
$\sigma_\omega = 1/\beta_1$	0.415	0.023	<0.001	0.370	0.460
C. Mixed Logit in WTP Space with Observed Heterogeneity in Risk Aversion & Noise (Log-likelihood = −5395.158)					
β_0:base	−0.366	0.038	<0.001	−0.440	−0.291
β_0:female	−0.357	0.047	<0.001	−0.450	−0.264
τ:base	1.139	0.029	<0.001	1.011	1.267
τ:female	−0.176	0.099	0.076	−0.370	0.018
σ_0	0.885	0.026	<0.001	0.828	0.943
σ_τ	1.108	0.046	<0.001	1.018	1.197
$E[\mu_{\omega n}]$:base	0.366	0.038	<0.001	0.291	0.440
$E[\mu_{\omega n}]$:female	0.357	0.047	<0.001	0.264	0.450
$E[\ln \sigma_{\omega n}]$:base	−1.139	0.029	<0.001	−1.267	−1.011
$E[\ln \sigma_{\omega n}]$:female	0.176	0.099	0.076	0.018	0.370
$SD[\mu_{\omega n}] = \sigma_0$	0.724	0.055	<0.001	0.616	0.832
$SD[\ln \sigma_{\omega n}] = \sigma_\tau$	0.415	0.023	<0.001	0.370	0.460

Notes: All models have been estimated using the Andersen, Harrison, Lau and Rutström [2014] data set. Standard errors have been adjusted for clustering at the subject level, except in panel C. The *gmnl* (version 1.1.0) command in *Stata* does not support clustered standard errors.

Online Appendix B: Supporting Results

Appendix B collates detailed estimation results that support our discussion in the main text, along with additional results from Monte Carlo experiments. The results for models with CRRA utility are based on data reported in Andersen, Harrison, Lau and Rutström [2014], and the results for models with non-parametric utility are based on data reported in Harrison and Rutström [2008; §2.6].

A. EUT with CRRA Utility

Tables B1 and B2 relate to the RP-EUT model with CRRA utility in equation (3), which is reproduced here

$$L_{nt}(\mu_\omega, \sigma_\omega) = \int \mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0] f(\omega_{nt} \,|\, \mu_\omega, \sigma_\omega) d\omega_{nt}. \tag{B1}$$

Table B1 reports the empirical estimates of μ_ω and σ_ω that generate the RP urns in Figure 1. Table B2 reports the finite sample properties of alternative estimators based on the Monte Carlo experiment that is summarized in Figure B1.

Figure B1 displays the ML and MSL estimates of μ_ω and σ_ω across the 1,000 simulated data sets. In each panel the data sets are re-numbered based on the ML estimate of the relevant parameter so that data set 1 has the smallest estimate and data set 1,000 has the largest. In the upper panel we observe that the two curves tracing the ML and MSL estimates of μ_ω are adjacent. The two sets of estimates are almost perfectly correlated (0.997), and their absolute difference is 0.007 on average and never exceeds 0.010. The lower panel displays the ML and MSL estimates of σ_ω across the same simulated data sets, and we again observe that the two curves are adjacent with a high degree of correlation (0.984). The absolute difference in the estimate of σ_ω is 0.004 on average and not greater than 0.012. The results thus suggest that the kernel smoothing approach approximates the analytic ML benchmark well in all simulated data sets.

The finite sample properties of the two procedures reported in Table B2 do not indicate that either procedure is distinctively better than the other. The MSL estimates show slightly larger, but still negligible, bias (-0.002 in μ_ω and 0.004 in σ_ω) compared to the ML estimates (< 0.001 in magnitude). At the asymptotic significance level of 5%, inferences based on the MSL estimates are correctly sized for μ_ω (5%) and slightly over-reject the true null hypothesis for σ_ω (7%). The ML estimates are slightly over-sized for both parameters (6%).

B. Partial RDU with Non-Parametric Utility

Tables B3 and B4 relate to the *partial* RP-RDU model with NP utility in equation (17), which is reproduced here

$$L_{nt}(\alpha_3, \alpha_4, \gamma, \varphi) = \int\int \mathbf{I}[\Delta RDU_{nt}(u_{3nt}, u_{4nt}, \varphi) > 0] g(v_{3nt}, v_{4nt} \,|\, \alpha_3, \alpha_4, \gamma) dv_{3nt} dv_{4nt}. \tag{B2}$$

To compare the results from our kernel smoothing approach with those from Wilcox's [2008][2011] analytic ML approach, we restrict the sample to three prize sets that his approach allows one to retain.

Table B3 reports the empirical estimates of α_3, α_4 and γ that generate the RP urns in Figure 2, along with the empirical estimates of the deterministic parameter φ. Table B4 reports the finite sample properties of alternative estimators based on the Monte Carlo experiment that is summarized in Figure B2.

Figure B2 shows that the MSL estimates at $\varkappa = 0.010$ are similar to the analytic ML estimates in a Monte Carlo experiment with 1,000 simulated data sets of the same size as the original data (2,985 choice observations). The DGP assumes that the decision maker evaluates each lottery according to the analytic ML estimates of the partial RP-RDU model. The original set of pairwise lotteries is retained in each simulated data set, but actual choices are replaced with simulated choices from the DGP. We compute two sets of results for α_3, α_4, γ and φ from each simulated data set using analytic ML and kernel-smoothed MSL estimation. Each panel in Figure B2 displays ML and MSL estimates of a particular parameter, with α_3 in the top left, α_4 in the top right, γ in the lower left, and φ in the lower right. The ML and MSL estimates of each parameter show a high degree of correlation, which is evident from the curves tracing the two types of estimates in each panel: the correlation coefficients are equal to {0.984, 0.970, 0.978, 0.985} for $\{\alpha_3, \alpha_4, \gamma, \varphi\}$. As discussed in relation to Figure 2, the two sets of estimates lead to substantively the same inferences about the RP urns in the original data set, where the absolute differences between the ML and MSL estimates of $\{\alpha_3, \alpha_4, \gamma, \varphi\}$ are equal to {0.093, 0.123, 0.128, 0.009}.[39] Across all simulated data sets, these ML-MSL differences are equal to {0.037, 0.038, 0.078, 0.015} on average, and do not exceed {0.127, 0.136, 0.150, 0.027}. Wilcox's analytic ML approach and our kernel smoothed MSL approach thus estimate practically the same RP urns in all simulated data sets.[40]

The finite sample properties of the two estimation procedures are reported in Table B4, and the results do not indicate that either procedure is distinctively better than the other. We do, however, find that both procedures display inferior size properties compared to EUT with CRRA utility: at the asymptotic significance level of 5%, the ML (MSL) estimator has sizes of 23%, 27%, 7%, and 7% (9%, 13%, 3%, and 15%) for α_3, α_4, γ, and φ, respectively. One might expect the inferior results since we are estimating twice as many parameters (4 instead of 2) and a more general preference structure (RDU vs EUT) from a smaller sample (2,985 vs 14,868 observations). That we also observe the inferior properties for the ML estimator indicates that the results stem from fundamental difficulties in estimating the RDU specification from the present data sets, not from the use of kernel smoothing.

Figures B3-B4 and Tables B5-B6 relate to the same *partial* RP-RDU model, but we now exploit our kernel smoothing approach by retaining all four prize sets in the estimation sample. The Monte Carlo experiment in this instance follows the same procedure as before, except that the DGP uses kernel-smoothed MSL estimates from the full sample of 3,736 observations, which have been computed by setting $\varkappa = 0.010$. We obtain similar point estimates compared to the restricted sample with three prize sets, but observe vastly improved size properties for the estimated parameters other than α_3: in the Monte Carlo experiment the MSL estimator has sizes of 26%, 5%, 5%, and 6% for α_3, α_4, γ, and φ. This improvement in the size properties for parameters other than α_3 makes intuitive sense: All of the additional 751 observations (that is, the difference between the sample sizes of 3,736 and 2,985) pertain to lottery pairs over the prize set $M_{-2} = \{m_1, m_3, m_4\}$, and therefore they do not add direct information about α_3 which concerns how large the utility level of prize m_3 tends to be relative to that of m_2.

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[39] As reported in Table B3, $\{\alpha_3, \alpha_4, \gamma, \varphi\}$ are estimated at {0.702, 0.830, 0.791, 1.011} by ML, and {0.795, 0.953, 0.663, 1.002} by MSL ($\varkappa = 0.010$). The differences are computed between these two vectors.

[40] The same inferences apply when we consider estimated means and standard deviations of the two RP parameters u_{3n} and u_{4n}. Across the 1,000 simulated choice sets, the correlation between the ML and MSL estimates is 0.974 for $E[u_{3nt}]$, 0.982 for $SD[u_{3nt}]$, 0.976 for $E[u_{4nt}]$, and 0.979 for $SD[u_{4nt}]$. The absolute differences in estimated means and standard deviations with the ML and MSL approaches are on average 0.020 and 0.044 for $E[u_{3nt}]$ and $SD[u_{3nt}]$, and 0.050 and 0.067 for $E[u_{4nt}]$ and $SD[u_{4nt}]$.

C. Full RDU with Non-Parametric Utility

Figures B5-B6 and Tables B7-B8 relate to the *full* RP-RDU model with NP utility in equation (19), which is reproduced here

$$L_{nt}(\alpha_3, \alpha_4, \gamma, \mu_\varphi, \sigma_\varphi) = \int\int\int \mathbf{I}[\Delta RDU_{nt}(u_{3nt}, u_{4nt}, \varphi_{nt}) > 0] g(v_{3nt}, v_{4nt} | \alpha_3, \alpha_4, \gamma) \times \xi(\ln(\varphi_{nt}) | m_\varphi, s_\varphi) dv_{3nt} dv_{4nt} d\varphi_{nt}. \tag{B3}$$

We use all prize sets in estimation of this model. Figure B5 displays RP urns for u_{3nt}, u_{4nt} and φ_{nt} evaluated at the empirical estimates of α_3, α_4, γ, m_φ and s_φ, and Table B7 reports these estimates. Figure B6 summarizes the estimates of those parameters across 926 (out of 1,000) simulated data sets in a Monte Carlo experiment, and Table B8 reports finite sample properties inferred therefrom. The BFGS algorithm failed to find a solution to the likelihood maximization problem in the remaining 74 data sets, which suggests that empirical identification of this model is fragile relative to other models that we report. The DGP uses kernel-smoothed MSL estimates at $\varkappa = 0.010$ from the full sample.

Using all four prize sets, we observe that the expected values of u_{3nt} and u_{4nt} in the *full* RP-RDU model are similar to the expected values of the same parameters in the *partial* RP-RDU model, and the expected value of φ_{nt} in the full model is similar to the point estimate of its deterministic counterpart in the partial model. The main effects of accounting for the full RP structure show up in reduced standard deviations of u_{3nt} and u_{4nt} since part of the random variation in preferences is now attributed to the dispersion of φ_{nt}. The Monte Carlo results, however, also show that the improvement in size properties from a larger sample with four prize sets is reversed when we introduce φ_{nt} as a third RP parameter, which suggests that having three different sources of stochastic choice behavior is perhaps too much to ask from a sample of this size.

D. RDU with CRRA Utility

Figures B7-B8 and Tables B9-B10 relate to the RP-RDU model with CRRA utility, which is specified as

$$L_{nt}(\mu_\omega, \sigma_\omega, \mu_\varphi, \sigma_\varphi) = \int\int \mathbf{I}[\Delta RDU_{nt}(\omega_{nt}, \varphi_{nt}) > 0] f(\omega_{nt} | \mu_\omega, \sigma_\omega) \times \xi(\ln(\varphi_{nt}) | m_\varphi, s_\varphi) d\omega_{nt} d\varphi_{nt}, \tag{B4}$$

where $\Delta RDU_{Lnt}(\omega_{nt}, \varphi_{nt})$ is the scaled RDU difference in equation (25); $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ is a logistic density function that represents the RP urn for the utility function; and $\xi(\ln(\varphi_{nt}) | m_\varphi, s_\varphi)$ is a log-normal density function that represents the RP urn for the PWF. In other words, this RP-RDU model generalizes the RP-EUT model with CRRA utility in equation (B1) by augmenting it with a random preference PWF that is used in equation (B3).

Figure B7 displays the RP urns for the utility function (top panel) and the PWF (bottom panel), evaluated at the empirical estimates of μ_ω, σ_ω, m_φ, and s_φ. Table B9 reports these estimates. Figure B8 summarizes the estimates of the four primitive parameters across 1,000 simulated data sets in a Monte Carlo experiment, and Table B10 reports finite sample properties inferred therefrom. The simulated data sets are generated by the same procedure as before, except that the DGP uses kernel-smoothed MSL estimates at $\varkappa = 0.007$ since we cannot estimate this model using the standard ML.

E. Random Preferences and Trembles

Tables B11 and B12 relate to the RP-Tremble-EUT model with CRRA utility, which is specified by replacing the generic preference index $\Delta V_{nt}(\boldsymbol{\alpha}_{nt})$ in equation (22) with the scaled EU difference in equation (B1) as follows

$$L_{nt}(\mu_\omega, \sigma_\omega, \tau) = (1-\tau)\int \mathbf{I}[\Delta EU_{nt}(\omega_{nt}) > 0] f(\omega_{nt} \mid \mu_\omega, \sigma_\omega) d\omega_{nt} + \tau 0.5. \tag{B5}$$

This model assumes a hybrid stochastic process which is a finite mixture of the RP model and the constant error model (also known as "trembles"). We use all available choice tasks in the sample for this model, which can accommodate violations of stochastic dominance by attributing dominated choices to the tremble component.

Table B11 reports the empirical estimates of μ_ω and σ_ω that generate the RP urns in Figure 3, along with the empirical estimates of τ. Table B12 reports the finite sample properties of alternative estimators based on the Monte Carlo experiment summarized in Figure B9.

Figure B9 displays analytic ML and kernel-smoothed MSL estimates of μ_ω, σ_ω and τ across 1,000 simulated data sets. The Monte Carlo simulations follow the same procedure as the earlier examples, *i.e.* we use the analytic ML estimates for the original data set as true parameter values, and generate each simulated data set by replacing observed choice indicators in the original data set with choice indicators simulated from this DGP. The three panels in Figure B9 show a remarkable overlap in parameter estimates between the analytic ML and kernel-smoothed estimation approaches: The correlation coefficients for the point estimates of μ_ω, σ_ω and τ are 0.994, 0.976 and 0.988. The absolute difference between the ML and MSL estimates is less than 0.002 for each parameter, and either approach has an empirical size of 6% for each parameter given an asymptotic significance level of 5%.

F. Random Preferences and Interpersonal Heterogeneity

Finally, Table B13 reports the empirical estimation results for the random coefficient RP-RDU model with CRRA utility that is summarized in Figure 4. This model accommodates between-subject variation in preferences by combining the RP-RDU model in equation (B4) with a random coefficient model of interpersonal heterogeneity. We replace the two representative RP urns in equation (B4) with two subject-specific RP urns $f(\omega_{nt} \mid \mu_{\omega n}, \sigma_{\omega n})$ and $\xi(\ln(\varphi_{nt}) \mid m_{\varphi n}, s_{\varphi n})$; and complete the random coefficient specification by assuming that the within-subject means ($\mu_{\omega n}$ and $m_{\varphi n}$) are distributed as normals between subjects, whereas the within-subject scales ($\sigma_{\omega n}$ and $s_{\varphi n}$) are distributed as folded-normals. The reported empirical estimates refer to the population means and standard deviations of primitive normals for these four between-subject distributions. We exclude dominated choice tasks from the estimation sample for this model because within-subject preference variations are now modeled as the pure RP model instead of the hybrid RP-Tremble model.

Table B1: RP-EUT with CRRA Utility - Empirical Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
			A. Analytic ML (*Log-likelihood* = −7501.476)		
μ_ω	0.535	0.033	<0.001	0.470	0.601
σ_ω	0.575	0.029	<0.001	0.518	0.632
			B. $\varkappa$ *= 0.037* (*Log-likelihood* = −7415.998)		
μ_ω	0.506	0.031	<0.001	0.445	0.567
σ_ω	0.463	0.029	<0.001	0.407	0.519
			C. $\varkappa$ *= 0.015* (*Log-likelihood* = −7465.641)		
μ_ω	0.523	0.034	<0.001	0.457	0.589
σ_ω	0.546	0.030	<0.001	0.488	0.604
			D. $\varkappa$ *= 0.007* (*Log-likelihood* = −7493.115)		
μ_ω	0.528	0.037	<0.001	0.456	0.600
σ_ω	0.565	0.031	<0.001	0.505	0.626

Notes: The estimation sample includes 14,868 observations (413 subjects making 36 choices) on non-dominated choice tasks from the experiment reported by Andersen, Harrison, Lau and Rutström [2014]. All standard errors have been adjusted for clustering at the subject level. The RP urn for the coefficient of RRA ω_{nt} is logistic density $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ with the mean parameter μ_ω and the scale parameter σ_ω.

Table B2: RP-EUT with CRRA Utility - Monte Carlo Results

Parameter	DGP	Bias	RMSE	Size
		A. Analytic ML		
μ_ω	0.535	<0.001	0.011	0.058
σ_ω	0.575	<0.001	0.011	0.057
		B. $\varkappa$ = 0.037		
μ_ω	0.535	−0.022	0.025	0.466
σ_ω	0.575	−0.061	0.062	0.996
		C. $\varkappa$ = 0.015		
μ_ω	0.535	−0.007	0.014	0.080
σ_ω	0.575	−0.004	0.012	0.087
		D. $\varkappa$ = 0.007		
μ_ω	0.535	−0.002	0.012	0.053
σ_ω	0.575	0.004	0.012	0.074

Notes: The results in each panel are based on 1,000 simulated data sets of 14,868 observations. Each simulated data set is identical to the empirical estimation sample in Table B1 except that actual choices made by subjects are replaced with simulated choices from the data generating process (DGP). A bias of <0.001 refers to a value in (0.000, 0.001). RMSE reports the root mean squared error of relevant parameter estimates. Size reports the rejection frequency of the Wald test of the true null; the asymptotic significance level is 5% and the test statistic is constructed using standard errors clustered at the subject level.

Table B3: Partial RP-RDU with Non-Parametric Utility - Empirical Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
			A. Analytic ML (Log-likelihood = −1866.702)		
α_4	0.830	0.139	<0.001	0.557	1.103
α_3	0.702	0.111	<0.001	0.485	0.919
γ	0.791	0.130	<0.001	0.536	1.047
φ	1.011	0.031	<0.001	0.949	1.072
$\mu[u_{4nt}]$	2.212	0.104	<0.001	2.008	2.417
$\sigma[u_{4nt}]$	0.979	0.103	<0.001	0.778	1.180
$\mu[u_{3nt}]$	1.556	0.044	<0.001	1.470	1.641
$\sigma]u_{3nt}]$	0.663	0.068	<0.001	0.530	0.796
			B. ϰ = 0.020 (Log-likelihood = −1872.674)		
α_4	1.123	0.244	<0.001	0.644	1.602
α_3	0.910	0.187	<0.001	0.544	1.275
γ	0.535	0.112	<0.001	0.315	0.754
φ	0.983	0.031	<0.001	0.922	1.044
$\mu[u_{4nt}]$	2.087	0.092	<0.001	1.906	2.268
$\sigma[u_{4nt}]$	0.763	0.093	<0.001	0.581	0.944
$\mu[u_{3nt}]$	1.486	0.039	<0.001	1.410	1.563
$\sigma]u_{3nt}]$	0.510	0.062	<0.001	0.389	0.631
			C. ϰ = 0.010 (Log-likelihood = −1880.838)		
α_4	0.953	0.180	<0.001	0.601	1.305
α_3	0.795	0.140	<0.001	0.520	1.070
γ	0.663	0.116	<0.001	0.435	0.890
φ	1.002	0.030	<0.001	0.942	1.062
$\mu[u_{4nt}]$	2.158	0.093	<0.001	1.976	2.340
$\sigma[u_{4nt}]$	0.876	0.090	<0.001	0.700	1.052
$\mu[u_{3nt}]$	1.527	0.040	<0.001	1.449	1.604
$\sigma]u_{3nt}]$	0.591	0.060	<0.001	0.473	0.709
			D. ϰ = 0.002 (Log-likelihood = −1893.346)		
α_4	0.917	0.174	<0.001	0.576	1.258
α_3	0.778	0.154	<0.001	0.476	1.080
γ	0.697	0.108	<0.001	0.485	0.908
φ	1.022	0.024	<0.001	0.974	1.069

$\mu[u_{4nt}]$	2.180	0.079	<0.001	2.026	2.335
$\sigma[u_{4nt}]$	0.907	0.065	<0.001	0.780	1.034
$\mu[u_{3nt}]$	1.542	0.041	<0.001	1.462	1.621
$\sigma]u_{3nt}]$	0.614	0.043	<0.001	0.529	0.699

Notes: The estimation sample includes 2,985 observations (63 subjects making 43 to 48 choices each) on three prize sets (M_{-1}, M_{-3} and M_{-4}) from the experiment reported by Harrison and Rutström [2008]. All standard errors have been adjusted for clustering at the subject level. The RP urn for utility levels u_{3nt} and u_{4nt} is based on McKay's [1934] bivariate gamma distribution. The assumed bivariate distribution implies that the marginal distribution of (u_{3nt} − 1) is univariate gamma with the shape parameter α_3 and the scale parameter γ, and that of (u_{4nt} − 1) is univariate gamma with the shape parameter ($\alpha_3 + \alpha_4$) and the scale parameter γ. μ[.] and σ[.] report the expected value and standard deviation of each utility level derived from the estimates of α_3, α_4 and γ.

Table B4: Partial RP-RDU with Non-Parametric Utility - Monte Carlo Results

Parameter	DGP	Bias	RMSE	Size
		A. Analytic ML		
α_4	0.830	−0.095	0.123	0.270
α_3	0.702	−0.075	0.106	0.229
γ	0.791	0.136	0.189	0.074
φ	1.011	−0.002	0.023	0.065
$\mu[u_{4nt}]$	2.212	0.031	0.064	0.052
$\sigma[u_{4nt}]$	0.979	0.093	0.133	0.087
$\mu[u_{3nt}]$	1.556	0.016	0.029	0.078
$\sigma]u_{3nt}]$	0.663	0.064	0.089	0.104
		B. $\varkappa$ = 0.020		
α_4	0.830	0.053	0.136	0.052
α_3	0.702	0.036	0.121	0.053
γ	0.791	−0.078	0.149	0.182
φ	1.011	−0.039	0.045	0.466
$\mu[u_{4nt}]$	2.212	−0.085	0.100	0.396
$\sigma[u_{4nt}]$	0.979	−0.085	0.129	0.220
$\mu[u_{3nt}]$	1.556	−0.043	0.049	0.499
$\sigma]u_{3nt}]$	0.663	−0.060	0.087	0.248
		C. $\varkappa$ = 0.010		
α_4	0.830	−0.060	0.114	0.129
α_3	0.702	−0.039	0.099	0.087
γ	0.791	0.058	0.149	0.030
φ	1.011	−0.017	0.028	0.148
$\mu[u_{4nt}]$	2.212	−0.019	0.058	0.106
$\sigma[u_{4nt}]$	0.979	0.025	0.103	0.041
$\mu[u_{3nt}]$	1.556	−0.004	0.024	0.075
$\sigma]u_{3nt}]$	0.663	0.020	0.068	0.040
		D. $\varkappa$ = 0.002		
α_4	0.830	−0.118	0.149	0.453
α_3	0.702	−0.087	0.121	0.443
γ	0.791	0.165	0.229	0.293
φ	1.011	−0.004	0.024	0.098
$\mu[u_{4nt}]$	2.212	0.031	0.068	0.120
$\sigma[u_{4nt}]$	0.979	0.109	0.156	0.250
$\mu[u_{3nt}]$	1.556	0.020	0.033	0.146

$\sigma[u_{3nt}]$	0.663	0.077	0.106	0.263

Notes: The results in each panel are based on 1,000 simulated data sets of 2,985 observations. Each simulated data set is identical to the empirical estimation sample in Table B3 except that actual choices made by subjects are replaced with simulated choices from the data generating process (DGP). RMSE reports the root mean squared error of relevant parameter estimates. Size reports the rejection frequency of the Wald test of the true null; the asymptotic significance level is 5% and the test statistic is constructed using standard errors clustered at the subject level.

Table B5: Partial RP-RDU with Non-Parametric Utility - Full Sample Empirical Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
			B. ϰ = 0.019 *(Log-likelihood = −2358.552)*		
α_4	0.899	0.187	<0.001	0.534	1.265
α_3	0.859	0.174	<0.001	0.518	1.201
γ	0.585	0.124	<0.001	0.342	0.827
φ	0.961	0.031	<0.001	0.900	1.023
$\mu[u_{4nt}]$	2.029	0.088	<0.001	1.857	2.200
$\sigma[u_{4nt}]$	0.776	0.097	<0.001	0.585	0.966
$\mu[u_{3nt}]$	1.503	0.042	<0.001	1.420	1.585
$\sigma]u_{3nt}]$	0.542	0.068	<0.001	0.409	0.675
			C. ϰ = 0.010 *(Log-likelihood = −2367.041)*		
α_4	0.792	0.148	<0.001	0.503	1.082
α_3	0.762	0.140	<0.001	0.488	1.035
γ	0.705	0.135	<0.001	0.442	0.969
φ	0.978	0.032	<0.001	0.916	1.040
$\mu[u_{4nt}]$	2.096	0.091	<0.001	1.917	2.275
$\sigma[u_{4nt}]$	0.879	0.102	<0.001	0.680	1.078
$\mu[u_{3nt}]$	1.537	0.044	<0.001	1.451	1.623
$\sigma]u_{3nt}]$	0.616	0.071	<0.001	0.477	0.754
			D. ϰ = 0.002 *(Log-likelihood = −2382.332)*		
α_4	0.708	0.063	<0.001	0.584	0.833
α_3	0.655	0.055	<0.001	0.546	0.763
γ	0.851	0.132	<0.001	0.591	1.110
φ	0.991	0.028	<0.001	0.936	1.047
$\mu[u_{4nt}]$	2.160	0.111	<0.001	1.941	2.378
$\sigma[u_{4nt}]$	0.993	0.122	<0.001	0.754	1.232
$\mu[u_{3nt}]$	1.557	0.043	<0.001	1.472	1.642
$\sigma]u_{3nt}]$	0.688	0.080	<0.001	0.532	0.844

Notes: The estimation sample includes 3,736 observations (63 subjects making 54 to 60 choices each) on all four prize sets from the experiment reported by Harrison and Rutström [2008]. Otherwise this model specification is identical to the partial RP-RDU model reported in Table B3.

Table B6: Partial RP-RDU with Non-Parametric Utility - Full Sample Monte Carlo Results

Parameter	DGP	Bias	RMSE	Size
		A. $\varkappa$ = 0.020		
α_4	0.792	0.112	0.153	0.143
α_3	0.762	−0.027	0.103	0.079
γ	0.705	−0.019	0.108	0.098
φ	0.978	−0.011	0.021	0.110
$\mu[u_{4nt}]$	2.096	0.008	0.045	0.049
$\sigma[u_{4nt}]$	0.879	−0.011	0.082	0.076
$\mu[u_{3nt}]$	1.537	−0.043	0.048	0.595
$\sigma]u_{3nt}]$	0.616	−0.035	0.062	0.168
		B. $\varkappa$ = 0.010		
α_4	0.792	0.011	0.085	0.050
α_3	0.762	−0.091	0.123	0.256
γ	0.705	0.100	0.152	0.053
φ	0.978	0.007	0.020	0.056
$\mu[u_{4nt}]$	2.096	0.073	0.087	0.284
$\sigma[u_{4nt}]$	0.879	0.090	0.123	0.096
$\mu[u_{3nt}]$	1.537	−0.006	0.021	0.074
$\sigma]u_{3nt}]$	0.616	0.037	0.066	0.050
		C. $\varkappa$ = 0.002		
α_4	0.792	−0.040	0.090	0.210
α_3	0.762	−0.125	0.149	0.549
γ	0.705	0.179	0.224	0.362
φ	0.978	0.016	0.027	0.171
$\mu[u_{4nt}]$	2.096	0.112	0.125	0.570
$\sigma[u_{4nt}]$	0.879	0.153	0.181	0.431
$\mu[u_{3nt}]$	1.537	0.016	0.028	0.143
$\sigma]u_{3nt}]$	0.616	0.082	0.103	0.331

Notes: The results in each panel are based on 1,000 simulated data sets of 3,736 observations. Each simulated data set is identical to the empirical estimation sample in Table B5 except that actual choices made by subjects are replaced with simulated choices from the data generating process (DGP). RMSE reports the root mean squared error of relevant parameter estimates. Size reports the rejection frequency of the Wald test of the true null; the asymptotic significance level is 5% and the test statistic is constructed using standard errors clustered at the subject level.

Table B7: RP-RDU with Non-Parametric Utility - Empirical Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
			A. $\varkappa$ = 0.019		
			(Log-likelihood = −2354.719)		
α_4	1.155	0.276	<0.001	0.614	1.697
α_3	1.112	0.253	<0.001	0.616	1.608
γ	0.392	0.101	<0.001	0.194	0.590
m_φ	−0.065	0.039	0.098	−0.143	0.012
s_φ	0.404	0.102	<0.001	0.203	0.605
$\mu[u_{4nt}]$	1.888	0.097	<0.001	1.697	2.079
$\sigma[u_{4nt}]$	0.590	0.095	<0.001	0.404	0.776
$\mu[u_{3nt}]$	1.435	0.048	<0.001	1.341	1.530
$\sigma]u_{3nt}]$	0.413	0.067	<0.001	0.281	0.545
$\mu[\varphi_{nt}]$	1.017	0.061	<0.001	0.897	1.136
$\sigma[\varphi_{nt}]$	0.428	0.138	<0.001	0.157	0.699
			B. $\varkappa$ = 0.010		
			(Log-likelihood = −2359.496)		
α_4	1.033	0.240	<0.001	0.563	1.503
α_3	1.001	0.222	<0.001	0.566	1.435
γ	0.449	0.100	<0.001	0.253	0.644
m_φ	−0.062	0.043	0.155	−0.146	0.023
s_φ	0.493	0.132	<0.001	0.235	0.751
$\mu[u_{4nt}]$	1.912	0.083	<0.001	1.750	2.074
$\sigma[u_{4nt}]$	0.640	0.082	<0.001	0.479	0.800
$\mu[u_{3nt}]$	1.449	0.042	<0.001	1.367	1.531
$\sigma]u_{3nt}]$	0.449	0.058	<0.001	0.334	0.563
$\mu[\varphi_{nt}]$	1.062	0.083	<0.001	0.899	1.224
$\sigma[\varphi_{nt}]$	0.556	0.205	<0.001	0.155	0.958
			C. $\varkappa$ = 0.002		
			(Log-likelihood = −2376.056)		
α_4	0.809	0.100	<0.001	0.614	1.004
α_3	0.767	0.129	<0.001	0.515	1.019
γ	0.622	0.150	<0.001	0.327	0.917
m_φ	−0.057	0.038	0.141	−0.132	0.019
s_φ	0.489	0.037	<0.001	0.415	0.562
$\mu[u_{4nt}]$	1.980	0.109	<0.001	1.767	2.193
$\sigma[u_{4nt}]$	0.781	0.136	<0.001	0.514	1.047
$\mu[u_{3nt}]$	1.477	0.043	<0.001	1.393	1.561
$\sigma]u_{3nt}]$	0.545	0.088	<0.001	0.372	0.718
$\mu[\varphi_{nt}]$	1.065	0.030	<0.001	1.005	1.125

$\sigma[\varphi_{nt}]$	0.553	0.045	<0.001	0.464	0.642

Notes: The estimation sample includes 3,736 observations (63 subjects making 54 to 60 choices each) on all four prize sets from the experiment reported by Harrison and Rutström [2008]. All standard errors have been adjusted for clustering at the subject level. The RP urn for utility levels u_{3nt} and u_{4nt} is based on McKay's [1934] bivariate gamma distribution. The assumed bivariate distribution implies that the marginal distribution of $(u_{3nt} - 1)$ is univariate gamma with the shape parameter α_3 and the scale parameter γ, and that of $(u_{4nt} - 1)$ is univariate gamma with the shape parameter $(\alpha_3 + \alpha_4)$ and the scale parameter γ. The RP urn for the parameter φ_{nt} that determines the shape of the probability weighting function is a log-normal distribution: $\ln(\varphi_{nt}) \sim N(m_\varphi, s_\varphi^2)$. $\mu[.]$ and $\sigma[.]$ report the expected value and standard deviation of each utility level derived from the estimates of α_3, α_4 and γ, or those of the PWF parameter derived from the estimates of m_φ and s_φ.

Table B8: RP-RDU with Non-Parametric Utility - Monte Carlo Results

Parameter	DGP	Bias	RMSE	Size
		A. $\varkappa$ = 0.019		
		(Log-likelihood = −2354.719)		
α_4	1.033	−0.003	0.173	0.070
α_3	1.001	−0.100	0.200	0.171
γ	0.449	0.028	0.110	0.049
m_φ	−0.062	−0.020	0.037	0.074
s_φ	0.493	0.070	0.158	0.065
$\mu[u_{4nt}]$	1.912	−0.025	0.071	0.086
$\sigma[u_{4nt}]$	0.640	0.008	0.091	0.054
$\mu[u_{3nt}]$	1.449	−0.036	0.047	0.234
$\sigma]u_{3nt}]$	0.449	−0.007	0.060	0.083
$\mu[\varphi_{nt}]$	1.062	0.032	0.096	0.062
$\sigma[\varphi_{nt}]$	0.556	0.138	0.307	0.025
		B. $\varkappa$ = 0.010		
		(Log-likelihood = −2359.496)		
α_4	1.033	−0.154	0.205	0.238
α_3	1.001	−0.212	0.255	0.371
γ	0.449	0.111	0.160	0.070
m_φ	−0.062	−0.026	0.044	0.099
s_φ	0.493	0.211	0.261	0.286
$\mu[u_{4nt}]$	1.912	−0.007	0.065	0.092
$\sigma[u_{4nt}]$	0.640	0.070	0.114	0.085
$\mu[u_{3nt}]$	1.449	−0.022	0.037	0.144
$\sigma]u_{3nt}]$	0.449	0.038	0.070	0.071
$\mu[\varphi_{nt}]$	1.062	0.133	0.183	0.058
$\sigma[\varphi_{nt}]$	0.556	0.449	0.596	0.090
		C. $\varkappa$ = 0.002		
		(Log-likelihood = −2376.056*)*		
α_4	1.033	−0.257	0.289	0.657
α_3	1.001	−0.293	0.324	0.714
γ	0.449	0.224	0.275	0.524
m_φ	−0.062	−0.021	0.048	0.333
s_φ	0.493	0.240	0.290	0.791
$\mu[u_{4nt}]$	1.912	0.046	0.082	0.373
$\sigma[u_{4nt}]$	0.640	0.159	0.196	0.544
$\mu[u_{3nt}]$	1.449	0.007	0.031	0.242
$\sigma]u_{3nt}]$	0.449	0.102	0.126	0.516
$\mu[\varphi_{nt}]$	1.062	0.166	0.218	0.606

$\sigma[\varphi_{nt}]$	0.556	0.538	0.699	0.746

Notes: The results in each panel are based on 926 simulated data sets of 3,736 observations. Each simulated data set is identical to the empirical estimation sample in Table B7 except that actual choices made by subjects are replaced with simulated choices from the data generating process (DGP). RMSE reports the root mean squared error of relevant parameter estimates. Size reports the rejection frequency of the Wald test of the true null; the asymptotic significance level is 5% and the test statistic is constructed using standard errors clustered at the subject level.

Table B9: RP-RDU with CRRA Utility - Empirical Estimates

Parameter	Estimate	Standard Error	p-value	95% Confidence Interval	
			A. $\varkappa = 0.037$		
			(Log-likelihood = −7351.484)		
μ_ω	0.702	0.067	<0.001	0.571	0.834
σ_ω	0.585	0.064	<0.001	0.460	0.710
m_φ	0.409	0.106	<0.001	0.201	0.617
s_φ	0.701	0.062	<0.001	0.579	0.823
$\mu[\varphi_{nt}]$	1.925	0.197	<0.001	1.538	2.311
$\sigma[\varphi_{nt}]$	1.533	0.248	<0.001	1.046	2.019
			B. $\varkappa = 0.015$		
			(Log-likelihood = −7383.696)		
μ_ω	0.658	0.064	<0.001	0.532	0.784
σ_ω	0.586	0.051	<0.001	0.486	0.686
m_φ	0.267	0.097	0.006	0.078	0.457
s_φ	0.871	0.054	<0.001	0.765	0.978
$\mu[\varphi_{nt}]$	1.910	0.210	<0.001	1.499	2.321
$\sigma[\varphi_{nt}]$	2.036	0.348	<0.001	1.354	2.719
			C. $\varkappa = 0.007$		
			(Log-likelihood = −7397.092)		
μ_ω	0.636	0.076	<0.001	0.487	0.784
σ_ω	0.571	0.042	<0.001	0.490	0.653
m_φ	0.197	0.113	0.082	-0.025	0.418
s_φ	0.906	0.052	<0.001	0.805	1.008
$\mu[\varphi_{nt}]$	1.835	0.234	<0.001	1.376	2.295
$\sigma[\varphi_{nt}]$	2.071	0.379	<0.001	1.328	2.814

Notes: The estimation sample includes 14,868 observations (413 subjects making 36 choices) on non-dominated choice tasks from the experiment reported by Andersen, Harrison, Lau and Rutström [2014]. All standard errors have been adjusted for clustering at the subject level. The RP urn for the coefficient of RRA ω_{nt} is logistic density $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ with the mean parameter μ_ω and the scale parameter σ_ω. The RP urn for the parameter φ_{nt} that determines the shape of the probability weighting function is a log-normal distribution: $\ln(\varphi_{nt}) \sim N(m_\varphi, s_\varphi^2)$. $\mu[.]$ and $\sigma[.]$ report the expected value and standard deviation of the PWF parameter derived from the estimates of m_φ and s_φ.

Table B10: RP-RDU with CRRA Utility - Monte Carlo Results

Parameter	DGP	Bias	RMSE	Size
		A. $\varkappa = 0.037$		
μ_ω	0.658	0.052	0.065	0.248
σ_ω	0.586	0.006	0.040	0.045
m_φ	0.267	0.163	0.175	0.690
s_φ	0.871	−0.139	0.164	0.386
$\mu[\varphi_{nt}]$	1.910	0.116	0.199	0.057
$\sigma[\varphi_{nt}]$	2.036	−0.304	0.478	0.225
		B. $\varkappa = 0.015$		
μ_ω	0.658	0.025	0.062	0.058
σ_ω	0.586	0.030	0.066	0.076
m_φ	0.267	0.063	0.114	0.094
s_φ	0.871	−0.091	0.129	0.292
$\mu[\varphi_{nt}]$	1.910	−0.006	0.182	0.058
$\sigma[\varphi_{nt}]$	2.036	−0.270	0.462	0.237
		C. $\varkappa = 0.007$		
μ_ω	0.658	0.017	0.063	0.079
σ_ω	0.586	0.027	0.068	0.096
m_φ	0.267	0.033	0.109	0.082
s_φ	0.871	−0.068	0.117	0.420
$\mu[\varphi_{nt}]$	1.910	−0.027	0.207	0.093
$\sigma[\varphi_{nt}]$	2.036	−0.216	0.469	0.299

Notes: The results in each panel are based on 1,000 simulated data sets of 14,868 observations. Each simulated data set is identical to the empirical estimation sample in Table B9 except that actual choices made by subjects are replaced with simulated choices from the data generating process (DGP). RMSE reports the root mean squared error of relevant parameter estimates. Size reports the rejection frequency of the Wald test of the true null; the asymptotic significance level is 5% and the test statistic is constructed using standard errors clustered at the subject level.

Table B11: RP-Tremble-EUT with CRRA Utility - Empirical Estimates

Parameter	Estimate	Standard Error	p-value	95% Confidence Interval	
			A. Analytic ML (Log-likelihood = −7784.824)		
μ_ω	0.523	0.029	<0.001	0.466	0.580
σ_ω	0.337	0.016	<0.001	0.305	0.369
τ	0.162	0.016	<0.001	0.131	0.193
			B. $\varkappa$ = 0.036 (Log-likelihood = −7777.816)		
μ_ω	0.517	0.028	<0.001	0.462	0.572
σ_ω	0.281	0.019	<0.001	0.245	0.318
τ	0.144	0.016	<0.001	0.112	0.176
			C. $\varkappa$ = 0.014 (Log-likelihood = −7784.791)		
μ_ω	0.522	0.029	<0.001	0.465	0.579
σ_ω	0.332	0.017	<0.001	0.298	0.366
τ	0.156	0.016	<0.001	0.124	0.187
			D. $\varkappa$ = 0.007 (Log-likelihood = −7785.653)		
μ_ω	0.524	0.029	<0.001	0.467	0.581
σ_ω	0.341	0.015	<0.001	0.312	0.370
τ	0.158	0.016	<0.001	0.127	0.189

Notes: The estimation sample includes 16,520 observations (413 subjects making 40 choices) on all choice tasks from the experiment reported by Andersen, Harrison, Lau and Rutström [2014]. All standard errors have been adjusted for clustering at the subject level. The RP urn for the coefficient of RRA, ω_{nt}, is logistic density $f(\omega_{nt} | \mu_\omega, \sigma_\omega)$ with mean parameter μ_ω and scale parameter σ_ω. The tremble parameter τ measures the probability that the subject makes an arbitrary choice instead of applying RP-EUT decision criteria.

Table B12: RP-Tremble-EUT with CRRA Utility - Monte Carlo Results

Parameter	DGP	Bias	RMSE	Size
		A. Analytic ML		
μ_ω	0.523	$<\lvert -0.001 \rvert$	0.010	0.064
σ_ω	0.337	<0.001	0.012	0.058
τ	0.162	$<\lvert -0.001 \rvert$	0.009	0.055
		B. $\varkappa$ = 0.036		
μ_ω	0.523	−0.011	0.015	0.202
σ_ω	0.337	−0.038	0.040	0.832
τ	0.162	−0.022	0.023	0.678
		C. $\varkappa$ = 0.014		
μ_ω	0.523	−0.004	0.011	0.069
σ_ω	0.337	−0.004	0.013	0.076
τ	0.162	−0.006	0.011	0.106
		D. $\varkappa$ = 0.007		
μ_ω	0.523	−0.001	0.010	0.062
σ_ω	0.337	−0.000	0.012	0.059
τ	0.162	−0.002	0.009	0.060

Notes: The results in each panel are based on 1,000 simulated data sets of 16,520 observations. Each simulated data set is identical to the empirical estimation sample in Table B11 except that actual choices made by subjects are replaced with simulated choices from the data generating process (DGP). A bias of <0.001 refers to a value in (0.000, 0.001), and that of $<\lvert -0.001 \rvert$ to a value in (−0.001, 0.000). RMSE reports the root mean squared error of relevant parameter estimates. Size reports the rejection frequency of the Wald test of the true null; the asymptotic significance level is 5% and the test statistic is constructed using standard errors clustered at the subject level.

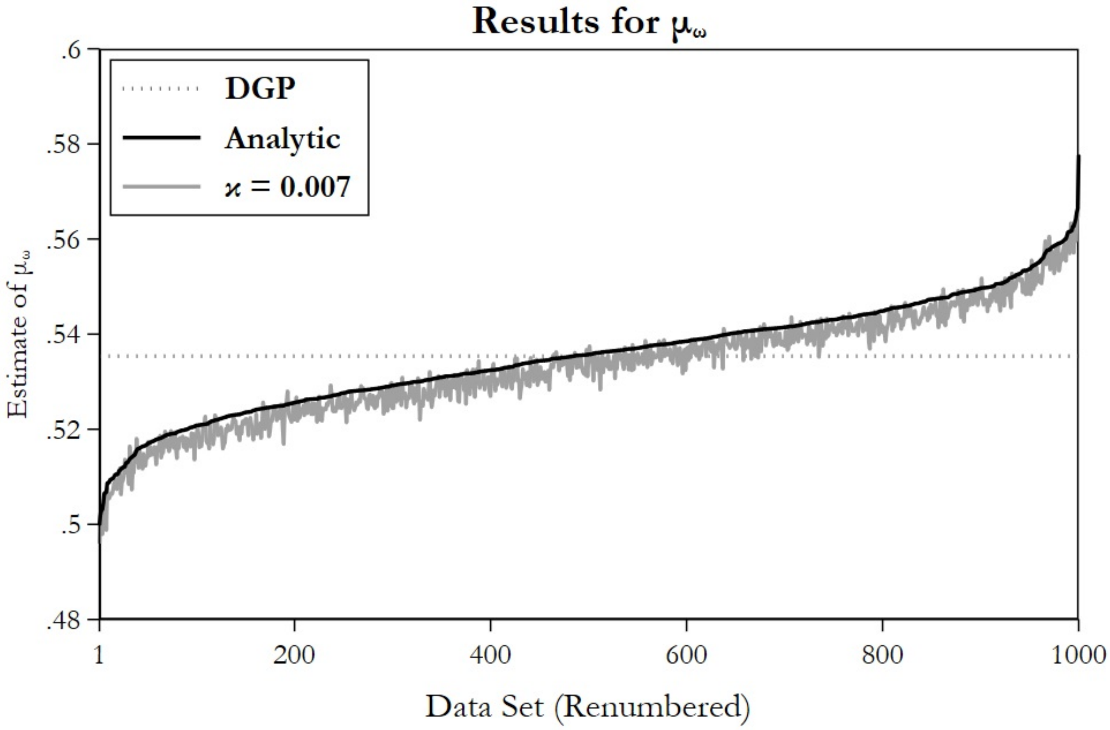
Table B13: Random Coefficient RP-RDU with CRRA Utility - Empirical Estimates

Parameter	Estimate	Standard Error	*p*-value	95% Confidence Interval	
			A. ϰ = 0.037 *(Log-likelihood = −-5267.919)*		
$E[\mu_{\omega n}]$	0.565	0.040	<0.001	0.486	0.644
$E[m_{\varphi n}]$	0.102	0.074	0.171	−0.044	0.247
$E[\sigma^{UN}_{\omega n}]$	−0.197	0.025	0.000	−0.246	−0.148
$E[s^{UN}_{\varphi n}]$	0.033	0.044	0.455	−0.053	0.119
$SD[\mu_{\omega n}]$	0.691	0.050	<0.001	0.593	0.788
$SD[m_{\varphi n}]$	1.021	0.071	<0.001	0.882	1.159
$SD[\sigma^{UN}_{\omega n}]$	0.250	0.026	<0.001	0.199	0.301
$SD[s^{UN}_{\varphi n}]$	0.061	0.073	0.406	−0.083	0.205
			B. ϰ = 0.015 *(Log-likelihood = −5228.946)*		
$E[\mu_{\omega n}]$	0.571	0.048	<0.001	0.476	0.665
$E[m_{\varphi n}]$	0.026	0.088	0.768	−0.147	0.199
$E[\sigma^{UN}_{\omega n}]$	−0.262	0.022	<0.001	−0.305	−0.219
$E[s^{UN}_{\varphi n}]$	−0.039	0.111	0.727	−0.256	0.179
$SD[\mu_{\omega n}]$	0.722	0.048	<0.001	0.628	0.816
$SD[m_{\varphi n}]$	1.114	0.062	<0.001	0.992	1.236
$SD[\sigma^{UN}_{\omega n}]$	0.311	0.036	<0.001	0.240	0.382
$SD[s^{UN}_{\varphi n}]$	0.261	0.036	<0.001	0.191	0.331
			C. ϰ = 0.007 *(Log-likelihood = −5235.721)*		
$E[\mu_{\omega n}]$	0.558	0.042	<0.001	0.476	0.640
$E[m_{\varphi n}]$	0.038	0.087	0.665	−0.132	0.207
$E[\sigma^{UN}_{\omega n}]$	−0.275	0.024	<0.001	−0.322	−0.227
$E[s^{UN}_{\varphi n}]$	0.080	0.044	0.069	−0.006	0.166
$SD[\mu_{\omega n}]$	0.765	0.063	<0.001	0.642	0.889
$SD[m_{\varphi n}]$	1.164	0.063	<0.001	1.041	1.287
$SD[\sigma^{UN}_{\omega n}]$	0.347	0.030	<0.001	0.288	0.406
$SD[s^{UN}_{\varphi n}]$	0.314	0.027	<0.001	0.261	0.367

Notes: The estimation sample includes 14,868 observations (413 subjects making 36 choices) on non-dominated choice tasks from the experiment reported by Andersen, Harrison, Lau and Rutström [2014]. All standard errors have been adjusted for clustering at the subject level. The RP urn for the coefficient of RRA ω_{nt} is logistic density $f(\omega_{nt}|\mu_{\omega n}, \sigma_{\omega n})$ with the mean parameter $\mu_{\omega n}$ and the scale parameter $\sigma_{\omega n}$. The RP urn for the parameter φ_{nt} that determines the shape of the probability weighting function is a log-normal distribution: $\ln(\varphi_{nt}) \sim N(m_{\varphi n}, s_{\varphi n}^2)$. $\mu[.]$ and $\sigma[.]$ report the expected value and standard deviation of the PWF parameter derived from the estimates of m_φ and s_φ. In the random coefficient model $\mu_{\omega n}$ and $m_{\varphi n}$ are assumed to follow normal distributions with between-subject means $E[\mu_{\omega n}]$ and $E[m_{\varphi n}]$ and standard deviations $SD[\mu_{\omega n}]$ and $SD[m_{\varphi n}]$. $\sigma_{\omega n}$ and $s_{\varphi n}$ are assumed to follow folded-normal distributions so that $\sigma_{\omega n} = |\sigma^{UN}_{\omega n}|$ and $s_{\varphi n} = |s^{UN}_{\varphi n}|$, where $\sigma^{UN}_{\omega n} \sim N(E[\sigma^{UN}_{\omega n}], SD[\sigma^{UN}_{\omega n}]^2)$ and $s^{UN}_{\varphi n} \sim N(E[s^{UN}_{\varphi n}], SD[s^{UN}_{\varphi n}]^2)$.

Figure B1: RP-EUT with CRRA Utility

Estimates in Monte Carlo Experiment



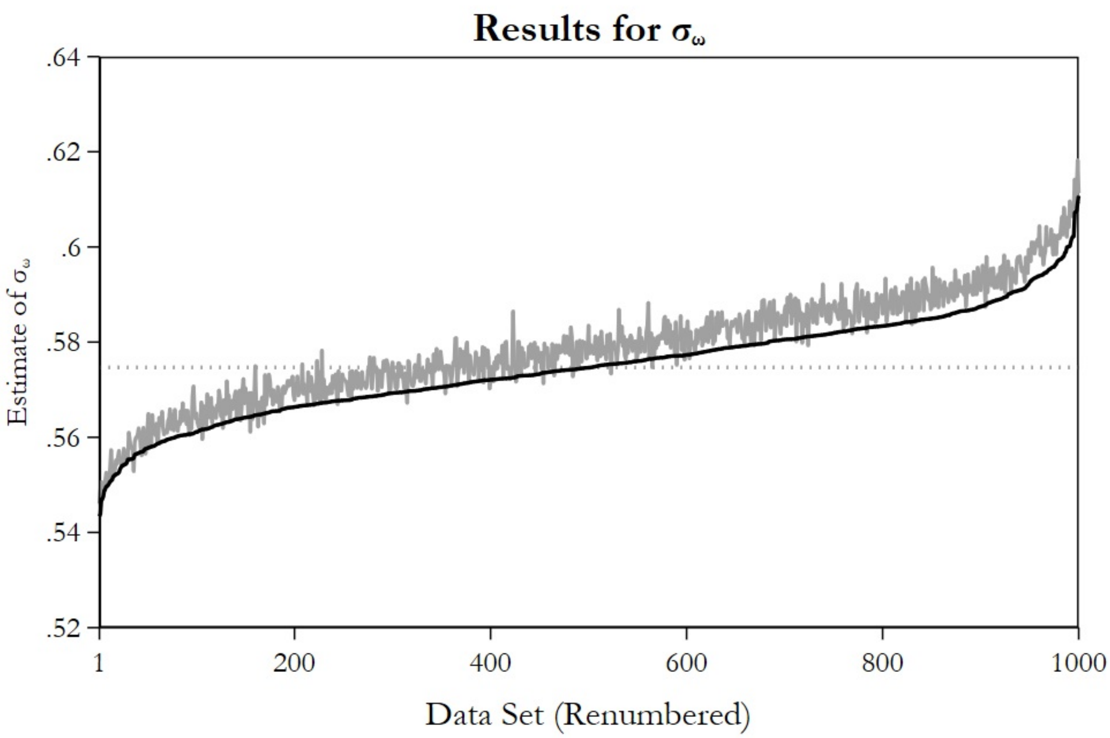


Figure B2: Partial RP-RDU with NP Utility

Estimates in Monte Carlo Experiment

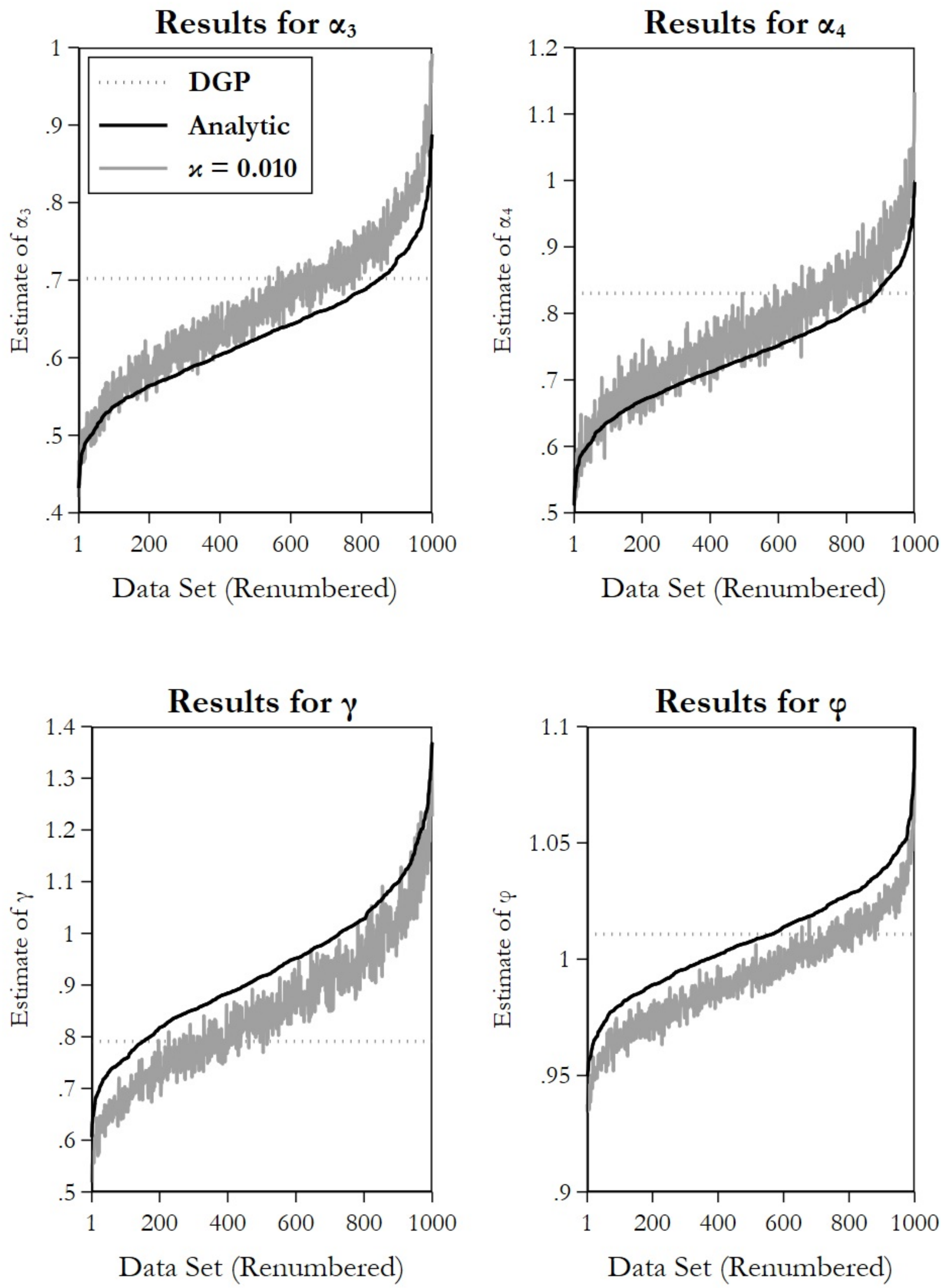
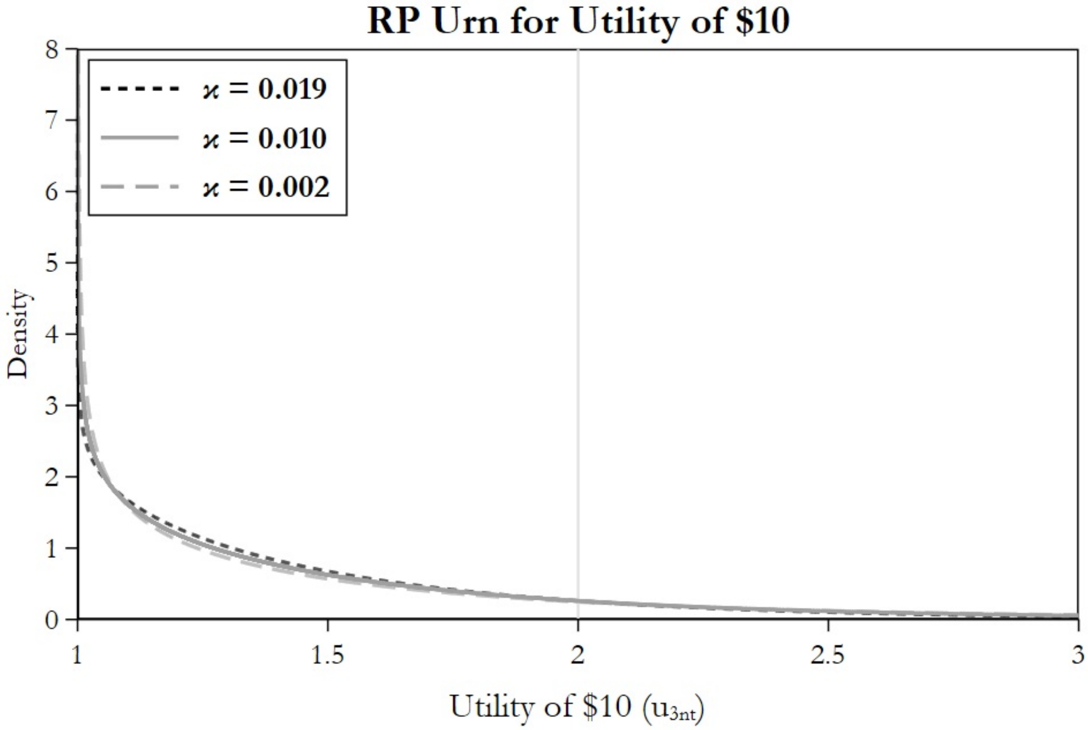


Figure B3: Partial RP-RDU with NP Utility

Representative Agent's RP Urns



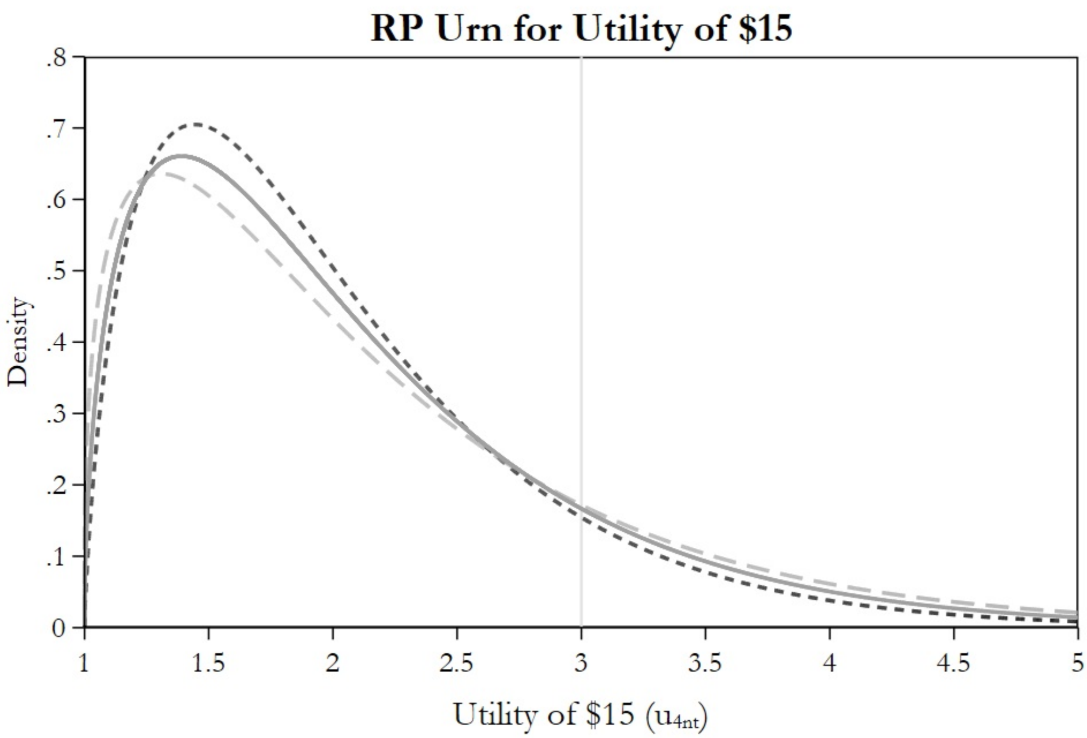


Figure B4: Partial RP-RDU with NP Utility

Estimates in Monte Carlo Experiment

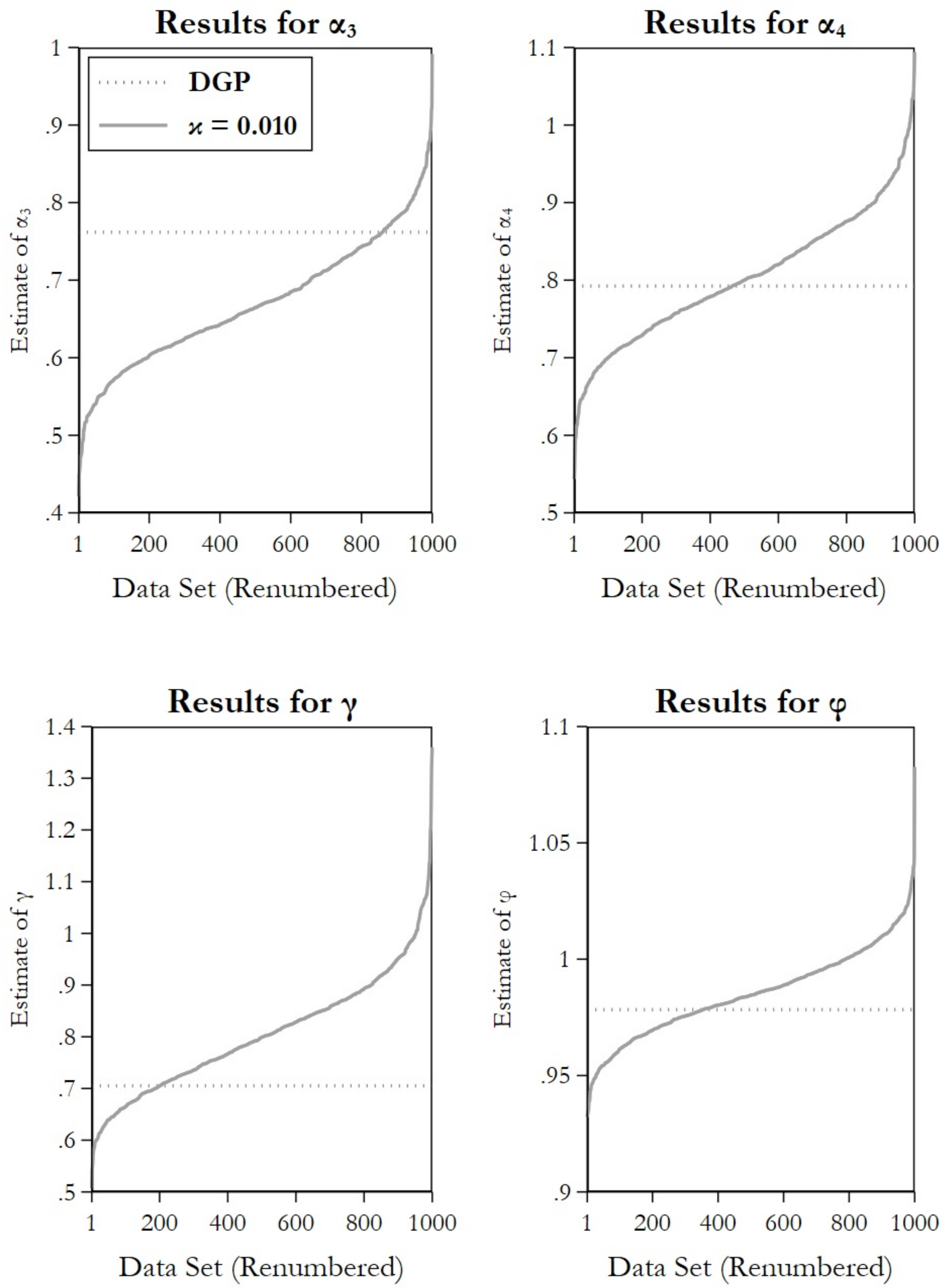


Figure B5: RP-RDU with NP Utility

Representative Agent's RP Urns

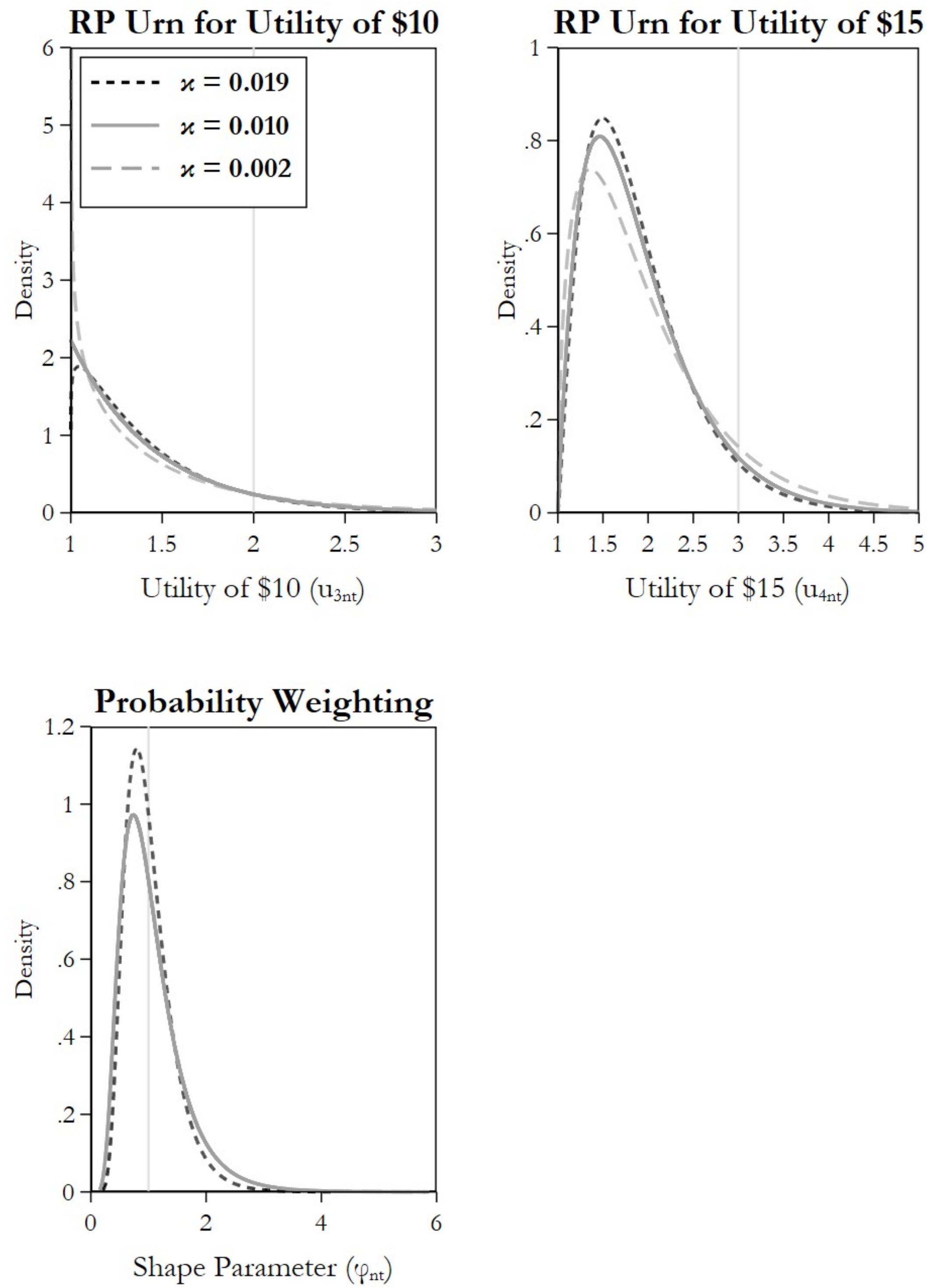


Figure B6: RP-RDU with NP Utility

Estimates in Monte Carlo Experiment

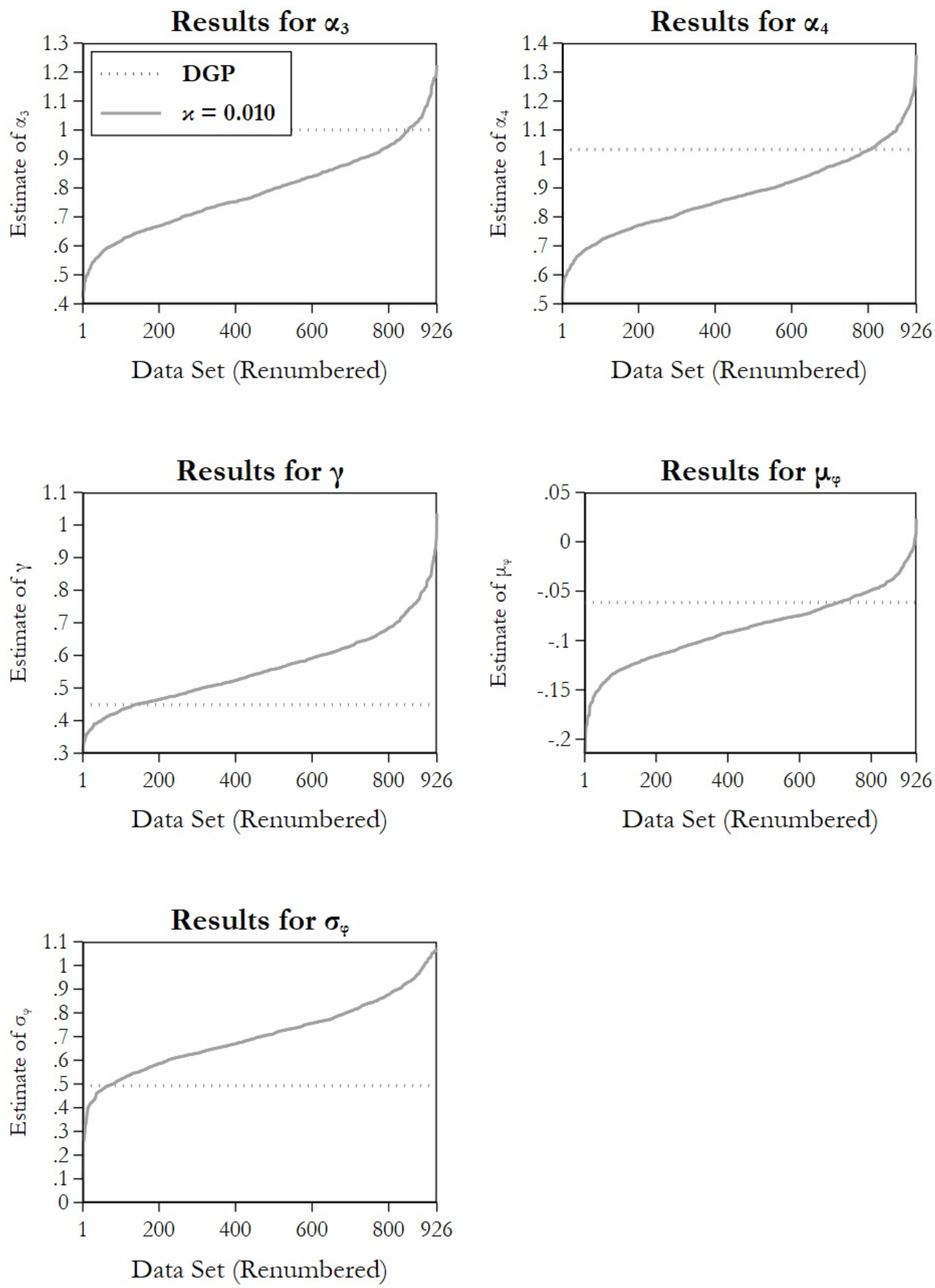


Figure B7: RP-RDU with CRRA Utility

Representative Agent's RP Urns

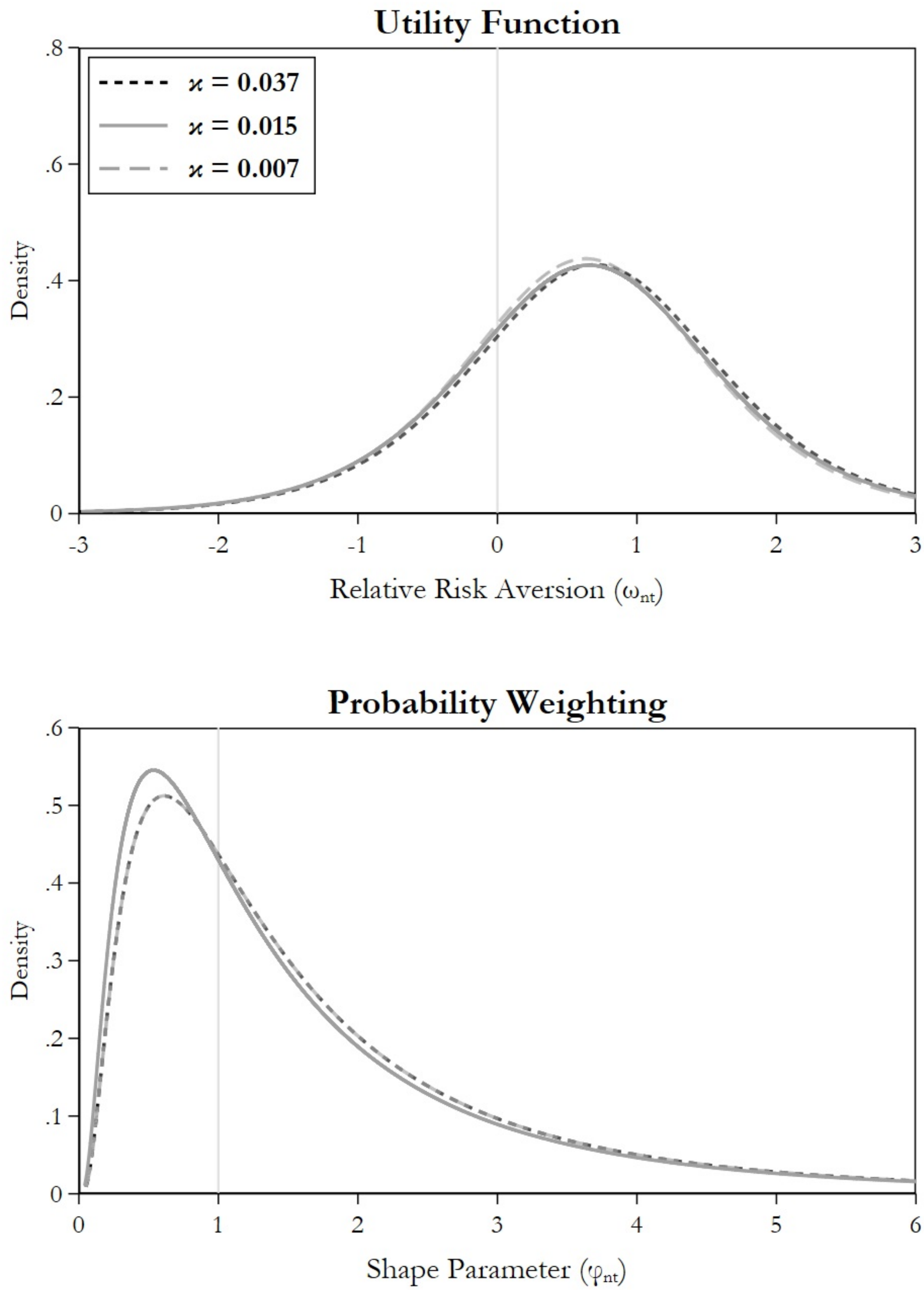


Figure B8: RP-RDU with CRRA Utility

Estimates in Monte Carlo Experiment

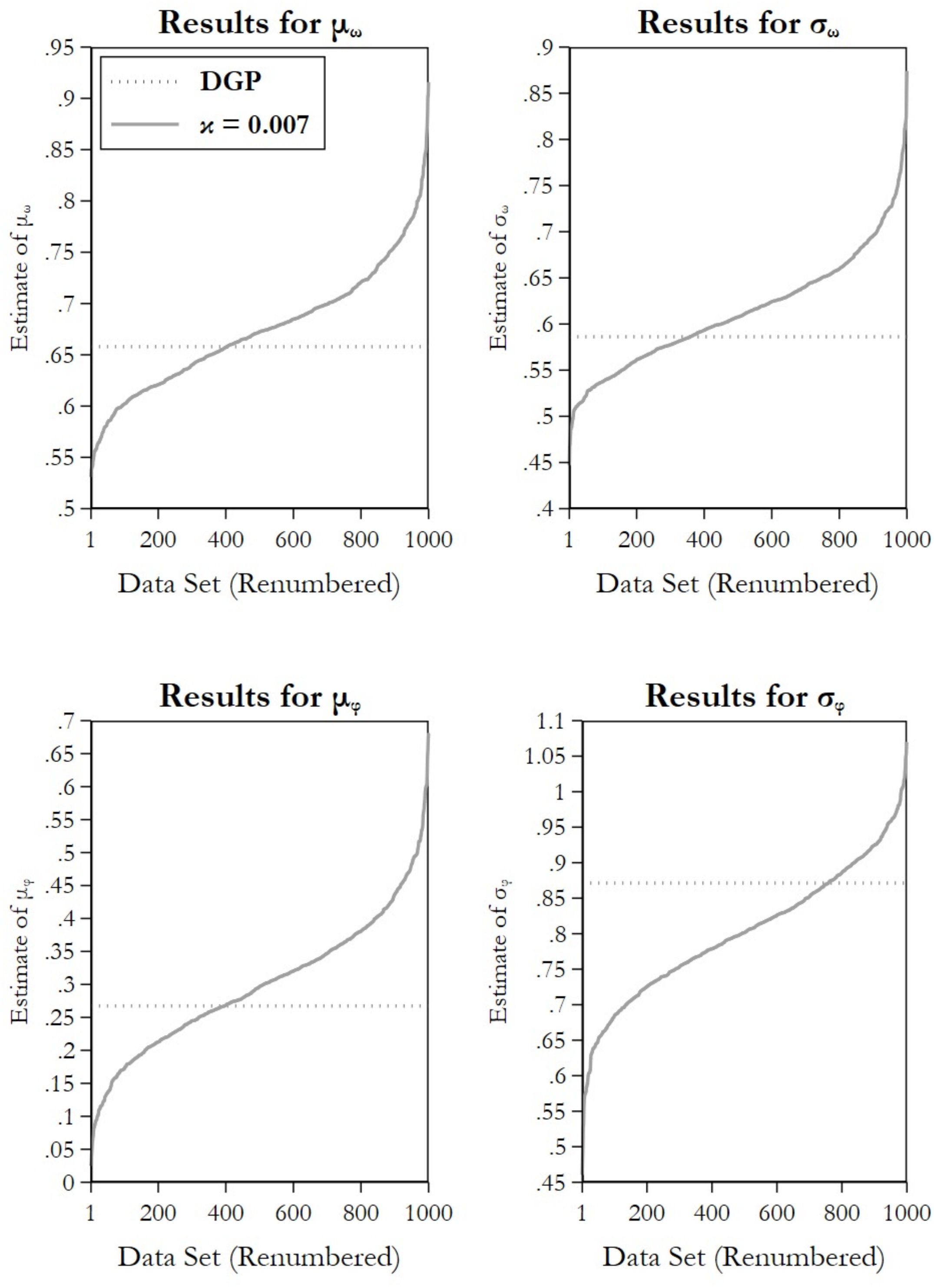


Figure B9: RP-Tremble-EUT with CRRA Utility

Estimates in Monte Carlo Experiment