Constant Discounting, Temporal Instability and Dynamic Inconsistency in Denmark: A Longitudinal Field Experiment

by

Glenn W. Harrison, Morten I. Lau and Hong Il Yoo[†]

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Abstract. Claims that individuals have dynamically inconsistent preferences are usually made by studying individual discount rates over different time delays, but where those discount rates are elicited at a single point in time. However, to test dynamic inconsistency one has to know if the same subject has a different discounting function *at a later point in time*. We evaluate data from a longitudinal field experiment undertaken with a nationally representative sample of the adult Danish population. We cannot reject the hypothesis of constant discounting at the population level, but we reject the hypotheses of temporal stability and dynamic consistency.

Keywords: Time Preferences, Dynamic Consistency, Longitudinal Experiments, Sample Selection and Attrition
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1. Introduction

Dynamic inconsistency is often cited as a behavioral trait, which highlights the importance of considering alternative formulations of intertemporal choice behavior that do not rely on constant discount rates. A simple instance of dynamic inconsistency, also known as time inconsistency, arises when the decision maker prefers a larger, later payment to a smaller, sooner payment when both payments are delayed, but prefers the smaller, sooner payment to the larger, later payment once enough time has passed to make the sooner payment immediately accessible. It is possible to attribute this type of preference reversal to genuine shifts in the decision maker's preferences between decision dates, but it is also possible to find theoretical explanations that do *not* require temporal instability in preferences. Prominent examples of the latter approach are models of present biased preferences, in which discount rates are declining over longer time delays between payments.

Inferences on dynamic inconsistency are usually made by studying individual discount rates over different horizons, but where those discount rates are elicited at a single point in time.[1] Thus individuals might be prompted to reveal their discount rate over a 1-month period starting at some reference point in time, and then reveal their discount rate over a 24-month period starting from the same reference point.[2] If the elicited discount rates vary over different time horizons, the individual is typically claimed to have preferences that imply dynamic inconsistency, by holding and acting on preferences at one point in time that contradict the preferences and decisions of the same individual at a later date. However, evaluation of dynamic inconsistency requires a longitudinal design that elicits the same individual's choices at two different points in time. While cross-sectional discrepancy between the two sets of choices may support the hypothesis of non-constant discounting, one cannot infer dynamic inconsistency from these choices without invoking the assumption of temporal stability in time preferences. The need to distinguish between the notions of dynamic consistency, constant discounting and temporal stability has been well-recognized in the literature on time preferences,[3] and Halevy [2015] provides a particularly clear statement of the theoretical interplay between the three.[4] Despite the

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[1] See Frederick, Loewenstein and O'Donoghue [2002] and Cohen, Ericson, Laibson and White [2020] for literature reviews on the measurement and elicitation of time preferences.

[2] An alternative approach in the literature is to vary the time delay to the sooner payment and keep the time period between the sooner and later payments constant, see Halevy [2015] for an example.

[3] For example, see Bommier [2006; p.1236] and Meier and Sprenger [2015; p.273]).

importance of temporal stability for inferences about dynamic consistency, there is very little data available from longitudinal experiments or other primary sources to *directly* test these properties of intertemporal preferences, and none that reflects a nationally representative population.[5]

We evaluate and test the hypotheses of constant discounting, temporal stability and dynamic consistency using data from a longitudinal field experiment with a *nationally representative sample of the adult Danish population*. The experiment is designed to elicit discount rates for assets of different time delays at different decision dates. Our econometric approach is structural in the sense that we directly estimate latent preference parameters that characterize a potentially non-constant discounting function, and use the estimated coefficients to draw statistical inferences. Estimating structural models and undertaking formal hypothesis tests with respect to underlying latent primitives raises an important point about statistical tests of the underlying theory. It is possible that two of the three properties each *individually* hold in a statistical sense, but that they do not *jointly* hold statistically. We demonstrate an instance of this issue as a special case of our analysis.

We model time preferences using a quasi-hyperbolic (also known as "β-δ") discounting function, which nests constant and non-constant discounting depending on the value of the present bias parameter β. We evaluate constant discounting by estimating the present bias parameter from choices in the initial and repeat experiments, respectively, and we evaluate temporal stability by comparing the estimated discounting functions from the initial and repeat experiments. To evaluate dynamic consistency we consider choices between two payments delivered at or after the repeat experiment. We then ask whether the discounting function estimated from the initial experiment predicts the same choice as the discounting function estimated from the repeat experiment. The degree of present bias in the discounting function from the initial experiment is therefore irrelevant to our evaluation of dynamic consistency, since neither payment option is immediate from the perspective of the initial experiment. However, the present bias parameter is directly relevant to our evaluation of temporal stability, which requires that the elicited discounting functions from the initial and repeat experiments display the same degree of present bias.

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[4] Halevy [2015] nicely clarifies the distinction and relation between three properties that characterize time preferences: stationarity, time invariance, and time consistency. These properties correspond to our definitions of constant discounting, temporal stability, and dynamic consistency, respectively.

[5] We review the existing literature in Section 5.A.

To control for utility curvature on inferred discount rates, we jointly estimate discounting and utility functions, following the approach by Andersen, Harrison, Lau and Rutström [2008], and extend their modeling framework to our longitudinal data set by accounting for within-individual correlation in risk and time preferences over time. Our objective of evaluating discounting functions at two different decision dates provides a distinct reason to stress the importance of controlling for utility curvature. In a cross-sectional analysis, incorrectly assuming linear utility may bias the estimation of discounting functions since neglected utility curvature may be mistaken for the effects of discounting. In a longitudinal analysis, it is thus possible that temporal instability in the utility function is mistaken for temporal instability in the discounting function, unless one jointly estimates both functions at both decision dates. We control for temporal instability in utility curvature under Rank-Dependent Utility Theory (RDU) following Quiggin [1982], as well as utility curvature under Expected Utility Theory (EUT).

We recruited a nationally representative sample of Danish adults between 19 to 75 years of age, and draw inferences relevant to the broad population. While the experimenter may invite a random representative sample of subjects from a field population of interest, whether those subjects show up to the experiment is their own decision. Selection bias arises when subjects with certain types of (latent) preferences are more likely to self-select into the observed sample. The longitudinal design also raises concerns about possible sample attrition bias, which arises when unobserved individual preferences are correlated with the decision to select into the repeat experiment.[6] To control for non-random selection and attrition effects that may bias our inferences with respect to the adult Danish population, we apply the direct likelihood approach of Harrison, Lau and Yoo [2020] and construct a statistical model that explicitly recognizes participants in the longitudinal experiment as self-selected members of the population in *both* waves.

Our structural model controls for endogenous sample selection and attrition bias by allowing the error terms in the selection and attrition equations to be correlated with unobserved heterogeneity in

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[6] It is common to claim that there is no selection or attrition bias if the sample and population means of some key observed characteristics are similar. However, such difference-in-means comparisons do not address the econometric origin of selection or attrition bias, namely *unobservable characteristics* that are correlated with selection decisions and preferences. Harrison, Lau and Yoo [2020] show that correction for non-random selection and attrition can have significant economic effects on inferred risk attitudes, even when difference-in-means tests may (mis)lead one to expect otherwise.

individual preferences. The model combines the random coefficient specification of population heterogeneity with joint estimation of risk and time preferences akin to Andersen, Harrison, Lau and Rutström [2008], Heckman's [1979] celebrated correction for selection bias, and correction for attrition bias in the style of Capellari and Jenkins [2004]. Estimation of selection and attrition processes requires data on those who accepted our invitations to the experiments as well as those who did not. We targeted a random sample of adult Danes obtained from the Danish Civil Registry and observe some basic socio-demographic characteristics on both groups of individuals, regardless of their decision to participate in the experiments. Our design builds in explicit randomization of incentives to participate by varying the recruitment fee in the initial experiment, which provides an exclusion restriction for the empirical identification of endogenous sample selection bias.

We generally find dynamic inconsistency in the Danish population between 2009 and 2010. This finding is remarkable since we find constant discounting in the *first* wave in 2009, which empirically illustrates the importance of distinguishing constant discounting from dynamic consistency. The estimated population means in the first wave for the baseline (annual) discount rate, δ, and the present bias parameter, β, are respectively equal to 0.109 and 1.002. We do *not* find comparable results on constant discounting in the *second* wave in 2010, and that is what is required, on one leg of the tripod of properties, to claim dynamic consistency. The estimated population means in the second wave for δ and β are equal to 0.075 and 0.989, respectively. We also reject the hypothesis of temporal stability of time preferences over the two waves. Hence we infer dynamic inconsistency in the general population. At a methodological level we also show that flexible specifications of risk preferences, and corrections for sample selection and attrition, matter for our inferences on constant discounting, temporal stability and dynamic consistency.

In Section 2 we discuss our longitudinal field experiment with a focus on sampling procedures and identification of latent risk and time preference parameters. In Section 3 we formalize hypotheses of constant discounting, temporal stability and dynamic consistency from our structural model of time preferences. Section 4 report our findings, based on estimation of the joint distribution of individual-specific preference parameters within and between waves of the experiment. The approach allows us to make inferences at the population level as well as the individual level. Section 5 connects our analysis to

previous literature. We conclude in Section 6 and provide a numerical example that illustrates the implications of our finding on dynamic inconsistency.

2. Data

Our data originate from a longitudinal, artefactual field experiment conducted in Denmark. The first wave was designed to study alternative specifications of discount functions (Andersen, Harrison, Lau and Rutström [2014]) and intertemporal risk aversion (Andersen, Harrison, Lau and Rutström [2018]). The second wave was designed to study dynamic consistency, temporal stability of risk and time preferences, and sample selection and attrition in risk and time preferences, which we address here.

A. Sampling Procedures

Between September 28 and October 22, 2009, we conducted an artefactual field experiment with 413 Danes. The sample was drawn to be representative of the adult population aged between 18 and 75 years as of January 1, 2009. We obtained a random sample of 50,000 Danes from the Danish Civil Registration Office, stratified the sample by geographic area, and sent out 1,996 invitations to a randomly selected sub-sample. The information from the Danish Civil Registration Office includes sex at birth, age, residential location, marital status, and whether the individual is an immigrant. Thus we are in the fortunate, and rare, position of knowing some basic demographic characteristics of the individuals who do *not* select into our experiment. At a broad level our final sample is representative of the population in terms of observable characteristics: the sample of 50,000 Danes had an average age of 49.8, 50.1% of them were married, and 50.7% were female; our final sample of 413 subjects had an average age of 48.7, 56.5% of them were married, and 48.2% were female.

The initial recruitment letter clearly explained that there would be fixed and stochastic earnings from participating in the experiment. The fixed amount is 500 kroner in one treatment and 300 kroner in another treatment, and subjects were randomly assigned to one of these two treatments.[7] The stochastic earnings in the recruitment letter refer to the risk aversion and discounting tasks. There were 40 risk aversion tasks and 130 discounting tasks in the experiment, where the risk aversion tasks preceded the discounting tasks in one treatment, and *vice versa* in another treatment. Between April 2010

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[7] The exchange rate at the time was close to 5 kroner per U.S. dollar.

and October 2010 we repeated the decision tasks with a sample of 182 subjects from the 413 subjects who participated in the first experiment.[8] Each subject was interviewed in private in the second experiment, and the meeting was conducted at a convenient location for them (e.g., their private residence or the hotel where the first experiment took place). All subjects were paid a fixed fee of 300 kroner for their participation in the second experiment.[9]

B. Decision Tasks

Individual discount rates are evaluated by asking subjects to make a series of choices over two outcomes that are paid at different dates. For example, the sooner option could be 3000 kroner now, and the later option could be 3300 kroner in one year. If the subject with a linear utility function chooses the sooner option, we can infer that the discount rate is above 10% for a one year time delay. If the same subject picks the later option instead, we can infer that the annual discount rate is below 10%. By varying the amount of the later option, we can identify the discount rate of the individual, conditional on knowing the utility of those amounts to the individual. One can also vary the time delay between the sooner and later options to identify the discounting function, and of course one can vary the delay to the sooner option. This method has been widely employed (e.g., Coller and Williams [1999], Harrison, Lau and Williams [2002], Andersen, Harrison, Lau and Rutström [2008], Eckel, Johnson and Montmarquette [2005] and Dohmen, Falk, Huffman and Sunde [2010]).

We consider time delays between the sooner and later options from 2 weeks to 1 year. Each subject was presented with choices over four different time delays in ascending or descending order, and those time delays were drawn at random from a set of thirteen intervals (2 weeks, and 1, 2, 3, 4, 5, 6, 7,

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[8] From the sample of 413 subjects in the first experiment, a random sub-sample of 354 subjects were invited to the second experiment. The sample response rate in the first wave was 24.1% with the high recruitment fee, and 18.1% with the low recruitment fee. This reduction in sample response rates in the first wave is statistically significant according to a Fisher Exact test, with a p-value less than 0.001. After participating in the first wave, the sample response rate in the second wave was slightly lower for those recruited into the first wave with the high recruitment fee compared to those recruited with the low fee. The sample response rates were 48.4% and 54.7% in the second wave, respectively, and are not statistically different according to a Fisher Exact test with a two-sided p-value of 0.24. The sampling procedures are documented in Harrison, Lau and Yoo [2020].

[9] We did not vary the recruitment fee in the second experiment because we offered to interview the subjects at home or the hotel where the first experiment was conducted. The experiments were very time consuming and expensive to conduct, and we paid subjects the low recruitment fee of 300 kroner in the second experiments to keep costs down, although we see the value in varying recruitment fees in the second stage as well.

8, 9, 10, 11 and 12 months).[10] We also varied the delay to the sooner option on a between-subjects basis: roughly half of the sample had decision tasks with no delay to the sooner option, and the other half had a delay of 30 days. Similarly, we varied the provision of implied annual interest rates for each choice on a between-subjects basis. Finally, we employed two different principals on a between-subjects basis (1500 and 3000 kroner) to assess the significance of magnitude effects on elicited discount rates. The four sets of treatments, the order of time delay, the delay to the sooner option, information on implied interest rates, and the level of the principal, give a 2×2×2×2 design. Each subject was randomly assigned to one of these sixteen combinations in each wave of the experiment.

The subjects were presented with 40 binary choices, in four sets of 10 with the same time delay between the sooner and later option. The annual interest rate varied between 5% and 50%, in increments of 5%, on the principal of 1500 kroner or 3000 kroner. We randomly selected one decision task for each subject using numbered dice and the subjects were paid their preferred sooner or later option in that task.[11]

Utility functions are evaluated from data in which subjects made a series of choices over two risky lotteries. For example, lottery A might give the individual a 10-90 chance of receiving 2000 kroner or 1600 kroner to be paid today, and lottery B might have a 10-90 chance of receiving 3850 kroner or 100 kroner today. We gave subjects 40 choices, in four sets of 10 with the same prize combinations. The prize sets employed were: [A1: 2000 and 1600; B1: 3850 and 100], [A2: 1125 and 750; B2: 2000 and

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[10] We over-sampled the first three time delays, since short time delays are important with respect to identification of alternative discounting functions. The shorter time delays were each chosen with probability 2/16=0.125, compared to the 1/16=0.0625 probability for each of the others. The subjects were randomly assigned to four different time delays in each wave of the experiment.

[11] The following language explained the payment procedures: "You will receive the money on the date stated in your preferred option. If you receive some money today, then it is paid out at the end of the experiment. If you receive some money to be paid in the future, then it is transferred to your personal bank account on the specified date. In that case you will receive a written confirmation from Copenhagen Business School which guarantees that the money is reserved on an account at Danske Bank. You can send this document to Danske Bank in a prepaid envelope, and the bank will transfer the money to your account on the specified date." Payments by way of bank transfer are common in Denmark, Copenhagen Business School is a well-known institution in Denmark, and Danske Bank is the largest financial enterprise in the country as measured by total assets. All payments to subjects were made by Danske Bank on the specified dates of the subjects' preferred options, and we treat all future payments as credible and certain in the intertemporal decision model. Halevy [2008] and Chakraborty, Halevy and Saito [2020], on the other hand, treat present payments as certain and future payments as uncertain in their evaluation of the interplay between risk and time preferences. They introduce a hazard rate on future payments, which means that the correction for probability weighting under RDU enters the additive intertemporal utility function since future payments are uncertain.

250], [A3: 1000 and 875; B3: 2000 and 75] and [A4: 2250 and 1000; B4: 4500 and 50]. The order of the four prize sets was randomized for each subject, with probabilities of high prizes varying in ascending order between 0.1 and 1 in increments of 0.1 within each set. We asked each subject to select their preferred option in each of the 40 decision tasks and then randomly selected one task for payment using numbered dice.[12]

3. Hypotheses

Each discounting task presents a choice between option A that pays Y_t in period t and option B that pays $Y_{t+\tau}$ in period $t+\tau$, where $\tau > 0$. Given an atemporal utility function $U(m)$ and a discounting function $D(t)$, the discounted utility of each option is specified as

$$PV_A = D(t) \times U(Y_t + \omega) + D(t + \tau) \times U(\omega) \tag{1}$$

$$PV_B = D(t) \times U(\omega) + D(t + \tau) \times U(Y_{t+\tau} + \omega) \tag{2}$$

where ω is a measure of background consumption.[13]

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[12] The experimental procedures were standard: Appendix A in Andersen, Harrison, Lau and Rutström [2014] documents an English translation of the instructions, and shows typical screen displays for the discounting and risk aversion tasks, as well as a list of parameter values for all choices. After all choices had been made the subject was asked a series of standard socio-demographic questions. Section 5.B discusses alternative procedures for eliciting time preferences. The large incentives and budget constraints precluded us from paying all subjects, so each subject was given a 10% chance of being paid for one choice in each set of 40 choices. The average payment was 242 kroner for the risk attitude choices and 201 kroner for the discounting choices, for a combined average of 443 kroner, or $91, in addition to the fixed fee of $60 or $100.

[13] Andersen, Harrison, Lau and Rutström [2008] show that the addition of background consumption $\omega > 0$ is a sufficient condition to avoid negative discount rates under exponential discounting. The background consumption parameter ω is set exogenously in our model. We follow Andersen, Harrison, Lau and Rutström [2008][2014] and set it to the average daily consumption of private non-durable goods per capita, which was 130 kroner at the time of our experiments in 2009. Andreoni and Sprenger [2012] use convex budget sets to elicit discounting and utility functions, and treat background consumption as an endogenous parameter that is estimated along with the $\{\beta, \delta\}$ parameters in the quasi-hyperbolic discounting function and a relative risk aversion parameter α under EUT. They do not place any sign restrictions on background consumption and report both positive and negative values for this parameter. To allow negative background consumption, one would need to restrict the relative risk aversion parameter $\alpha > 0$ in their power specification, which is equivalent to $r < 1$ in our utility function. This inequality constraint may be innocuous if one is interested in estimating the population mean of the r parameter, which happens to be smaller than 1. However, when estimating the population distribution of risk attitudes, as long as there are *some* individuals whose r parameters exceed 1, the constraint is not innocuous, regardless of whether the mean is smaller or greater than 1.

Our experiment includes discounting tasks with variation in the delay to the sooner option. The resulting basket of choices between an immediate payment and a future payment, and between two future payments, allows us to identify and estimate a quasi-hyperbolic (QH) discounting function

$$D(t) = 1 \qquad\qquad \text{if } t = 0 \qquad\qquad (3)$$
$$= \beta \times 1/(1 + \delta)^t \quad \text{if } t > 0$$

where β is a present bias parameter and δ is a *baseline* discount rate for someone with no present bias ($\beta = 1$).[14] More generally, δ can be interpreted as a long-run discount rate, regardless of the value of β.[15] We denote time delay t in years (e.g., $t = 0.5$ for a 6-month horizon), and specify δ on an annualized basis. We assume *a priori* that decision makers display long-run delay aversion ($\delta > 0$), and let β take values on either side of 1 to allow for present bias ($\beta < 1$) as well as future bias ($\beta > 1$). We assume an additive intertemporal utility function, following convention.

Consider an individual who at time t_1 makes a choice between a sooner reward delivered at time $t_1 + t$ and a later reward delivered at time $t_1 + t + \tau$, where $t \geq 0$ is the delay to the sooner option, and $\tau > 0$ is the time delay between the sooner and later payments. To facilitate the discussion, let t_1 be the date of the initial experiment and t_2 be the date of the second experiment. Assume for the moment that the sooner and later payments are constant nominal values regardless of the payment date and time delay. Denote by $y\{t_1 + t, t_1 + t + \tau \,|\, t_1\}$ a binary indicator of whether the later payment is preferred at time t_1. The indicator $y\{t_1 + t, t_1 + t + \tau \,|\, t_2\}$ is similarly defined with respect to time t_2.

The quasi-hyperbolic discounting function $D(t)$ in equation (3) takes values of β_1 and δ_1 in the first wave (t_1) and β_2 and δ_2 in the second wave (t_2) of the longitudinal experiment. We subscript $D(t)$ as $D_w(t)$ to emphasize that it is based on β_w and δ_w, where $w \in \{1, 2\}$. Define the relative discounting factor, $R_w(t, t+\tau) \equiv D_w(t+\tau)/D_w(t)$. The individual is more likely to choose the later payment when the present value favors the later instead of the sooner payment. The evaluation of sooner and later payments in the statistical model is invariant to using the discount factors of $D_w(t)$ over time delay t and $D_w(t+\tau)$ over time delay $t+\tau$ or alternative discount factors of 1 over horizon t and $R_w(t, t+\tau)$ over horizon $t+\tau$. It follows that when one considers two different pairs of numerator and denominator

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[14] The QH specification was introduced by Phelps and Pollak [1968] for a social planning problem, and applied to model individual behavior by Elster [1979; p.71] and then Laibson [1997].

[15] The discount rate under the QH specification is $d^*(t)$ that solves $1/(1 + d^*)^t = \beta \times 1/(1 + \delta)^t$. The solution is given by $d^*(t) = (1 + \delta) / \beta^{(1/t)} - 1$. For any $\beta > 0$, $d^*(t)$ converges to δ as the horizon t increases, and in this sense δ can be interpreted as a long-run discount rate.

discount factors, both pairs favor the same payment as long as the implied relative discount factors are identical. We use $R_w(t_1+t, t_1+t+\tau \,|\, \mathbf{t_1})$ to denote the relative discount factor associated with the choice $y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}\}$, and vary the time indices to address other cases.

In terms of binary preference relations, *constant discounting* at the time of the initial experiment refers to the property that choices made at time t_1 depend only on the time delay between the two payments, so $y\{t_1, t_1+\tau \,|\, \mathbf{t_1}\} = y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}\}$ where $t > 0$. In our structural model, this property can be stated as a single hypothesis H_0: $\beta_1 = 1$. With a delay to the sooner option $t > 0$, choice $y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}\}$ is between two future payments and the relative discounting factor is given by $R_1(t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}) = 1/(1 + \delta_1)^\tau$. With no delay to the sooner option, choice $y\{t_1, t_1+\tau \,|\, \mathbf{t_1}\}$ is between an immediate reward and a future reward such that $R_1(t_1, t_1+\tau \,|\, \mathbf{t_1}) = \beta_1/(1 + \delta_1)^\tau$. The two relative discounting factors are identical and favor the same payment when $\beta_1 = 1$, which is when the discounting function displays neither present nor future bias.

Temporal stability refers to the property that choices do not vary from one point in time to another, as long as each payment date is adjusted to maintain the same time delay relative to the decision date, *i.e.*, $y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}\} = y\{t_2+t, t_2+t+\tau \,|\, \mathbf{t_2}\}$. This can be structurally stated as a joint hypothesis H_0: $\beta_1 = \beta_2$ and $\delta_1 = \delta_2$. To focus on the pure effects of discounting, we hold constant (or "partial out," to use a linear regression analogy) the effects of the utility function that may vary between the evaluation points, as the joint estimation of discounting and utility functions allows us to do. Consider first discounting tasks where $t > 0$. Choice $y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}\}$ in the initial experiment and choice $y\{t_2+t, t_2+t+\tau \,|\, \mathbf{t_2}\}$ in the repeat experiment give relative discounting factors of $R_1(t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}) = 1/(1 + \delta_1)^\tau$ and $R_2(t_2+t, t_2+t+\tau \,|\, \mathbf{t_2}) = 1/(1 + \delta_2)^\tau$, respectively. Both relative discounting factors favor the same reward when $\delta_1 = \delta_2$. Without any delay to the sooner payment, the relative discounting factor is $R_1(t_1, t_1+\tau \,|\, \mathbf{t_1}) = \beta_1/(1 + \delta_1)^\tau$ for the initial choice $y\{t_1, t_1+\tau \,|\, \mathbf{t_1}\}$, and $R_2(t_2, t_2+\tau \,|\, \mathbf{t_2}) = \beta_2/(1 + \delta_2)^\tau$ for the repeat choice $y\{t_2, t_2+\tau \,|\, \mathbf{t_2}\}$. On its own, equation $R_1(t_1, t_1+\tau \,|\, \mathbf{t_1}) = R_2(t_2, t_2+\tau \,|\, \mathbf{t_2})$ has two unknown parameters on each side, and may hold for some configuration of $\{\beta_1, \delta_1\}$ and $\{\beta_2, \delta_2\}$ despite $\beta_1 \neq \beta_2$ and $\delta_1 \neq \delta_2$. But given the condition $\delta_1 = \delta_2$ for tasks without a front-end delay, the two relative discounting factors favor the same reward when $\beta_1 = \beta_2$.

Finally, *dynamic consistency* refers to the property that choices do not vary from one point to another when each payment date is fixed, *i.e.*, $y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1}\} = y\{t_1+t, t_1+t+\tau \,|\, \mathbf{t_1+t}\}$. This can be

structurally stated as a joint hypothesis H_0: $\beta_2 = 1$ and $\delta_1 = \delta_2$. While this property also pertains to comparisons of choices made at two points in time, each payment date is now fixed in time. Consider first decisions with a delay to the sooner payment $t > 0$ in the repeat experiment. Since the choice is between two future payments from the perspective of the initial and repeat experiment, one may expect the present bias parameters β_1 and β_2 to be irrelevant. Indeed, the relative discounting factors, $R_1(t_2+t, t_2+t+\tau \mid \mathbf{t_1}) = 1/(1 + \delta_1)^\tau$ for choice $y\{t_2+t, t_2+t+\tau \mid \mathbf{t_1}\}$ in the initial experiment and $R_2(t_2+t, t_2+t+\tau \mid \mathbf{t_2}) = 1/(1 + \delta_2)^\tau$ for choice $y\{t_2+t, t_2+t+\tau \mid \mathbf{t_2}\}$ in the repeat experiment, favor the same payment when $\delta_1 = \delta_2$. Now suppose that the delay to the sooner payment in the repeat experiment is removed. This makes the sooner payment immediate from the perspective of the repeat experiment, but it remains delayed from that of the initial experiment. As one may expect, β_1 remains irrelevant but β_2 becomes relevant to relative discounting factors in the repeat experiment: $R_1(t_2, t_2+\tau \mid \mathbf{t_1}) = 1/(1 + \delta_1)^\tau$ for the initial choice $y\{t_2, t_2+\tau \mid \mathbf{t_1}\}$, and $R_2(t_2, t_2+\tau \mid \mathbf{t_2}) = \beta_2/(1 + \delta_2)^\tau$ for the repeat choice $y\{t_2, t_2+\tau \mid \mathbf{t_2}\}$. On its own, $R_1(t_2, t_2+\tau \mid \mathbf{t_1}) = R_2(t_2, t_2+\tau \mid \mathbf{t_2})$ may be satisfied by some configuration of $\beta_2 \neq 1$ and $\delta_1 \neq \delta_2$. But given the condition $\delta_1 = \delta_2$ in the case with a positive delay to the sooner payment in the repeat experiment, the two relative discounting factors favor the same reward when $\beta_2 = 1$.

Our structural approach fully accommodates the interplay of constant discounting, temporal stability and dynamic consistency: Each of our three hypotheses may hold without the other two, and a combination of any two hypotheses implies the third one. Applying the logic behind our approach to a wider range of non-constant discounting functions shows that the quasi-hyperbolic discounting function is a rare, if not unique, functional form that allows one to test each hypothesis of constant discounting, temporal stability and dynamic consistency separately. For example, a hyperbolic discounting function $D_w(t) = 1/(1 + \varkappa_w t)$ allows one to test for temporal stability in the $\varkappa_w$ parameter and nothing else. Given any non-zero value of $\varkappa_w$, the relative discount factor, $R_w(t, t+\tau) = [1 + \varkappa_w(t+\tau)]/(1 + \varkappa_w t)$, implies non-constant discounting and dynamic inconsistency. A Weibull discounting function $D_w(t) = \exp\{-\delta_w t^{\zeta_w}\}$ leads to a relative discount factor of $R_w(t, t+\tau) = \exp\{-\delta_w[(t+\tau)^{\zeta_w} - t^{\zeta_w}]\}$, which does not allow one to test for constant discounting and dynamic consistency separately. When applied to a choice between payments dated t_2 and $t_2+\tau$, the relative discount factor becomes $R_1(t_2, t_2+\tau \mid \mathbf{t_1}) = \exp\{-\delta_1[(t_2-t_1+\tau)^{\zeta_1} - (t_2-t_1)^{\zeta_1}]\}$ for the initial choice at t_1, and $R_2(t_2, t_2+\tau \mid \mathbf{t_2}) = \exp\{-\delta_2[\tau^{\zeta_2}]\}$ for the repeat choice at t_2. Thus,

in the Weibull case, constant discounting at the time of the initial experiment, $\zeta_1 = 1$, is a necessary condition for dynamic consistency, $R_1(t_2, t_2+\tau \mid \mathbf{t_1}) = R_2(t_2, t_2+\tau \mid \mathbf{t_2})$.

Although the three hypotheses of constant discounting, temporal stability and dynamic consistency pertain to discounting functions, explicit controls for utility curvature in each wave of the experiment are important to avoid biased evaluations of those hypotheses. We use the risk aversion tasks to identify the utility function $U(m)$ and assume that it has the functional form

$$U(m) = m^{(1-r)}/(1-r), \tag{4}$$

where the r parameter is an index of utility curvature, with concave utility when $r > 0$ and convex utility when $r < 0$. Under EUT the r parameter is interpreted as the coefficient of relative risk aversion.

Our identification strategy employs the more general RDU model that attributes risk preferences to the effects of rank-dependent probability weighting as well as utility curvature. We jointly estimate the utility function in (4) with the probability weighting function (PWF)

$$w(P) = \exp\{-(-\ln P)^\varphi\}, \tag{5}$$

where the φ parameter determines the shape of the PWF, which follows an inverse-S shape over probabilities if $\varphi < 1$ and an S shape if $\varphi > 1$. An inverse-S shape over-weights probabilities ($w(P) < P$) when P is relatively small, and under-weights probabilities ($w(P) > P$) when P is relatively large.[16] The order of over-weighting and under-weighting is reversed for an S-shaped function. EUT is a special case of RDU which assumes $\varphi = 1$, hence $w(P) = P$ everywhere. As with the discounting parameters, we allow the values of the risk preference parameters r and φ to vary between waves.

4. Results

We are interested in evaluating and testing hypotheses with respect to three properties of individual time preferences: constant discounting, temporal stability, and dynamic consistency. We use maximum simulated likelihood to estimate the full statistical model that captures behavioral noise in decision making,[17] unobserved preference heterogeneity, endogenous selection into the first wave of the

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[16] Given our functional form assumption, the small and large probabilities are defined relative to P = 0.368, which is the fixed point where $w(P) = P$ for any value of φ.
  [17] Our stochastic specification adopts the contextual utility model of Wilcox [2011] for risk aversion tasks and a Fechner error specification for discounting tasks. We discuss alternatives in the literature in Section 5.B.

longitudinal field experiment, and endogenous panel attrition between the two waves of the same experiment. The statistical model is documented in Appendix A.

Our model accommodates unobserved preference heterogeneity by specifying the time and risk preference parameters for each wave $w \in \{1, 2\}$ as random parameters $\{\beta_{nw}, \delta_{nw}, r_{nw}, \varphi_{nw}\}$, where n indexes different individuals in the population. Then, we estimate population means, medians and standard deviations of the individual-specific preference parameters as well as within-wave and between-wave correlations in those parameters. This approach of estimating the joint distribution of the preference parameters allows us to evaluate discounting functions at both the population level and the individual level in a coherent manner. At the population level, we test whether discounting functions evaluated at the population means of the preference parameters display each property of interest. At the individual level, we derive the population share of individuals who have discounting functions that display each property of interest, based on the estimated joint distribution function. We also use the estimated correlation matrix to study within-individual correlation in the same parameter between the two waves, and within-individual correlation in different parameters as at the same wave.

The statistical procedure addresses the panel dimension of the data set at both the modeling and inferential stages. At the modeling stage, our random parameter specification induces panel correlation across repeated observations on the same individual, in an analogous fashion to mixed logit models for repeated choice data. At the inferential stage, we adjust all standard errors and test statistics for clustering at the individual level.

We use Johnson's [1949] "S_B" distribution to capture the population distribution of each time preference parameter. An S_B distribution is a logit transformation of a normal distribution. This transformation produces a flexible parametric distribution that is capable of approximating a wide range of shapes in population distributions (e.g., uniformity, unimodality, bimodality, and left and right skewness) without requiring us to impose any shape restriction *a priori*. The primary tradeoff is that we cannot obtain analytic solutions for the population means, standard deviations and correlation coefficients describing the S_B-distributed time preference parameters.[18] We simulate these population

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[18] We choose more restrictive but analytically tractable distributions for the risk preference parameters r_{nw} and φ_{nw}, whose roles in our analyses are akin to control variables in everyday regression contexts. Specifically, we use normal and log-normal distributions to capture the population distributions of r_{nw} and φ_{nw}, respectively.

moments using 10,000 draws from the estimated joint distribution of the underlying normal parameters, and compute associated standard errors and confidence intervals using the bootstrapping procedure of Krinsky and Robb [1986]. All moments that we report for the time preference parameters, β_{nw} and δ_{nw}, refer to their S_B distributions rather than the underlying normal distributions.

A. Constant Discounting, Temporal Stability, and Dynamic Consistency

The upper-left panel of Figure 1 displays the estimated population distribution of the baseline discount rate δ_{nw} for each wave w, based on the empirical model that corrects for endogenous selection and attrition bias. The estimated coefficients in the model are reported in Table 1. The estimated distribution for wave 1 is unimodal and right-skewed, neither of which has been imposed *a priori*. There is an evident degree of preference heterogeneity, although most individuals seem to have baseline discount rates in the interval between 0% and 10% per annum. This informal description is comparable with the estimated mean, median and standard deviation, which are 10.9%, 4.4% and 14.5%, respectively. The estimated population distribution for wave 2 takes a similar shape with slightly higher density in the interval between 0% and 10%, and the estimated mean, median and standard deviation are 7.5%, 4.1% and 9.2%, respectively. All the estimated means, medians and standard deviations are significantly greater than 0, with *p*-values < 0.001. Henceforth, unless we say otherwise, all point estimates in our discussion have *p*-values less than 0.001.

The lower-left panel of Figure 1 displays the estimated population distribution of the present bias parameter β_{nw} for each wave, based on the same empirical model that corrects for endogenous selection and attrition bias. The estimated distribution for wave 1 is unimodal and left-skewed, and the interval between 0.95 and 1.05 captures most decision makers in the population. While the estimated mean and median is 1.002 and 1.005, respectively, the population distribution displays a statistically significant standard deviation of 0.025. The estimated distribution for wave 2 is more skewed to the left, with an estimated mean, median and standard deviation of 0.989, 0.992 and 0.030, respectively.[19]

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[19] Formally, the marginal distribution of the baseline discount rate parameter, $\delta_{nw} = 0.7/[1 + \exp(-\delta^*_{nw})]$, is characterized by the mean and standard deviation of the underlying normal parameter δ^*_{nw}. Similarly, the marginal distribution of the present bias parameter, $\beta_{nw} = 0.7 + 0.4/[1 + \exp(-\beta^*_{nw})]$, is characterized by the mean and standard deviation of the underlying normal parameter β^*_{nw}. Using $\chi^2(2)$-distributed Wald tests, we reject the hypothesis that the two marginal distributions in wave 1 and 2 are identical for each of parameters δ_{nw} and β_{nw}, with *p*-values less than 0.001.

We now move to more formal statistical tests of properties describing time preferences at the population level, and consider first *the hypothesis of constant discounting*, H_0: $E(\beta_{nw}) = 1$. The estimated population mean of the present bias parameter for wave 1 is equal to 1.002, and we cannot reject H_0: $E(\beta_{n1}) = 1$, despite the small standard error of 0.002 (p-value = 0.317). The estimated population mean for wave 2 is equal to 0.989 and close to unity, but we nevertheless reject H_0: $E(\beta_{n2}) = 1$, with a p-value < 0.001. Hence, we cannot reject the null hypothesis that discount rates are constant in wave 1, but we formally reject the hypothesis in wave 2, although the estimated population mean of the present bias parameter is very close to 1.

Consider next *the hypothesis of temporal stability*, H_0: $E(\beta_{n1}) = E(\beta_{n2})$ and $E(\delta_{n1}) = E(\delta_{n2})$. The estimated difference in the population means of the present bias parameter, $E(\beta_{n2}) - E(\beta_{n1})$, is equal to −0.013, with a 95% confidence interval of [−0.019, −0.007], and the estimated difference in the population means of the baseline discount rate, $E(\delta_{n2}) - E(\delta_{n1})$, is equal to −0.033, with a 95% confidence interval of [−0.048, −0.012]. The confidence intervals are consistent with relatively small standard errors for the estimated differences in populations means of the two discounting parameters, and we reject both H_0: $E(\beta_{n1}) = E(\beta_{n2})$ and H_0: $E(\delta_{n1}) = E(\delta_{n2})$ with a p-value < 0.001. We also reject the joint null hypothesis of temporal stability (p-value < 0.001).[20]

Finally, consider *the hypothesis of dynamic consistency*, H_0: $E(\beta_{n2}) = 1$ and $E(\delta_{n1}) = E(\delta_{n2})$. This null hypothesis entails constant discounting in wave 2 *and* temporal stability in the baseline discount rate across the two waves. The joint test of the two constraints does *not* entail constant discounting in wave 1, since the relevant payment options are delayed from the perspective of wave 1, which implies that β_{n1} does not influence the ordering of discounted utilities. Although the estimated coefficients roughly

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[20] Two correlation coefficients are directly relevant to temporal stability in time preferences: corr(δ_{n1}, δ_{n2}) and corr(β_{n1}, β_{n2}) capture within-individual correlation over time in the baseline discount rate and present bias parameter, respectively. Each correlation coefficient may be used as a measure of individual-level temporal stability in the respective parameter. With corrections for endogenous selection and attrition bias, the estimated value of corr(δ_{n1}, δ_{n2}) is 0.354 with a p-value < 0.001, and the estimated value of corr(β_{n1}, β_{n2}) is 0.273 with a p-value = 0.018. Thus temporal stability holds for each parameter in the sense that if some individuals display greater long-run delay aversion or present bias than the average person in wave 1, they also tend to do so in wave 2. However, this directional prediction is not deterministic, since we reject the hypothesis of perfect positive correlation for either parameter (p-value < 0.001). We discuss comparisons of these results with previous literature in Section 5.C.

satisfy $E(\beta_{n2}) = 1$ and $E(\delta_{n1}) = E(\delta_{n2})$, we reject each null hypothesis, as well as the joint hypothesis of dynamic consistency, with a *p*-value < 0.001.[21]

Our econometric approach also allows us to undertake a more individualistic evaluation of constant discounting, temporal stability and dynamic consistency, by considering the population shares of decision makers whose discounting functions agree with each hypothesis. Since we have estimated the population distributions of all preference parameters, including all relevant correlation coefficients, we can derive population shares for each type of discounting behavior by computing the following marginal and joint probabilities: $\Pr(|\beta_{nw} - 1| < \varepsilon_\beta)$ for constant discounting at wave w; $\Pr(|\beta_{n2} - \beta_{n1}| < \varepsilon_\beta$ and $|\delta_{n2} - \delta_{n1}| < \varepsilon_\delta)$ for temporal stability between the two waves; and $\Pr(|\beta_{n2} - 1| < \varepsilon_\beta$ and $|\delta_{n2} - \delta_{n1}| < \varepsilon_\delta)$ for dynamic consistency. For hypotheses involving present bias parameters, the absolute tolerance $\varepsilon_\beta > 0$ defines small deviations from rigid predictions that we will allow for when classifying discounting behavior into different types.[22] For hypotheses involving baseline discount rates, the tolerance is given by $\varepsilon_\delta > 0$ instead.[23] We consider $\varepsilon_\beta = 0.025$ and $\varepsilon_\delta = 0.05$ as benchmark tolerances, and examine the sensitivity of our results to alternative configurations. Setting $\varepsilon_\delta = 0.05$ is arguably a natural benchmark for the baseline discount rate, since the smallest increment in annual interest rates across our discounting tasks for a particular time horizon is 5 percent. A benchmark tolerance for ε_β is less obvious and we set it equal to 0.025, which is close to the estimated standard deviation of the population distribution for the present bias parameter in both waves.

Table 2 reports the estimated population shares of decision makers that fall into different types of discounting behavior, based on the benchmark tolerance configuration. We find that a majority of Danes display *constant discounting* in both waves of the experiment: 66.9% of the decision makers have constant discount rates in wave 1, and 59.8% have constant discount rates in wave 2. The estimated population share is significantly different from 50% in wave 1 (*p*-value = 0.022), but we cannot reject that it is equal to 50% in wave 2 (*p*-value = 0.138).

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[21] We obtain the same conclusions when we consider stronger tests of temporal stability and dynamic consistency that also include utility curvature. In that case the test for temporal stability is H_0: $E(\beta_{n1}) = E(\beta_{n2})$, $E(\delta_{n1}) = E(\delta_{n2})$ and $E(r_{n1}) = E(r_{n2})$; and the test for dynamic consistency is H_0: $E(\beta_{n2}) = 1$, $E(\delta_{n1}) = E(\delta_{n2})$ and $E(r_{n1}) = E(r_{n2})$.

[22] For instance, our evaluation classifies individual n's behavior as constant discounting in wave w if $(1-\varepsilon_\beta) < \beta_{nw} < (1+\varepsilon_\beta)$, whereas the rigid prediction requires that $\beta_{nw} = 1$.

[23] Since we adopt continuous population distributions to model preference heterogeneity, setting $\varepsilon_\beta = \varepsilon_\delta = 0$ leads to the trivial conclusion that all three population shares are equal to 0.

The decision makers who display *temporal stability* in the baseline discount rate and the present bias parameter are estimated to make up 27.9% of the population, with a 95% confidence interval of [20.9%, 36.4%]. Hence, a majority of the population (72.1%) displays temporal instability in at least one of the two parameters. When we look at each parameter separately, those with temporally stable baseline discount rates make up 53.0% of the population, and those with temporally stable present bias parameters also make up 53.0%. The two parameters in the quasi-hyperbolic discounting function thus show a similar degree of temporal (in)stability. The population share of those with temporal instability in both parameters is estimated to be 21.9%.

Finally, the estimated share of decision makers who display *dynamic consistency* is equal to 31.5%, with a 95% confidence interval of [24.5%, 40.6%]. Dynamic consistency entails constant discounting in wave 2 along with temporally stability in baseline discount rates between the two waves. The share of decision makers with constant discounting in wave 2 is 59.8%, and that of those with temporally stable baseline discount rates is 53.0%. Constant discounting thus seems to be a more important source of dynamic consistency than temporal stability in baseline discount rates, although our inference concerning dynamic consistency is based on a joint probability that takes into account the estimated correlation coefficients for different preference parameters.

Figure 2 illustrates the sensitivity of these findings to different values of the two noise tolerances. The three panels display the estimated shares of decision makers who reveal constant discounting, temporal stability and dynamic consistency, respectively. The estimated population shares increase with the size of the tolerances, by construction, but the tendency is the same as before: when we look at the shares of decision makers with these traits we find more empirical support for constant discounting than temporal stability and dynamic consistency.[24]

B. *Are Risk and Time Preferences Correlated?*

Turning to risk preferences, the upper-left panel in Figure 3 shows the estimated population distributions of the utility curvature parameter r_{nw} across the two waves, with controls for non-random

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[24] Table C2 in Appendix C reports estimated population shares of decision makers who reveal constant discounting, temporal stability and dynamic consistency, based on an alternative structural model with EUT to identify utility curvature. Using the same tolerance configurations as above, we find smaller population shares for each of those three types of discounting behavior compared to the model with RDU.

selection and attrition bias. The population distribution of the utility curvature parameter shifts to the right in wave 2 compared to wave 1, but the apparent increase in risk aversion is *not* statistically significant (p-value = 0.097) at the 5% significance level. The estimated population mean is equal to 0.951 with a p-value of 0.066 in wave 1, and equal to 1.076 with a p-value of 0.089 in wave 2. We also observe that the population distribution in wave 2 has a smaller standard deviation than the distribution in wave 1; the estimated standard deviation is 0.725 in wave 1 and 0.597 in wave 2, and we cannot reject the null hypothesis that the estimated difference in the two coefficients is equal to 0 at the 5% level (p-value = 0.088). Hence, we find *temporal stability* with respect to the population mean, and also with respect to the standard deviation of the utility curvature parameter.[25]

The estimated population distributions of the shape parameter φ_{nw} in the PWF are displayed in the lower-left panel of Figure 3. The distributions control for selection and attrition bias, and we observe insignificant differences in the estimated population distributions of the shape parameter between the two waves. The estimated difference in the population mean between the two waves is statistically insignificant (p-value = 0.758), and we also find that the standard deviation of the population distribution is temporally stable (p-value = 0.520).[26]

Turning to the association between risk and time preferences, we find that the baseline discount rate is negatively correlated with utility curvature in both waves: the estimated values of $\text{corr}(\delta_{n1}, r_{n1})$ and $\text{corr}(\delta_{n2}, r_{n2})$ are -0.378 and -0.349, respectively, and both coefficients are significantly smaller than 0 (p-value < 0.001).[27] The negative correlation coefficients thus suggest that those with higher baseline discount rates tend to be less risk averse, *ceteris paribus*.[28] The present bias parameter is significantly correlated with utility curvature in wave 1, but not in wave 2. In wave 1 more present-biased individuals have less concave utility: the estimated value of $\text{corr}(\beta_{n1}, r_{n1})$ is 0.183 and significantly greater than 0 (p-

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[25] The estimated correlation coefficient between the two wave-specific population distributions of the utility curvature parameter, $\text{corr}(r_{n1}, r_{n2})$, is equal to 0.668, and we reject the hypothesis that they are independent (p-value < 0.001).

[26] The estimated between-wave correlation of the shape parameter, $\text{corr}(\varphi_{n1}, \varphi_{n2})$, is 0.871 with a standard error of 0.063, which suggests a strong degree of temporal stability at the individual level.

[27] Table B1 in Appendix B reports the estimated correlation coefficients.

[28] This finding is not an algebraic artefact. The discounted utility difference between sooner and later payment options changes in favor of the sooner option, as δ_{nw} (i.e., long-run delay aversion) increases or r_{nw} (i.e., concavity of the utility function) decreases. But this algebraic property says nothing about whether individuals with $\delta_{nw} > E(\delta_{nw})$ tend to have $r_{nw} < E(r_{nw})$.

value < 0.001). In wave 2, however, the estimated value of corr(β_{n2}, r_{n2}) is 0.005 and we cannot reject that the two distributions are independent (p-value = 0.949).[29]

The estimated correlation coefficient between the baseline discount rate and the shape parameter for the PWF in wave 1, corr(δ_{n1}, φ_{n1}), is 0.268. The corresponding coefficient in wave 2, corr(δ_{n2}, φ_{n2}), is 0.342. Both coefficients are significantly greater than 0. The implications of these positive correlation coefficients are less straightforward to evaluate. When the probability of the best outcome increases, the PWF switches from being concave to convex when $\varphi_{nw} < 1$, and *vice versa* when when $\varphi_{nw} > 1$. We find that $E(\varphi_{n1})$ and $E(\varphi_{n2})$ are equal to 2.171 and 2.091, respectively, which means that the average person has an S shaped PWF. If the average decision maker under-weights (over-weights) a particular probability, someone with $\varphi_{nw} > E(\varphi_{nw})$ will under-weight (over-weight) it even further. The positive correlation coefficients thus suggest that the effects of S-shaped probability weighting tend to be more pronounced for those with higher baseline discount rates. The results are more mixed with respect to the correlation between the present bias and shape parameters. The value of corr(β_{n1}, φ_{n1}) is −0.213 and significantly smaller than 0, but corr(β_{n2}, φ_{n2}) is equal to 0.098 and insignificant (p-value = 0.189). Hence, in wave 1, more present-biased individuals tend to have PWFs that display more pronounced S shapes. This association, however, almost vanishes in wave 2.

C. Controlling for Endogenous Selection and Attrition

Our model accounts for endogenous sample selection and panel attrition by specifying selection and attrition equations in the style of Heckman [1979] and Capellari and Jenkins [2004], and allowing the error terms in those equations to be correlated with each other as well as each of the time and risk preference parameters. Our experimental design provides natural candidates for exclusion restrictions to assist empirical identification of the selection and attrition equations. The initial invitation letter randomized individuals to different recruitment fees, and the longitudinal design allows us to observe each subject's additional earnings from the first experiment. We assume that the recruitment fees

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[29] The estimated pattern of correlation coefficients between risk and time preferences under EUT is generally the same as the results under RDU, in terms of sign and statistical significance. The only exception is the estimate of corr(β_{n2}, r_{n2}), which is 0.210 and significantly greater than 0 under EUT (p-value = 0.002). The estimate of corr(β_{n1}, r_{n1}) remains positive and significant, and we thus find that more present-biased individuals tend to be less risk-averse in both waves.

influence the initial decision to accept the first invitation, but not the decision to accept the second invitation once we control for additional earnings from the first experiment. We maintain the usual hypothesis that the recruitment fees do not affect the subject's evaluation of time-dated payments and lottery pairs directly. Finally, subjects had to travel to hotel meeting rooms to participate in the first experiment, whereas each subject chose their own preferred venue for the second experiment.

These features of our experimental design allow us to distinguish the selection equation from the attrition equation. We include the recruitment fee only in the selection equation, and the actual earnings from the first experiment only in the attrition equation.[30] In addition, we augment the selection equation with each subject's home-to-hotel distance (in miles) and its square value, and attrition equation with self-reported income that is only available for those who participated in the first experiment; both equations also include the individual's age and gender. The distinctive theoretical structures that we have placed on the discounting choice model (namely, discounted utilities with quasi-hyperbolic discounting functions) and the risky choice model (RDU) immediately differentiate the two models from each other, as well as from the selection and attrition equations.

The sample selection equation reported in Table 1 shows that recruitment fees and the distance from an individual's home to a session venue significantly affect the decision to participate in the experiments.[31] The law of demand effectively applies to the participation decisions, with the propensity to self-select into the experiments increasing significantly when the recruitment fee is raised from 300 kroner to 500 kroner for participation in wave 1. We also find a statistically significant and U-shaped association between the self-selection index and the home-to-hotel distance, with a negative and diminishing marginal effect of distance up to a turning point at 30.6 miles. As one may expect, people

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[30] Adding the recruitment fee to the attrition equation does not change our results and conclusions with respect to constant discounting, temporal stability and dynamic consistency. Table D1 in Appendix D reports the estimated parameters for the model where the recruitment fee is included in both the selection and attrition equation.

Additional earnings in the initial experiment include payments for choices in three sets of decision tasks which elicit individual risk attitudes, discount rates and correlation aversion, respectively. Additional earnings are thus negatively correlated with individual discount rates in the first experiment. We obtain virtually the same results when we restrict the measure of additional earnings to payments for choices in the risk aversion tasks. Table D2 in Appendix D reports the estimated parameters for the model where additional earnings in the attrition equation refer to payments for choices in the risk aversion tasks.

[31] The coefficient estimates for the sample selection and attrition equations have been re-scaled so that the reported results can be interpreted in the same manner as the coefficient estimates for the usual censored probit models that standardize the variance of each component equation to unity.

who live farther away from the session venues are less likely to participate, and people who live closer are more sensitive to a small increase in distance.[32] Looking at personal characteristics, middle-aged and older subjects are more likely to participate in the first wave than younger age groups.[33] Table 1 also shows that it is difficult to explain panel retention in terms of the subject's observed characteristics, though the point estimates suggest that those who earned more from the initial experiment are more likely to return to the second experiment if invited.[34]

The hypothesis of no endogenous selection bias involves 9 parametric constraints: it states that the error term in the selection equation is uncorrelated with the error term in the attrition equation *and* the population distributions of the 8 preference parameters in the structural model. We reject this hypothesis with a *p*-value < 0.001, so the result is consistent with significant sample selection. The hypothesis of no endogenous attrition bias implies that the error term in the attrition equation is uncorrelated with the 8 preference parameters, which is also rejected (*p*-value < 0.001). This result is also consistent with significant sample attrition. The estimated correlation coefficient between the error terms in the selection and attrition equations is equal to −0.273 with a standard error of 0.055, which further implies that one cannot correct for attrition bias without also correcting for selection bias.[35]

From Figure 1 we can see the effects of controlling for selection and attrition bias on baseline discount rates and present bias parameters. The left panels show the estimated population distributions from a model with corrections for endogenous selection and attrition, and the right panels show the corresponding results from a model without such corrections. Although we model the time preference parameters using S_B distributions that can display a variety of shapes, the corrected and uncorrected

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[32] The marginal effect of distance is positive after the turning point, but this is an artefact of the quadratic specification that has limited practical significance, since only 20 out of the 1996 invitees lived outside a 30.6 mile radius from a venue.

[33] Most variables have self-evident definitions. The "young" age group is defined as those less than 30 years, the "middle" age group is defined as those between 40 and 50 years, and the "old" age group is those over 50 years. Lower income is defined in variable "IncLow" by a household income in 2008 below 300,000 kroner, and higher income is defined in variable "IncHigh" by a household income of 500,000 kroner or more.

[34] Removing personal characteristics from the selection and attrition equations does not change our overall results. Table D3 in Appendix D reports the estimated parameters for the model where the recruitment fee and home-to-hotel distance are included in the selection equation, and the recruitment fee and revised measure of additional earnings are included in the attrition equation.

[35] The attrition equation can display selection bias because an individual's attrition outcome is observed only if the individual participated in the initial experiment, just as the individual's discounting and risky choices are.

population distributions happen to take on similar shapes. The corrected and uncorrected point estimates are also similar. The estimated coefficients in the model without corrections for sample selection and attrition bias are reported in Table 3. The estimated mean and standard deviation of the uncorrected distribution for the baseline discount rate are 11.1% and 12.8% in wave 1, and 9.5% and 12.3% in wave 2. For the present bias parameter, the estimated mean and standard deviation is 0.998 and 0.030 in wave 1, and 0.997 and 0.021 in wave 2.

Incorrectly assuming away endogenous selection *and* attrition bias changes our population-level inferences on constant discounting in wave 2, but not in wave 1. Based on the estimated population means, we do not reject constant discounting in wave 1 (*p*-value = 0.317) or in wave 2 (*p*-value = 0.317). Moreover, we cannot reject the hypothesis of temporal stability at the 10% significance level (*p*-value = 0.051), whereas temporal stability was rejected at the 1% significance level in the model with corrections. Although we do not reject constant discounting in wave 2 and cannot reject temporal stability at the 10% significance level, we continue to reject the hypothesis of dynamic consistency in the model without corrections (*p*-value = 0.009).[36]

The results illustrate the important methodological point that inferences on constant discounting and temporal stability must be made jointly to infer dynamic consistency. In this instance the two *t*-tests of single properties lead us to conclude that we have constant discounting in wave 2 and temporal stability between the two waves, but the joint Wald test of both properties leads us to conclude that we have dynamic *inconsistency*.

In Table 4 we use the results from the model without corrections for sample selection and attrition to evaluate the population shares of decision makers who reveal constant discounting, temporal stability and dynamic consistency. Using the same tolerance configuration used for the model with corrections ($\varepsilon_\beta = 0.025$ and $\varepsilon_\delta = 0.05$), we continue to find constant discounting for a majority of the population in both waves. But in contrast to the corrected estimates, the uncorrected estimates suggest

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[36] Incorrectly assuming away sample selection, but still correcting for sample attrition is relatively easy to do. In a longitudinal experiment there is often considerable "baseline" data collected on individuals, which can form the basis for corrections for attrition even if one does not, typically by definition, know anything about those that do not even turn up to the baseline. However, this special case also leads to some important differences in inferences compared to correcting for both selection and attrition jointly. It leads to incorrectly rejecting the null of constant discounting in wave 1, and incorrectly failing to reject the null of temporal stability. We do *correctly* reject the joint hypothesis of dynamic consistency at the 5% level, but *incorrectly* fail to also reject it at the 1% level.

that constant discounting is more prevalent in wave 2 than wave 1. The estimated population share is 58.9% in wave 1 and 77.5% in wave 2, with confidence intervals of [47.6%, 72.1%] and [63.6%, 90.1%], respectively. The population share of those with temporally consistent discounting functions continues to be relatively small; the estimated population share is 31.8%, with a confidence interval of [21.3%, 42.8%]. Finally, the population share of those with dynamically consistent time preferences is equal to 42.0%. This estimate is 10.5 percentage points larger than the corresponding share in the corrected model, and we cannot reject the null hypothesis that half of the population have dynamically consistent preferences (p-value = 0.089).

D. *Restrictions on Probability Weighting*

The best known theory of decision making under risk is EUT, which allows one to estimate a non-linear utility function and use it to control for the effects of utility curvature on the inferred discounting function. Our modeling framework employs RDU, which generalizes EUT, and we find statistically significant probability weighting in the population as a whole. Specifically, our model nests EUT as a special case that constrains the shape parameter of the PWF, φ_{nw}, to 1 for every person n and wave w. As Table 1 shows, we reject this hypothesis: the estimated standard deviation of φ_{nw} is significantly greater than 0 in each wave, which suggests that φ_{nw} is heterogeneous.

Nevertheless, given the prominence of EUT, Appendix C reports the EUT-based estimation results for comparisons with the RDU-based results that we have presented so far. In the EUT-based results, the estimated population means of the discounting parameters are very close to the null hypothesis values under constant discounting, temporal stability and dynamic consistency: the point estimates of $E(\delta_{n1})$, $E(\delta_{n2})$, $E(\beta_{n1})$, and $E(\beta_{n2})$ are 12.4%, 13.3%, 0.993, and 1.003, respectively. But we obtain relatively small standard errors for these estimates. We reject constant discounting in wave 1 (p-value < 0.001), although that result for wave 1 does not matter for the test of dynamic consistency. We do not reject constant discounting in wave 2 (p-value = 0.262). We do reject temporal stability (p-value = 0.007). However, despite rejecting temporal stability, we do not reject dynamic consistency (p-value = 0.372). Hence one would come to the wrong conclusion about dynamic consistency if one constrained our specification by imposing EUT instead of RDU.

5. Related Literature

A. Dynamic Consistency

Halevy [2015] provides a particularly clear statement of the *theoretical* interplay between constant discounting, temporal stability and dynamic consistency. He *evaluates* that interplay using a very different approach than we use. His identification and modeling strategy is based on binary preference relations over pairs of sooner and later payments that are relevant to analysts who are interested in making *non-structural* inferences by studying aggregate and individual-level *choice patterns*. The time delay between sooner and later payments was one week in all pairwise choices, and identification is based on variation in decision dates and time delays to sooner payments.[37] He used undergraduate students from a Canadian university as subjects in his experiments, an appropriate convenience sample to initially examine behaviour in this, and other, settings.

Halevy [2015] recruited 149 students in the first wave of the main experiment, of which 130 also participated in the second wave. The sooner payment was either 10 or 100 Canadian dollars. Later payments varied between \$9.90 and \$10.00 in increments of \$0.10 in the low stakes treatment; and between \$99 and \$110 in increments of \$1 in the high stakes treatment.[38] The observed patterns of intertemporal choices by his subjects do *not* reject constant discounting, *do* reject temporal stability, and *partly* reject dynamic consistency. Inferences about time *preferences* are not the same as inferences about binary *choice patterns*, however, unless one assumes away "behavioral noise." It is perfectly possible for observed choice patterns to be driven by behavioral errors that are correlated with latent time preferences. In effect, there are severe limitations as to what one can say about statistical significance of latent individual preferences by only comparing observed choice patterns across treatments.

Once one moves away from relying on direct inspection of choice patterns for inferences about dynamic consistency, it is possible to relax the temporal sequencing of the experimental design. Our

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[37] The one week time delay between sooner and later payments may not be sufficient to detect present bias if subjects do not discriminate between the two payments over the brief time interval (e.g., a subject might view the "present" as today, this week or this month, at least under some models of discounting other than the QH model, as noted by Takeuchi [2011]). This issue pertains to the delay between the two payment dates, and is not related to, or mitigated by, the two treatments with variation in the front-end delay to the sooner payment.

[38] To facilitate comparisons of implied interest rates in decision tasks with other studies and field alternatives, it is convenient to state these on an annual basis. Weekly interest rates between −1% and 10% are equivalent annual interest rates between −67.8% and 14,104%.

design presented subjects with various time horizons, between 2 weeks and 1 year, for receipt of the later option. We also varied receipt of the earlier option between 0 days and 30 days. This design allows estimation of a rich set of discounting models (e.g., Exponential, Quasi-hyperbolic, Smoothly Hyperbolic). These models define the discount rates of the subjects, at a particular point of time, over horizons between today and 13 months into the future. What matters for our design is that the re-visit to the subject occurs within that interval, not at any particular point in that interval. This is true as a matter of economic theory once we estimate the discounting function over the entire time horizon, and as a matter of how we estimate our structural models (detailed in Appendix A). It also happens to be practically essential for field experiments, where it is logistically difficult and prohibitively costly to locate and interview subjects on specific days over extended periods of time.

Some studies use willingness to pay for commitment devices as indicators of subjective awareness of behavioral tendencies to act in a dynamically inconsistent manner (e.g., Augenblick, Niederle and Sprenger [2015] and Giné, Goldberg, Silverman and Yang [2018]). Although some of those devices may be useful in restricting intertemporal choice, O'Donoghue and Rabin [2015; p. 277ff.] correctly note that they are not measures of dynamic inconsistency. The effects of commitment devices are, at best, suggestive of the effects of dynamic consistency, but could also reflect other behavioral traits such as low willpower.

Only a handful of experimental studies with non-student subjects have longitudinal designs that enable statistical tests of temporal stability alongside tests for constant discounting. Even then, those studies focus on relatively narrow segments of the general population; do not account for the interplay of interpersonal heterogeneity, endogenous sample selection and endogenous panel attrition; and do not consider formal or informal tests of dynamic consistency.[39]

Meier and Sprenger [2015] recruited a sample of low-to-moderate income earners who obtained a free tax preparation service at a particular site run by the City of Boston. They consider quasi-

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[39] Although Sadoff, Samek and Sprenger [2020] allude to dynamic inconsistency, their approach falls into a different strand of literature, since they do not elicit discounting behavior. The population in their study refers to customers at two grocery stores in low-income areas of Chicago and Los Angeles, and the notion of dynamic consistency refers to disparities in the demand for a basket of groceries from the order date to the delivery date. They test this hypothesis by examining whether the taste coefficients in a rank-ordered logit model change over time. One cannot disentangle dynamic consistency from temporal stability in this setting. Indeed, in the stated preference literature where the rank-ordered logit model originates from, the same type of parametric restriction is used to test temporal stability: see Doiron and Yoo [2017].

hyperbolic discounting and find empirical support for non-constant discounting and temporal stability. The population of interest in Kirby *et al.* [2002] are residents in two villages in a tropical rain forest area of Bolivia. They assume away behavioral noise and algebraically derive hyperbolic discount rates, which precludes statistical tests of constant discounting, and find that within-subject correlation in the hyperbolic discounting parameter is 0.32 or less over the four quarters in their longitudinal design. Finally, Dean and Sautmann [2020] recruited a sample of household heads who participated in a children's health care program in a peri-urban area of Bamako, Mali. The sooner option was either paid immediately or delayed by one week, and the time horizon between the sooner and later options was one week in all decision tasks. Summarizing the choice patterns observed at weekly intervals for three consecutive weeks, they report that 70% to 76% of the subjects display constant discounting in each week, and the within-subject correlation in choices ranges from 0.67 to 0.72 across the weekly waves of the experiment.[40]

In general, we contribute to this literature with the first set of longitudinal findings based on a nationally representative sample of the general population; a modeling framework which jointly addresses preference heterogeneity, selection bias and attrition bias; and a coherent inferential approach that allows one to use the same set of preference parameter estimates to test for constant discounting, temporal stability and dynamic consistency at both individual and population levels.

B. General Issues in the Elicitation of Time Preferences
Time Preferences for Populations or Individuals?

As documented in Appendix A, our estimation method does not require that each subject responds to the same set of questions in both waves of the experiment. We fit a joint structural model to a pooled sample of every individual's selection outcome and, where applicable, attrition outcome and choices during the experiment. Our model incorporates a random coefficient specification to account for interpersonal preference heterogeneity, along with selection and attrition equations to distinguish subjects from non-participants. An alternative approach to addressing interpersonal heterogeneity is to

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[40] Annual interest rates vary between 4% and 1,899,088% in Meier and Sprenger [2015], between 15.14% and $10^{24} \times 6.94\%$ in Kirby *et al.* [2002], and between -99.99% and $10^{42} \times 2.91\%$ in Dean and Sautmann [2020].

fit a structural model to observations on each subject separately.[41] This approach precludes formal statistical inferences about the population from which the subjects are drawn as well as corrections for selection and attrition biases.

Time Preferences and the Modeling of Behavioral Noise

Our stochastic specification adopts the contextual utility model of Wilcox [2011] for risk aversion tasks and a Fechner error specification for discounting tasks. Apesteguia and Ballester [2018] argue that the random preference model (RP) is a more attractive stochastic specification to use in structural estimation since it generates choice probabilities which vary monotonically in a risk or time preference parameter. As Wilcox [2008; §4.1.1] already noted, however, the RP model generates implausible predictions in other respects. For example, the RP model predicts that the probability of choosing a lottery rather than its mean-preserving spread remains constant regardless of their variance difference, as long as both alternatives are defined over the same set of three outcomes. Moreover, the stochastic monotonicity property of the RP model is irrelevant to our analysis which jointly estimates the QH discounting function with RDU. This property only applies to the standalone, non-joint, analysis of a decision model with a one-dimensional preference parameter (Lau and Yoo [2021]), such as an exponential discounting function or EUT with a CRRA utility function.[42]

Time Preferences and Arbitrage Opportunities

There are several issues that arise when one considers the opportunities that subjects have when they arbitrage their choices in experiments with choices they might make in the field or with their income and wealth outside the experiment. It is not appropriate to consider all of these issues here,

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[41] This approach is very demanding of the size of the sample of choices that each individual makes, and of course the extra demands of joint estimation of risk preferences and time preferences. A better approach would be to use Bayesian Hierarchical Models, which allow inferences from the pooled sample of individuals to provide informative prior distributions for estimation of individual-level parameters (e.g., Gao, Harrison and Tchernis [2022]). This approach would also allow individual subjects to have different or smaller choice sets, and to have between-subject treatments.

[42] The two-parameter RP model reported by Apesteguia, Ballester and Gutierrez-Daza [2022] jointly estimates a one-parameter RP model for choice data under risk and a separate one-parameter RP model for choice data over time delay. In their study risk preferences are modeled as EUT with a CRRA utility function and time preferences are modeled as exponential discounting.

other than to consider their effects on risk or time preferences that might be considered as sources of temporal instability and hence as confounds to our inferences about dynamic stability.

One possible source of arbitrage was identified by Coller and Williams [1999] and studied extensively in the field by Harrison, Lau and Williams [2002]. It arises when a subject faces borrowing and savings interest rates that are identical, even if they might differ from individual to individual: in the extreme characterization of "perfect capital markets" they are the same for everyone. It follows from the Fisher Separation Theorem that we would then be unable to identify individual time preferences from observed choices in an experiment. However, we do not live in that perfect world. It has been common for experiments on time preferences, such as Coller and Williams [1999] for university students, and Harrison, Lau and Williams [2002], Andersen, Harrison, Lau and Rutström [2008][2014][2018] for adult Danes, to collect information on individual borrowing and savings interest rates. The gap between them makes it possible to infer time preferences without concerns for arbitrage.[43]

A second possible source of arbitrage arises from the possibility that individuals might perfectly integrate their (actual or expected) income from experiments with their personal wealth. This would have an effect on the appropriate argument for the utility function, in turn serving as a possible confound for inferences about risk preferences and/or time preferences. Of course, to the extent that one can control for asset integration by proxies for income or wealth, they can be corrected for. An opportunity for a direct test arises in Denmark, due to the ability to link our experimental data directly to the Danish Registry, which contains administrative data on various types of income and (financial) wealth of the subject. Andersen, Cox, Harrison, Lau, Rutström and Sadiraj [2018] undertake this direct test and find that subjects do not perfectly integrate experimental income with field wealth, and that one can safely ignore that potential confound, at least for the Danish population.

A third possible source of arbitrage mentioned in the literature, such as Cubitt and Read [2007] and Cohen, Ericson, Laibson and White [2020], arises from the relationship between choices over time-dated money and intertemporal consumption. When one thinks of *utility* discount rates, it becomes more natural for some economists to think in terms of the utility of consumption flows rather than stocks of

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[43] When the borrowing and savings rates might influence responses in experiments, or inferences about standard errors of estimated discount rates, standard econometric methods allow one to "censor" observed choices in those experiments since one knows the censoring threshold from the elicited data on those rates. Coller and Williams [1999] document how one does this.

money. In that case, Cubitt and Read [2007] show that, even if one assumes linear utility functions for simplicity of exposition, discount rates over money do not necessarily elicit discount rates over consumption.

One can make structural assumptions about the link between time-dated money and consumption flows, and hence "translate" inferences about the former into inferences about the latter. This is what Andersen, Harrison, Lau and Rutström [2008] did, with a "dual-self" model of decision-making following Fudenberg and Levine [2006]. This approach does not *test* the proposition in question. Instead it makes assumptions, whether or not they are *a priori* plausible, that are hard to test, and that allow one to infer discount rates over consumption flows if true.

It is also possible to ask if the "worst case" preferences that generate differences between discount rates over money and consumption are plausible. This worst case requires that the elasticity of substitution between consumption flows in two periods is "sufficiently" low. Assumptions about this elasticity allow one to bound the possible difference between the two discount rates, and those bounds can become very tight for plausible elasticities. A more direct response is to ask if this "worst case" behavior is indeed observed, when one modifies the basic experimental design to allow it to show itself. A simple modification of the canonical experimental task we used channels a constructive suggestion by Cubitt and Read [2007; p.384]. The constructive suggestion referred to is to simply allow each subject to form a portfolio between the smaller-sooner amount of money and the larger-later amount of money, rather than having to choose one or the other. In other words, to form an *interior* portfolio from the two choice outcomes. There is evidence[44] that one "almost never" observes the preferences, at the individual subject level, that generate the problems posed by Cubitt and Read [2007; p.384]. This suggestion was popularized by Andreoni and Sprenger [2010], with some other extensions.[45] If individual subjects chose an interior solution, the worst-case conditions required for arbitrage between monetary allocations and consumption allocations to be an issue arise; if individual subjects choose one of the extreme solutions, this arbitrage issue does not apply as a matter of theory.

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[44] Presented in Harrison and Swarthout [2011].
[45] Andreoni and Sprenger [2010] extended the method proposed here by giving the subject 100 tokens to allocation between the sooner and later time period, and then *varying the exchange rate* between tokens and money for sooner or later amounts.

<u>Time Preferences Over Real Effort or Over Real Money?</u>

Several studies have made the general point that time preferences should only be elicited over allocations of flows of real effort by the subject, rather than allocations of time-dated stocks of real money. We take the general view that both types of allocations are of interest in different applications, and that there is no formal or informal basis to exclude one in favor of the other. A general concern with these real effort experiments is that they tend to involve somewhat boring tasks or popular games, each of which comes with potential confounds that would be expected *a priori* to vary from subject to subject. In that respect the fungibility of money provides greater control.

Augenblick, Niederle and Sprenger [2015] use a longitudinal design and evaluate constant discounting over money and work effort. They pool intertemporal choices from both waves of their longitudinal experiment with undergraduate students and structurally estimate quasi-hyperbolic discounting functions over money and work effort.[46] They find that subjects are more susceptible to present bias in work effort tasks compared to monetary decision tasks. However, they do not utilize the longitudinal dimension of their data to test for temporal stability and dynamic consistency, since they explicitly *assume* temporal stability.

Augenblick and Rabin [2019] use a longitudinal design with similar work effort tasks (identification of Greek letters) to study intertemporal choice. They asked the subjects to state how many computer based tasks their were willing to complete at different dates and piecemeal payments. All payments to subjects were made at specific dates, and the quasi-hyperbolic discounting function in their intertemporal model is identified from variation in decision and work effort dates while payment dates are kept fixed. They do not evaluate dynamic consistency but test for constant discounting

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[46] Augenblick, Niederle and Sprenger [2015] elicited time preferences from choice allocations in convex budget sets similar to those in Andreoni and Sprenger [2012]. There were four sets of monetary decision tasks in their design. In the first week of the experiment, the subjects were asked to allocate $20 between sooner payments now and/or later payments in four weeks; sooner payments now and/or later payments in seven weeks; and sooner payments in four weeks and/or later payments in seven weeks. Four weeks later the subjects were asked to repeat the first set of decision tasks with sooner payments now and/or later payments in four weeks. These tasks were repeated with different interest rates. In the work effort tasks the subjects were endowed with 50 replications of specific computer based tasks that involved identification of Greek letters and completion of partial Tetris games. In the first week of the experiment, the subjects were asked to allocate the computer based tasks between work effort in one week and/or work effort in two weeks at various intertemporal exchange rates, and one week later they were asked to allocate the computer based tasks between work effort now and/or work effort in one week.

assuming non-linear (dis)utility over work effort and linear utility over income from the work effort tasks.

C. Comparison of Results

Chuang and Schechter [2015] report in a literature review that the "correlation of time preferences over time" in longitudinal experiments with real incentives varies between 0.004 to 0.75. In each of those experiments subjects were asked to complete a structured list of choices between sooner and later payments, and an algebraic transformation of each subject's switching point from sooner to later payments was used as a measure of their time preferences. Thus, those correlation coefficients are effectively descriptive statistics for raw data.

It is difficult to compare the magnitudes of those correlation coefficients across studies, because the algebraic transformation in question varies from study to study. Dean and Sautmann [2020] compute correlation coefficients for switching points observed in two different periods directly, without taking any further transformation of the data. Before computing the correlation coefficients, Kirby et al. [2002] and Kirby [2009] transform each switching point into the parameter $\varkappa_w$ from the hyperbolic discounting function $D_w(t) = 1/(1 + \varkappa_w t)$ reviewed in Section 3. Wölbert and Riedl [2014] instead transform each switching point into an exponential discount rate. Meier and Sprenger [2015] also undertake this type of algebraic calculation, because their structural model does not incorporate correlated preference parameters. Exploiting variations in switching points across tasks with and without front-end delays, they compute that $\text{corr}(\delta_{n1}, \delta_{n2})$ and $\text{corr}(\beta_{n1}, \beta_{n2})$ are equal to 0.246 and 0.364, respectively, which happen to be similar to our structural estimates. Similarly, Yoon [2020] implements a cross-sectional experiment, using algebraic transforms to infer discount rates assuming linear utility. These attempts to derive discounting functions from switching points rely on assumptions that the utility function is linear, that there is no sample selection into the experiment, and that there is no behavioral error in the subject's decision making.

6. Conclusions

At a substantive level, we find dynamic inconsistency in the Danish population between 2009 and 2010. In an analysis that allows for small classification errors, we find that 68.5% of the decision makers display dynamic inconsistency, by violating constant discounting in the second wave of the experiment and/or temporal stability in baseline discount rates between the first and second waves. The results refer to a stratified sample of the adult population in Denmark, and the statistical analysis recognizes preference heterogeneity within that population while controlling for endogenous sample selection and attrition.

A simple numerical example illustrates this finding. Imagine a smaller, sooner (SS) outcome of 1000 kroner in 1 year and a larger, later (LL) outcome of 1100 kroner in 2 years. Using the estimated population means for β and δ from Table 1 for wave 1, we calculate present values of the SS (LL) outcome today of 903.5 (896.2) kroner, making the SS outcome more attractive. If wave 2 occurs 6 months from the present, and using the estimated population means for wave 2, these present values would be 953.9 (976.1) kroner, making the LL outcome more attractive, and reversing the preferences in wave 1. If wave 2 occurs 1 year from the present instead, these present values for wave 2 become 1000 (1012.0), again leading to the preference reversal. Figure 4 illustrates this simple example. The horizontal line is the present value of the SS outcome of 1000 kroner and the upward sloping line is the present value of LL outcome. The dashed, vertical line indicates the indifference point between the SS and LL outcomes, which moves from 1109 kroner when the two options are evaluated today, to 1075 kroner when the two options are evaluated after six months, and to 1087 kroner when they are evaluated after 1 year. Dynamic inconsistency in this simple example thus occurs at annual interest rates between 7.5% and 10.9%.[47]

At a methodological level, we demonstrate the careful interplay of theory, experimental design and econometric inference needed to draw our substantive conclusion on dynamic inconsistency. We

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[47] One can of course look at other numerical examples where the preference reversal moves in the opposite direction. Suppose the baseline discount rate is temporally stable with $\delta_1 = \delta_2 = 0.109$ and the present bias parameter in wave 2, β_2, is equal to 0.989, such that constant discounting is violated in wave 2. The decision maker will make the same choice when the two delayed outcomes are evaluated today and after 6 months, and select the LL outcome when the annual interest rate is higher than 10.9%. However, the decision maker will select the SS outcome at annual interest rates below 12.1% when the two outcomes are evaluated after 1 year, leading to the classical example of preference reversal in intertemporal choice at annual interest rates between 10.9% and 12.1%.

know from the literature on individual discounting that one needs to pay attention to issues such as utility over income (or goods) to draw reliable inferences about time preferences, whether that issue was attended to by experimental design (e.g., Laury, McInnes and Swarthout [2012] and Andreoni and Sprenger [2012]) or joint estimation across several experimental tasks (e.g., Andersen, Harrison, Lau and Rutström [2008]). We also need to consider ways in which discount rates may be non-constant, such as quasi-hyperbolic discounting functions (e.g., Phelps and Pollak [1968], Elster [1979] and Laibson [1997]), and we need to consider corrections for sample selection and attrition in analyses of longitudinal data to infer non-biased preferences of a population (e.g., Harrison, Lau and Yoo [2020]).

Inferences about dynamic consistency requires careful consideration of all these issues. The need to test for constant discounting *and* temporal stability in order to draw inferences about dynamic consistency is stressed by Halevy [2015], which serves as a warning against claiming that non-constant discounting necessarily implies dynamic consistency. Our results show that it is critical to test joint hypotheses of non-constant discounting and temporal stability, which requires longitudinal data and, for now, a structural model of time preferences.

Our econometric analysis is facilitated by access to a remarkable combination of Danish civil registry and experimental data. However, this is not to say that our structural approach to address selection and attrition bias is applicable only in this rich data environment. Longitudinal data sets normally provides sufficient information to correct for endogenous attrition bias, and correction for endogenous selection bias does not necessarily require access to administrative data. In some cases, experiments on risk and time preferences have been combined with large household surveys, such as the Living Standard Survey in Vietnam (Tanaka, Camerer and Ngyuen [2010]), the UK Household Longitudinal Study (Galizzi, Machado and Miniaci [2016]) and the American Life Panel (Dimmock, Kouwenberg, Mitchell and Peijnenburg [2021]). One may then merge the sample of experimental participants with the sample of non-participants in the main survey, and evaluate the effects of self-selection into the experiment relative to the sampling frame of the main survey.

Even when experimental data are not directly linked to survey data, it is customary to comment on potential selection bias by comparing the experimental sample to an unrelated household survey sample in terms of average socio-demographic characteristics. If one is willing to accept the validity of such informal comparisons, one may as well consider the same survey sample as a sample of non-

participants and formally test for endogenous selection. A unique feature of our experimental design is exogenous variation in recruitment fees that we use as an exclusion restriction to estimate the selection equation: without a deliberate design, it is difficult to find an equally attractive restriction. This challenge has not deterred the use of sample selection models in other empirical studies of economic behavior. In our view, formal correction for endogenous selection and attrition bias is an important step that one should consider in any field experiment when those studies intend to make inferences for broader segments of the population.

Table 1: Estimates of Structural Parameters
With Full Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Selection equation: $\beta_1/\sqrt{Var(u_{n1})}$					
female	-0.046	0.063	0.464	-0.171	0.078
young	0.187	0.117	0.112	-0.044	0.417
middle	0.326	0.111	0.003	0.108	0.544
old	0.395	0.102	<0.001	0.195	0.595
high_fee	0.189	0.066	0.004	0.060	0.318
dist	-0.033	0.006	<0.001	-0.044	-0.021
dist^2	0.001	<0.001	<0.001	<0.001	0.001
constant	-0.827	0.104	<0.001	-1.032	-0.623
Attrition equation: $\beta_2/\sqrt{Var(u_{n2})}$					
female	-0.074	0.125	0.552	-0.319	0.171
young	-0.308	0.236	0.191	-0.770	0.154
middle	-0.269	0.217	0.216	-0.694	0.156
old	-0.221	0.204	0.278	-0.621	0.178
IncLow	-0.192	0.170	0.258	-0.525	0.141
IncHigh	-0.122	0.141	0.387	-0.399	0.155
earnings	0.054	0.037	0.147	-0.019	0.126
constant	0.678	0.230	0.003	0.228	1.129
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.109	0.007	<0.001	0.096	0.124
δ_2	0.075	0.009	<0.001	0.062	0.097
β_1	1.002	0.002	<0.001	0.999	1.006
β_2	0.989	0.003	<0.001	0.983	0.995
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.044	0.008	<0.001	0.029	0.059
δ_2	0.041	0.006	<0.001	0.029	0.053
β_1	1.005	0.002	<0.001	1.001	1.008
β_2	0.992	0.002	<0.001	0.987	0.996
Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2					
$\sigma_{\delta 1}$	0.145	0.007	<0.001	0.131	0.159
$\sigma_{\delta 2}$	0.092	0.009	<0.001	0.078	0.113
$\sigma_{\beta 1}$	0.025	0.004	<0.001	0.018	0.033
$\sigma_{\beta 2}$	0.030	0.004	<0.001	0.022	0.039

$\varrho_{\delta 1\ \delta 2}$	0.354	0.059	<0.001	0.246	0.474
$\varrho_{\beta 1\ \beta 2}$	0.273	0.131	0.036	-0.006	0.502

Means of r and φ parameters in wave 1 and wave 2

r_1	0.951	0.066	<0.001	0.821	1.081
r_2	1.076	0.089	<0.001	0.902	1.250
φ_1	2.171	0.195	<0.001	1.787	2.554
φ_2	2.091	0.236	<0.001	1.627	2.554

Standard deviations and correlation coefficients of r and φ parameters in wave 1 and wave 2

σ_{r1}	0.725	0.056	<0.001	0.616	0.835
σ_{r2}	0.597	0.067	<0.001	0.466	0.728
$\sigma_{\varphi 1}$	3.579	0.870	<0.001	1.875	5.283
$\sigma_{\varphi 2}$	2.864	0.876	0.001	1.147	4.581
$\varrho_{r1\ r2}$	0.668	0.078	<0.001	0.515	0.821
$\varrho_{\varphi 1\ \varphi 2}$	0.871	0.063	<0.001	0.747	0.995

Notes: Table B1 in Online Appendix B reports other correlation coefficients.

Table 2: Individual-level Discounting Behavior
With Full Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
A. Constant discounting					
$\Pr(|\beta_1-1|<0.025)$	0.669	0.074	<0.001	0.538	0.823
$\Pr(|\beta_2-1|<0.025)$	0.598	0.066	<0.001	0.481	0.739
B. Temporal stavbility					
$\Pr(|\Delta\beta|<0.025)$	0.530	0.058	<0.001	0.430	0.659
$\Pr(|\Delta\delta|<0.050)$	0.530	0.025	<0.001	0.477	0.575
$\Pr(|\Delta\beta|<0.025; |\Delta\delta|<0.05)$	0.279	0.040	<0.001	0.209	0.364
$\Pr(|\Delta\beta|<0.025; |\Delta\delta|>0.05)$	0.251	0.026	<0.001	0.206	0.308
$\Pr(|\Delta\beta|>0.025; |\Delta\delta|<0.05)$	0.251	0.032	<0.001	0.182	0.306
$\Pr(|\Delta\beta|>0.025; |\Delta\delta|>0.05)$	0.219	0.032	<0.001	0.153	0.280
C. Dynamic consistency					
$\Pr(|\beta_2-1|<0.025; |\Delta\delta|<0.05)$	0.315	0.041	<0.001	0.245	0.406
$\Pr(|\beta_2-1|<0.025; |\Delta\delta|>0.05)$	0.283	0.032	<0.001	0.225	0.349
$\Pr(|\beta_2-1|>0.025; |\Delta\delta|<0.05)$	0.215	0.035	<0.001	0.136	0.274
$\Pr(|\beta_2-1|>0.025; |\Delta\delta|>0.05)$	0.187	0.035	<0.001	0.119	0.257

Table 3: Estimates of Structural Parameters
Without Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.111	0.008	<0.001	0.095	0.128
δ_2	0.095	0.007	<0.001	0.080	0.109
β_1	0.998	0.002	<0.001	0.994	1.002
β_2	0.997	0.003	<0.001	0.991	1.001
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.059	0.005	<0.001	0.049	0.068
δ_2	0.042	0.008	<0.001	0.025	0.058
β_1	1.001	0.002	<0.001	0.997	1.006
β_2	0.997	0.002	<0.001	0.993	1.002
Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2					
$\sigma_{\delta 1}$	0.128	0.009	<0.001	0.111	0.147
$\sigma_{\delta 2}$	0.123	0.011	<0.001	0.100	0.143
$\sigma_{\beta 1}$	0.030	0.004	<0.001	0.023	0.039
$\sigma_{\beta 2}$	0.021	0.003	<0.001	0.015	0.028
$\varrho_{\delta 1 \delta 2}$	0.373	0.034	<0.001	0.318	0.453
$\varrho_{\beta 1 \beta 2}$	0.292	0.146	0.045	-0.062	0.502
Means of r and φ parameters in wave 1 and wave 2					
r_1	0.797	0.072	<0.001	0.656	0.938
r_2	0.955	0.073	<0.001	0.811	1.099
φ_1	3.337	0.307	<0.001	2.736	3.938
φ_2	2.653	0.272	<0.001	2.120	3.186
Standard deviations and correlation coefficients of r and φ parameters in wave 1 and wave 2					
σ_{r1}	0.620	0.039	<0.001	0.543	0.696
σ_{r2}	0.586	0.071	<0.001	0.447	0.725
$\sigma_{\varphi 1}$	3.328	0.779	<0.001	1.802	4.855
$\sigma_{\varphi 2}$	1.945	0.664	0.003	0.643	3.247
$\varrho_{r1 r2}$	0.679	0.077	<0.001	0.528	0.830
$\varrho_{\varphi 1 \varphi 2}$	0.248	0.160	0.120	-0.065	0.562

Notes: Table B2 in Online Appendix B reports other correlation coefficients.

**Table 4: Individual-level Discounting Behavior
without Controls for Sample Selection and Attrition**

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
A. Constant discounting					
$\Pr(|\beta_1-1|<0.025)$	0.589	0.063	<0.001	0.476	0.721
$\Pr(|\beta_2-1|<0.025)$	0.775	0.068	<0.001	0.636	0.901
B. Temporal stavbility					
$\Pr(|\Delta\beta|<0.025)$	0.577	0.072	<0.001	0.434	0.717
$\Pr(|\Delta\delta|<0.050)$	0.496	0.026	<0.001	0.449	0.552
$\Pr(|\Delta\beta|<0.025;|\Delta\delta|<0.05)$	0.318	0.055	<0.001	0.213	0.428
$\Pr(|\Delta\beta|<0.025;|\Delta\delta|>0.05)$	0.260	0.021	<0.001	0.216	0.298
$\Pr(|\Delta\beta|>0.025;|\Delta\delta|<0.05)$	0.179	0.039	<0.001	0.107	0.256
$\Pr(|\Delta\beta|>0.025;|\Delta\delta|>0.05)$	0.244	0.040	<0.001	0.164	0.324
C. Dynamic consistency					
$\Pr(|\beta_2-1|<0.025;|\Delta\delta|<0.05)$	0.420	0.047	<0.001	0.330	0.512
$\Pr(|\beta_2-1|<0.025;|\Delta\delta|>0.05)$	0.354	0.034	<0.001	0.276	0.406
$\Pr(|\beta_2-1|>0.025;|\Delta\delta|<0.05)$	0.076	0.026	0.003	0.031	0.130
$\Pr(|\beta_2-1|<0.025;|\Delta\delta|>0.05)$	0.149	0.044	0.001	0.067	0.240

Figure 1: Population Distributions of Discounting Parameters

Baseline Discount Rate

Present Bias
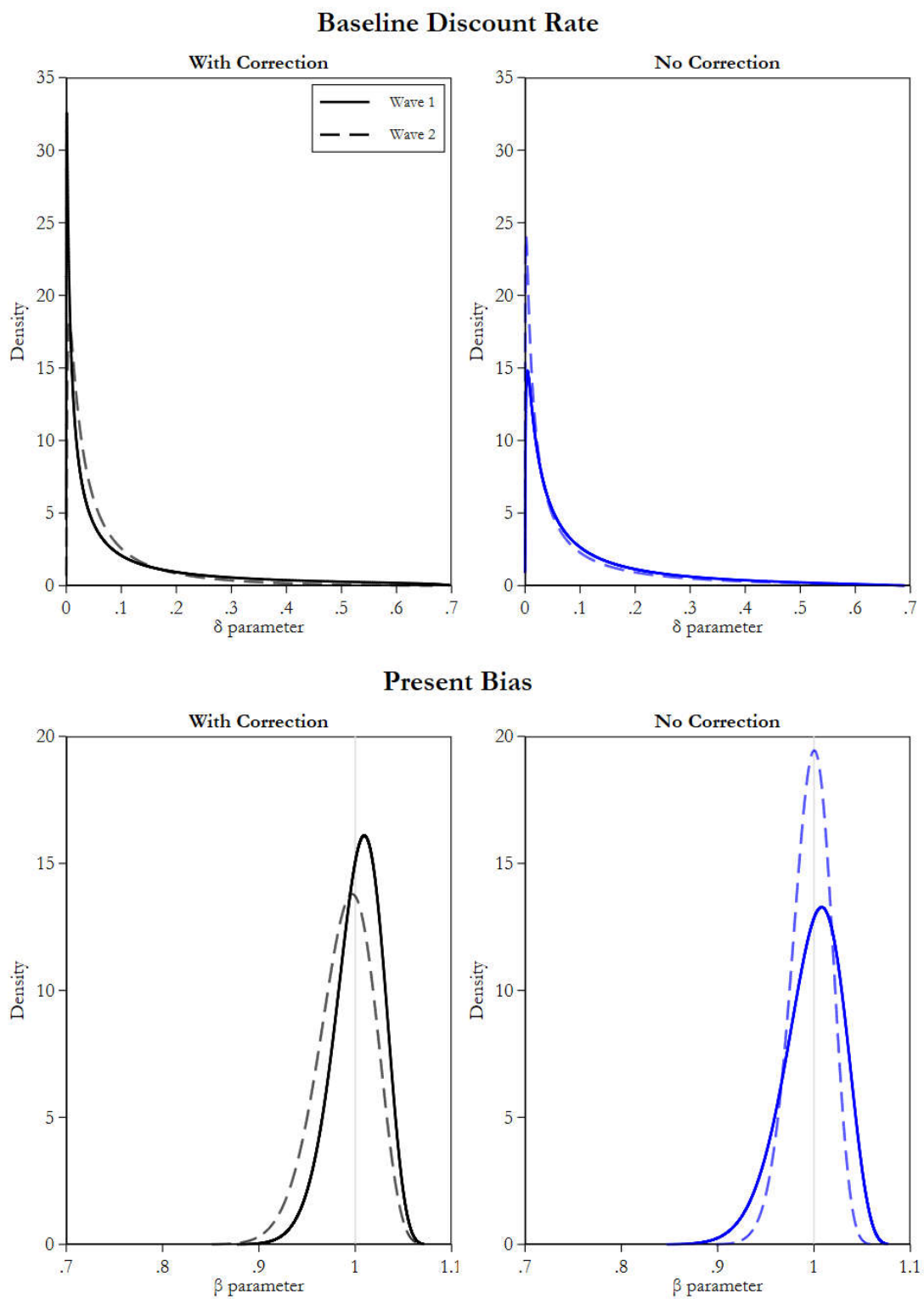
Figure 2: Cumulative Probabilities

Constant Discounting
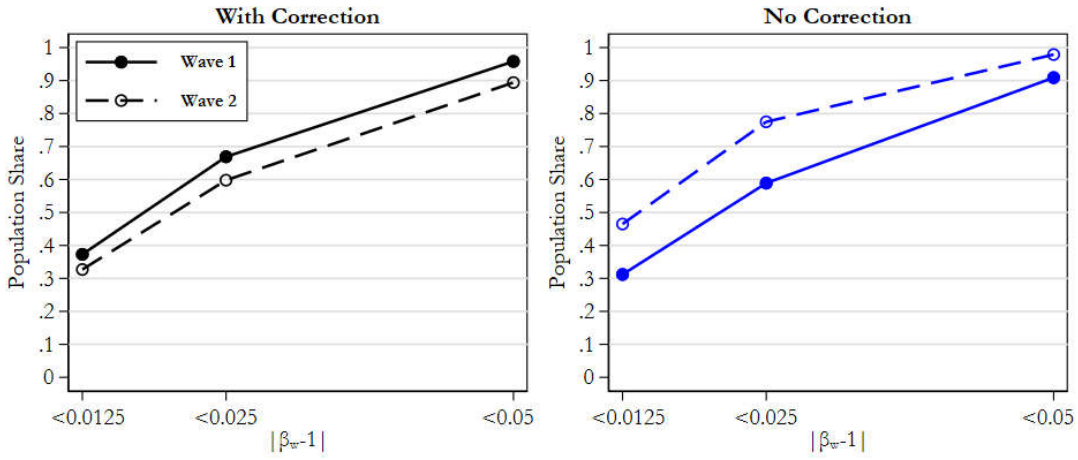
Temporal Stability
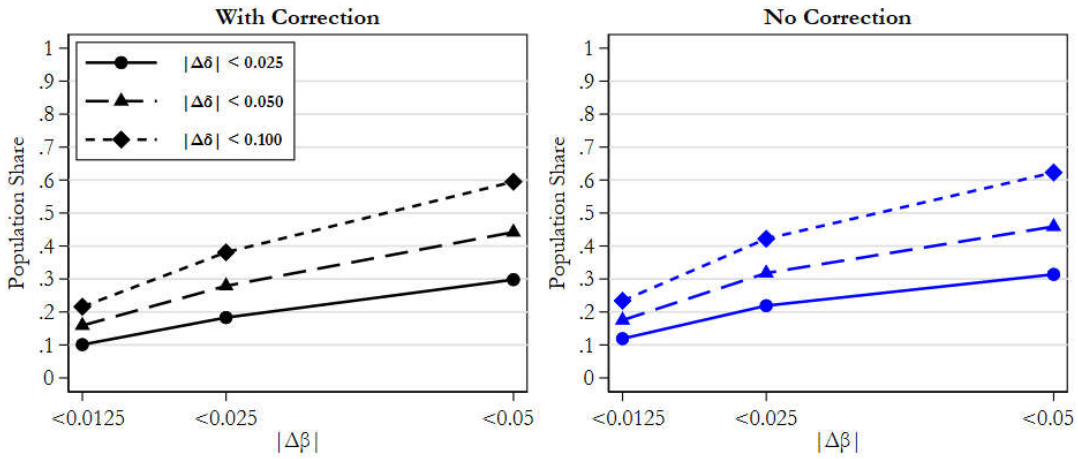
Dynamic Consistency
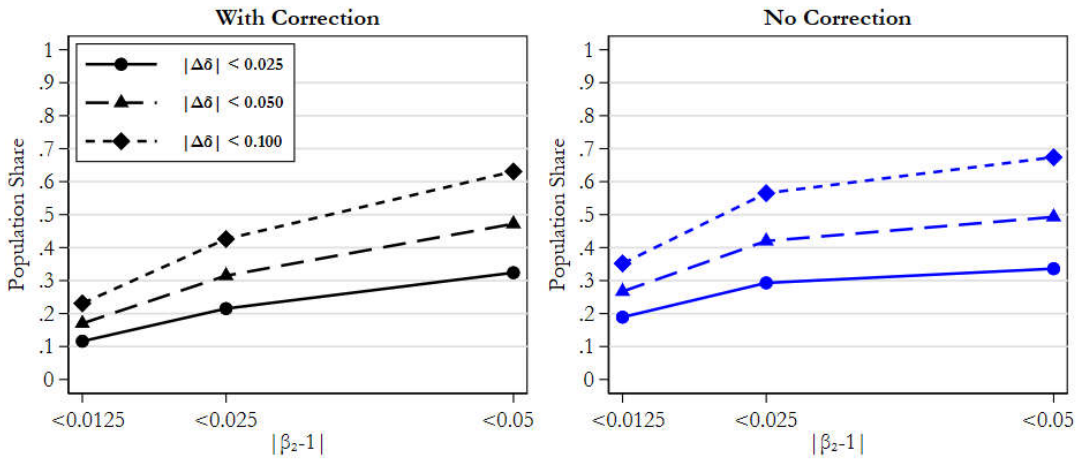
Figure 3: Population Distributions of Risk Parameters

Utility Curvature

Probability Weighting
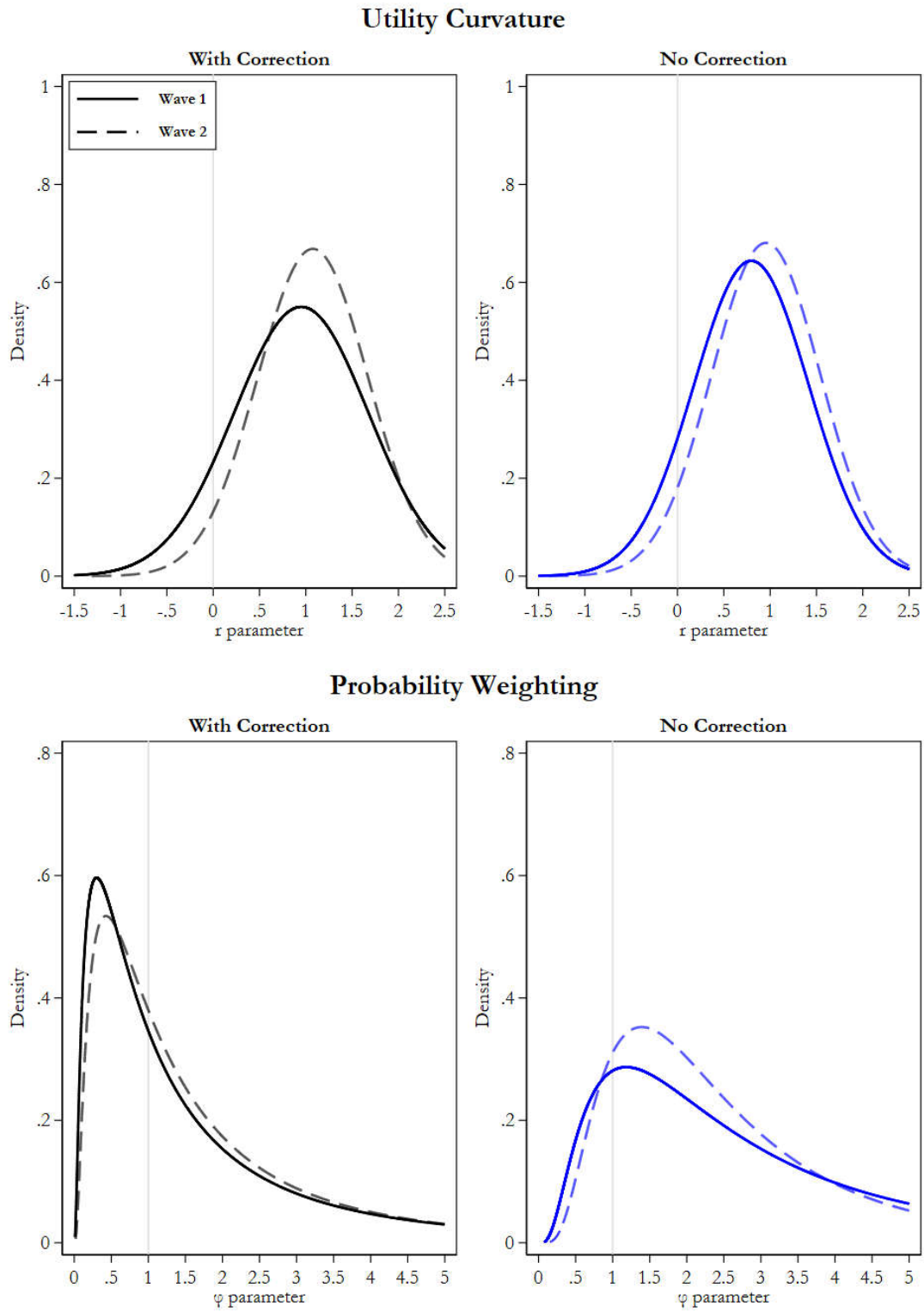
Figure 4: Intertemporal Choice under Risk Neutrality

Evaluated at estimated population means

Present value today

$\delta_1 = 0.109$ and $\beta_1 = 1.002$

Indifference at 1109 kr

Present value in 6 months

$\delta_2 = 0.075$ and $\beta_2 = 0.989$

Indifference at 1075 kr

Present value in 12 months

$\delta_2 = 0.075$ and $\beta_2 = 0.989$

Indifference at 1087 kr
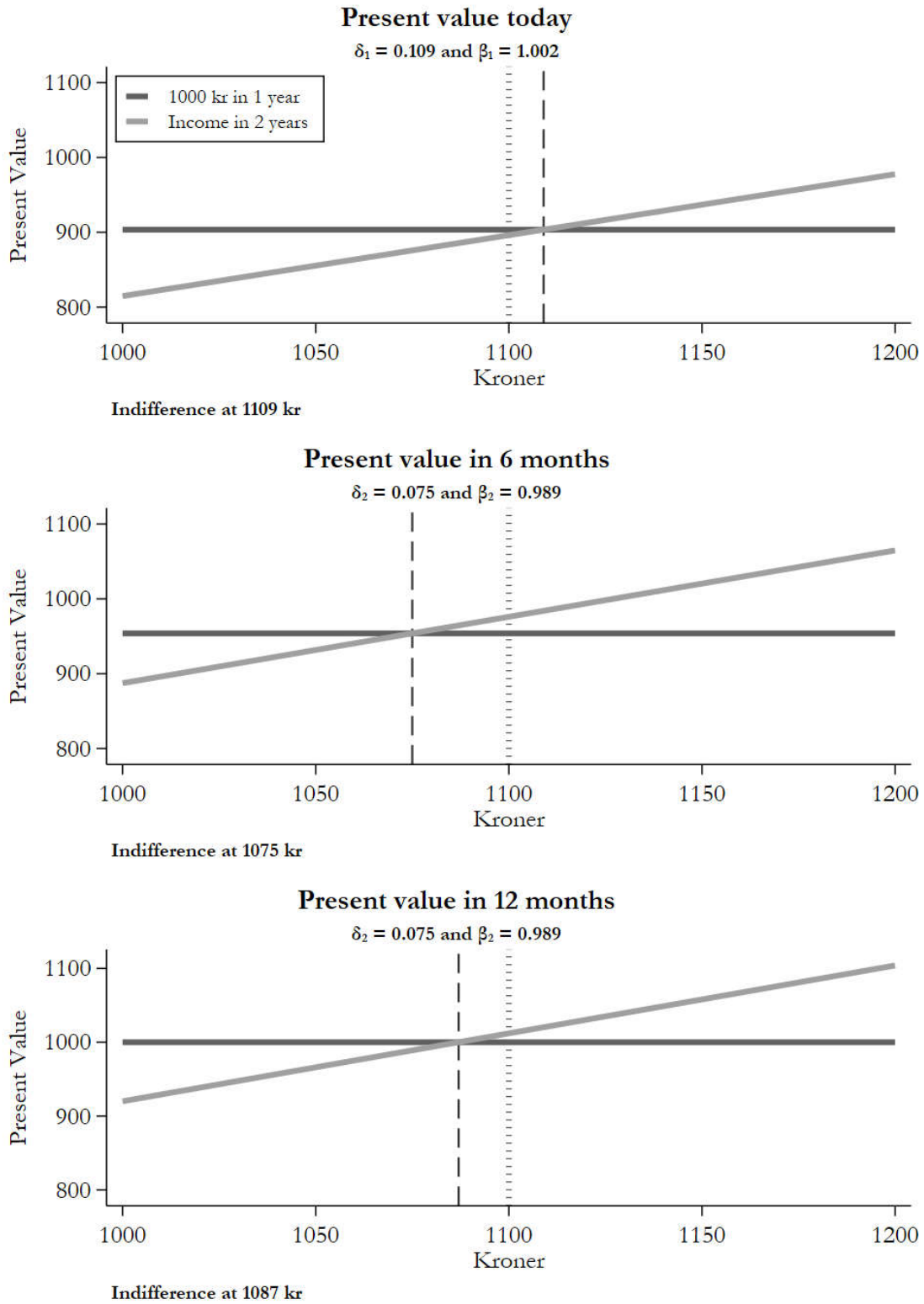
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Online Appendix A: Econometrics

We first write out a structural model of discounting choices and then consider extensions to address unobserved preference heterogeneity and corrections for endogenous selection and attrition bias.

A. Basics

Consider first the estimation of time preferences in a simple framework that does not involve unobserved preference heterogeneity, endogenous sample selection and attrition. Each discounting task presents a choice between option A that pays Y_t in period t and option B that pays $Y_{t+\tau}$ in period t+τ, where $\tau > 0$. Given an atemporal utility function U(m) and a discounting function D(t), the discounted utility of each option is specified as

$$PV_A = D(t) \times U(Y_t + \omega) + D(t + \tau) \times U(\omega) \tag{A1}$$
$$PV_B = D(t) \times U(\omega) + D(t + \tau) \times U(Y_{t+\tau} + \omega) \tag{A2}$$

where ω is a measure of background consumption.[48]

Let y denote a binary indicator of the subject's choice between the sooner payment A (y = 0) or the later payment B (y = 1). Let $\mathbf{I}$(.) denote an indicator function, and assume that the choice depends on the difference in discounted utilities, as well as a behavioral error ν that is normally distributed, N(0, γ^2), such that $y = \mathbf{I}(PV_B - PV_A + \gamma > 0)$. The likelihood of each choice can then be specified as

$$\Pr(\beta, \delta, r, \gamma) = [1 - \Phi(\nabla PV)]^{(1-y)} \times \Phi(\nabla PV)^y \tag{A3}$$

where $\Phi(\nabla PV)$ is the standard normal cumulative distribution evaluated at the index value $\nabla PV = (PV_B - PV_A)/\gamma$. We now turn to the specifications of the discounting and atemporal utility functions that determine the discounted utilities of the sooner and later options.

Our experiment includes discounting tasks with variation in the delay to the sooner option. The resulting basket of choices between an immediate payment and a future payment, and between two future payments, allows us to identify and estimate a quasi-hyperbolic (QH) discounting function

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[48] Andersen, Harrison, Lau and Rutström [2008] show that the addition of background consumption $\omega > 0$ is a sufficient condition to avoid negative discount rates under exponential discounting. The background consumption parameter ω is set exogenously in our model. We follow Andersen, Harrison, Lau and Rutström [2008][2014] and set it to the average daily consumption of private non-durable goods per capita, which was 130 kroner at the time of our experiments in 2009. Andreoni and Sprenger [2012] use convex budget sets to elicit discounting and utility functions, and treat background consumption as an endogenous parameter that is estimated along with the {β, δ} parameters in the quasi-hyperbolic discounting function and a relative risk aversion parameter α under EUT. They do not place any sign restrictions on background consumption and report both positive and negative values for this parameter. To allow negative background consumption, one would need to restrict the relative risk aversion parameter $\alpha > 0$ in their power specification, which is equivalent to $r < 1$ in our utility function. This inequality constraint may be innocuous if one is interested in estimating the population mean of the r parameter, which happens to be smaller than 1. However, when estimating the population distribution of risk attitudes, as long as there are *some* individuals whose r parameters exceed 1, the constraint is not innocuous, regardless of whether the mean is smaller or greater than 1.

$$D(t) = 1 \qquad\qquad \text{if } t = 0 \tag{A4}$$
$$ = \beta \times 1/(1 + \delta)^t \quad \text{if } t > 0$$

where β is a present bias parameter and δ is a *baseline* discount rate for someone with no present bias ($\beta = 1$). More generally, δ can be interpreted as a long-run discount rate, regardless of the value of β. We denote time delay t in years (e.g., $t = 0.5$ for a 6-month horizon), and specify δ on an annualized basis. We assume *a priori* that decision makers display long-run delay aversion ($\delta > 0$), and let β take values on either side of 1 to allow for present bias ($\beta < 1$) as well as future bias ($\beta > 1$). We assume an additive intertemporal utility function, following convention.

The risk aversion tasks allow us to identify the utility function $U(m)$, and apply the joint likelihood strategy of Andersen, Harrison, Lau and Rutström [2008]. Each risk aversion task presents a choice between lottery A and lottery B, where lottery $L \in \{A, B\}$ pays prize $M_{L,1}$ with probability P and prize $M_{L,2}$ with probability $(1 - P)$. Assume that the subject evaluates each lottery according to the Rank Dependent Utility (RDU) model of Quiggin [1982] such that the RDU evaluation of lottery L is

$$RDU_L = w(P) \times U(M_{L,1}) + (1 - w(P)) \times U(M_{L,2}) \tag{A5}$$

where $U(m)$ has a constant relative risk aversion (CRRA) functional form

$$U(m) = m^{(1-r)}/(1-r) \tag{A6}$$

and $w(.)$ is a probability weighting function due to Prelec [1998]

$$w(P) = \exp\{-(-\ln P)^\varphi\}. \tag{A7}$$

The r parameter in $U(m)$ is an index of utility curvature, with concave utility when $r > 0$ and convex utility when $r < 0$. Under Expected Utility Theory (EUT), which assumes $w(P) = P$, the r parameter is interpreted as the coefficient of relative risk aversion. The φ parameter in $w(P)$ determines the shape of the probability weighting function, which follows an inverse-S shape over probabilities if $\varphi < 1$ and an S shape if $\varphi > 1$. An inverse-S shape over-weights probabilities ($w(P) < P$) when P is relatively small, and under-weights probabilities ($w(P) > P$) when P is relatively large; an S shaped probability weighting function under-weights probabilities for small P and over-weights probabilities for large P.

Let y denote a binary indicator of the subject's choice between lottery A ($y = 0$) and lottery B ($y = 1$). Assume that the choice depends on the RDU difference as well as a normally distributed behavioral error ε, such that $y = \mathbf{I}[(RDU_B - RDU_A) + \varepsilon > 0]$. Denote U_{max} and U_{min} as the maximum and minimum of the four outcome utilities, $U(M_{A,1})$, $U(M_{A,2})$, $U(M_{B,1})$ and $U(M_{B,2})$. We adopt the contextual utility model of Wilcox [2011], under which the standard deviation of ε is equal to $(U_{max} - U_{min})$ times a constant μ. The likelihood of the resulting choice observation can then be specified as

$$\Pr(r, \varphi, \mu) = [1 - \Phi(\nabla RDU)]^{(1-y)} \times \Phi(\nabla RDU)^y \tag{A8}$$

where the index ∇RDU is given by $(RDU_B - RDU_A)/[\mu \times (U_{max} - U_{min})]$.

We now turn to the joint likelihood of multiple discounting choices and risk aversion choices made by the same subject. The data follows a multi-level panel structure. At the upper level we have two "waves" of data on subjects participating in both the initial (wave 1) and the repeat (wave 2) experiment. At the lower level, within each wave, we have 80 observations per subject since each subject completed

40 discounting tasks and 40 risk aversion tasks. We subscript the marginal likelihood of a discounting choice in (A3) as $Pr_{njw}(\beta_w, \delta_w, r_w, \gamma)$ henceforth, to emphasize that it describes subject n's choice in the jth discounting task of wave w. Similarly, we subscript the marginal likelihood of a risk aversion choice in (A8) as $Pr_{njw}(r_w, \varphi_w, \mu)$. For simplicity of exposition only, assume for now that the preference parameters $\alpha_w = \{\beta_w, \delta_w, r_w, \varphi_w\}$ may vary across waves, but not across subjects. The conditional likelihood of all observations by subject n is then given by

$$CL_n(\alpha_1, \alpha_2, \gamma, \mu) = \Pi_j Pr_{nj1}(\beta_1, \delta_1, r_1, \gamma) \times \Pi_j Pr_{nj1}(r_1, \varphi_1, \mu) \qquad \text{if } s_{n2} = 0 \qquad (A9)$$
$$= \Pi_w [\Pi_j Pr_{njw}(\beta_w, \delta_w, r_w, \gamma) \times \Pi_j Pr_{njw}(r_w, \varphi_w, \mu)] \qquad \text{if } s_{n2} = 1$$

where s_{n2} is an indicator of whether subject n participated in only the first wave ($s_{n2} = 0$) or both waves ($s_{n2} = 1$).

Equation (A9) presents a restrictive structural model that does not consider unobserved preference heterogeneity, within-individual correlation across repeated observations, selection bias, and attrition bias. Our econometric analysis relaxes all these restrictions, and we return to these extensions after discussing how one can test the hypotheses of constant discounting, temporal stability, and dynamic consistency with this structure.

B. *Unobserved Preference Heterogeneity*

We first extend the model to address interpersonal preference heterogeneity, by adopting a random parameter specification. Let $\alpha_{n1} = \{\beta_{n1}, \delta_{n1}, r_{n1}, \varphi_{n1}\}$ and $\alpha_{n2} = \{\beta_{n2}, \delta_{n2}, r_{n2}, \varphi_{n2}\}$ denote structural parameters describing subject n's preferences in wave 1 and 2. We consider each of these 8 parameters as a draw from a distinct statistical distribution that describes the population distribution of that parameter in a particular wave w. To accommodate within-individual correlation in preferences, we allow for an unrestricted set of correlations among the 8 distributions. These include, *inter alia*, $corr(\delta_{n1}, \delta_{n2})$ that captures correlation of baseline discount rates between the two waves, and $corr(\delta_{n1}, r_{n1})$ that is a measure of correlation between utility curvature and baseline discount rates in wave 1. To accommodate temporal instability in preferences, we allow the joint distribution of α_{n1} to be different from the joint distribution of α_{n2}.

The baseline discount rate δ_{nw} is assumed to be positive so that the subject displays aversion to time delay, at least over the long run. To impose this sign constraint we specify the marginal distribution as logit-normal, also known as Johnson's [1949] "S_B" distribution. This distribution offers qualitatively similar flexibility as a Beta distribution, with the added advantage of being amenable to a multivariate extension. We set $\delta_{nw} \in (0, 0.7)$, based on the required sign constraint and the highest annual interest rate of 50% offered in our discounting tasks, which translates into annual effective rates of 60% with quarterly compounding and 65% with daily compounding. The present bias parameter is also assumed to follow a logit-normal distribution, and we set $\beta_{nw} \in (0.7, 1.1)$.[49] As to the RDU parameters, we follow

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[49] The selection of lower and upper bounds for β_{nw} is informed by individual-level maximum likelihood estimates that Andersen, Harrison, Lau and Rutström [2014] computed using data from the first wave of the experiment. They focus squarely on the issue of constant discounting in the first wave and do not exploit the longitudinal dimension of the data to evaluate temporal stability and dynamic consistency, nor utilize information from the Danish Civil Registry to correct for selection bias.

Harrison, Lau, and Yoo [2020], and model r_{nw} and φ_{nw} using normal and log-normal distributions, respectively.[50]

Given our distributional assumptions, we can invert each $\boldsymbol{\alpha}_{nw} = \{\beta_{nw}, \delta_{nw}, r_{nw}, \varphi_{nw}\}$ into a vector of normally distributed random variables. Denote by $T^{-1}(\boldsymbol{\alpha}_{nw}) = \{\beta^*_{nw}, \delta^*_{nw}, r_{nw}, \ln(\varphi_{nw})\}$ this inverse transform, where β^*_{nw} and δ^*_{nw} are the underlying "normal" components of "logit-normal" β_{nw} and δ_{nw}. Specifically, $\beta_{nw} = 0.7 + 0.4 \times \Lambda(\beta^*_{nw})$ and $\delta_{nw} = 0.7 \times \Lambda(\delta^*_{nw})$, where $\Lambda(.)$ is the standard logistic distribution function. Denote by $h(\boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2} | \boldsymbol{\theta})$ the density function that describes the joint population distribution of the 8 structural parameters, as a function of distributional parameters $\boldsymbol{\theta}$ that include the [8 × 1] mean vector and [8 × 8] variance-covariance matrix of multivariate normal $\{T^{-1}(\boldsymbol{\alpha}_{n1}), T^{-1}(\boldsymbol{\alpha}_{n2})\}$.

To estimate the heterogeneous preferences model, the joint likelihood of subject n's choices can be specified by "integrating out" the random parameters, in the same manner as one integrates out the random intercept from a random effects probit model (e.g. Wooldridge [2010; p.613]). Specifically, the likelihood function is given by

$$L_n(\boldsymbol{\theta}, \gamma, \mu) = \iint CL_n(\boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2}, \gamma, \mu)h(\boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2}; \boldsymbol{\theta})d\boldsymbol{\alpha}_{n1}d\boldsymbol{\alpha}_{n2} \tag{A10}$$

where $CL_n(.)$ is the conditional likelihood function under the baseline model, as defined in (A9). The unconditional likelihood function $L_n(\boldsymbol{\theta}, \gamma, \mu)$ does not have a closed-form expression, but can be approximated using simulation methods (Train [2009; p.144-145]). We compute maximum simulated likelihood (MSL) estimates of preference parameters $\boldsymbol{\theta}$ and behavioral noise parameters γ and μ by maximizing a simulated analogue to the sample log-likelihood function $\Sigma_n L_n(\boldsymbol{\theta}, \gamma, \mu)$, where subject index n iterates over 413 subjects who participated in the first experiment or both experiments. Note that unlike the baseline model's conditional likelihood function, $L_n(\boldsymbol{\theta}, \gamma, \mu)$ is a multivariate integral, the log of which does not break down into the sum of marginal log-likelihood functions at the choice level. The structural model with random parameters thus allows for panel correlation across repeated observations on the same individual, in a similar manner as the usual random effects probit model.

The MSL estimator of $\boldsymbol{\theta}$ is consistent under the assumption of exogenous sample selection and panel attrition, which is a strong assumption to maintain considering that the subjects self-selected to participate into both waves of the experiment. We therefore relax that assumption next.

C. Sample Selection and Attrition

To correct for selection bias, we take the initial pool of 1,996 invited individuals as a random sample from the overall adult population, and model the selection process that led to the 413 subjects observed in the initial experiment. From this sample of 413 subjects, 354 subjects were invited to participate in the repeat experiment. To correct for attrition bias, we take those 354 subjects as a random sample from the sub-population that would have self-selected into the first experiment if invited, and model the attrition process that led to 182 subjects in the repeat experiment. This general strategy is consistent with our experimental design, under which the experimenter exogenously

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[50] Harrison, Lau, and Yoo [2020] introduce econometric methods to estimate structural models of decision making under risk with correction for selection and attrition bias. The study is based on the same risk preference experiment as the present analysis, but does not consider the joint estimation of risk and time preferences.

determines whether someone is invited to the initial experiment, and subsequently which subjects in the initial experiment are invited to the repeat experiment.

The extension to address selection and attrition biases can be specified as a system of binary response models. Let s_{nw} be an indicator of whether subject n accepted the invitation to the experiment in wave w ($s_{nw} = 1$) or not ($s_{nw} = 0$). For those who were not invited to the second experiment, we set $s_{n2} = -1$. Assume that each observed outcome s_{nw} is determined by a latent propensity s^*_{nw}, such that $s_{n1} = \mathbf{I}[s^*_{n1} > 0]$, and $s_{n2} = \mathbf{I}[s^*_{n1} > 0 \cap s^*_{n2} > 0]$ if subject n was invited to the second experiment. The latent propensities are specified as

$$s^*_{n1} = \mathbf{X}_{n1}\boldsymbol{\pi}_1 + u_{n1} = \mathbf{X}_{n1}\boldsymbol{\pi}_1 + (a_{n1} + e_{n1}) \tag{A11}$$
$$s^*_{n2} = \mathbf{X}_{n2}\boldsymbol{\pi}_2 + u_{n2} = \mathbf{X}_{n2}\boldsymbol{\pi}_2 + (a_{n2} + e_{n2}) \tag{A12}$$

where $\mathbf{X}_{nw}$ is a vector of explanatory variables including a constant, $\boldsymbol{\pi}_w$ is a conformable vector of coefficients to estimate, and u_{nw} is a random disturbance. u_{nw} is further decomposed into an error component a_{nw} that is potentially correlated across selection and attrition processes, and an idiosyncratic error term e_{nw} that is orthogonal to a_{nw}. Assume that the correlated components a_{n1} and a_{n2} are bivariate normal, and each idiosyncratic error e_{nw} is independently normal. Without loss of generality, we normalize this system by setting $E(a_{nw}) = E(e_{nw}) = 0$, $Var(a_{n1}) = Var(e_{n1}) = Var(e_{n2}) = 1$ and $Var(a_{n2}) = 1 + \sigma_{s1s2}$, where $\sigma_{s1s2} = Cov(u_{n1}, u_{n2}) = Cov(a_{n1}, a_{n2})$.

We link the latent selection and attrition processes to the structural model by allowing error components a_{n1} and a_{n2} to be correlated with the structural parameters $\boldsymbol{\alpha}_{n1} = \{\beta_{n1}, \delta_{n1}, r_{n1}, \varphi_{n1}\}$ and $\boldsymbol{\alpha}_{n2} = \{\beta_{n2}, \delta_{n2}, r_{n2}, \varphi_{n2}\}$. This type of correlation captures the essence of the self-selection problem: individuals with certain types of preferences may be more likely to self-select into each wave of the experiment. For example, if $corr(a_{n1}, \delta_{n1}) < 0$ then those who participate in the experiment tend to have a smaller baseline discount rate in the first wave than the average Dane. Note also that when viewed in isolation from the structural model, (A11) and (A12) are akin to the "selection" and "outcome" equations of the usual probit model with sample selection: when $corr(a_{n1}, a_{n2}) \neq 0$, one cannot correct for attrition bias without correcting for selection bias because attrition outcomes are only observed for those who self-selected into the initial experiment.

Denote by $q(a_{n1}, a_{n2}, \boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2}; \boldsymbol{\Theta})$ the density function that describes the population joint distribution of selection errors, attrition errors and 8 structural parameters. The distributional parameters $\boldsymbol{\Theta}$ include all parameters in $\boldsymbol{\theta}$ that characterize the joint distribution of the 8 structural parameters, $h(\boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2} | \boldsymbol{\theta})$, as well as additional covariance parameters that are unique to the [10 × 10] variance-covariance matrix of $\{a_{n1}, a_{n2}, T^{-1}(\boldsymbol{\alpha}_{n1}), T^{-1}(\boldsymbol{\alpha}_{n2})\}$.

The joint likelihood of observations on individual n under the full model that accounts for preference heterogeneity, selection bias and attrition bias can be then specified as

$$L_n(\boldsymbol{\Theta}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \gamma, \mu) = \int\int\int\int F_n(a_{n1}, a_{n2}, \boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \gamma, \mu)q(a_{n1}, a_{n2}, \boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2}; \boldsymbol{\Theta})da_{n1}da_{n2}d\boldsymbol{\alpha}_{n1}d\boldsymbol{\alpha}_{n2} \tag{A13}$$

where the kernel $F_n(.)$ is obtained by augmenting the baseline model's conditional likelihood function, $CL_n(.)$ in equation (A9), with the marginal probabilities of selection and attrition conditional on particular draws of a_{n1} and a_{n2}. Specifically,

$$F_n(a_{n1}, a_{n2}, \boldsymbol{\alpha}_{n1}, \boldsymbol{\alpha}_{n2}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \gamma, \mu) \tag{A14}$$
$$= 1 - \Phi(z_{n1}) \qquad\qquad \text{if } s_{n1} = 0$$

$$
\begin{aligned}
&= \Phi(z_{n1}) \times \textstyle\prod_j \Pr_{nj1}(\beta_{n1}, \delta_{n1}, r_{n1}, \gamma) \times \textstyle\prod_j \Pr_{nj1}(r_{n1}, \varphi_{n1}, \mu) && \text{if } s_{n1} = 1,\ s_{n2} = -1 \\
&= \Phi(z_{n1}) \times [1 - \Phi(z_{n2})] \times \textstyle\prod_j \Pr_{nj1}(\beta_{n1}, \delta_{n1}, r_{n1}, \gamma) \times \textstyle\prod_j \Pr_{nj1}(r_{n1}, \varphi_{n1}, \mu) && \text{if } s_{n1} = 1,\ s_{n2} = 0 \\
&= \Phi(z_{n1}) \times \Phi(z_{n2}) \times \textstyle\prod_w [\textstyle\prod_j \Pr_{njw}(\beta_{nw}, \delta_{nw}, r_{nw}, \gamma) \times \textstyle\prod_j \Pr_{njw}(r_{nw}, \varphi_{nw}, \mu)] && \text{if } s_{n1} = 1,\ s_{n2} = 1
\end{aligned}
$$

where $z_{nw} = \mathbf{X}_{nw}\boldsymbol{\pi}_w + a_{nw}$. The exact form of the likelihood function $L_n(\boldsymbol{\Theta}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \gamma, \mu)$ thus varies for those who rejected the first invitation ($s_{n1} = 0$), those who participated in the first experiment but did not receive the second invitation ($s_{n1} = 1$, $s_{n2} = -1$), those who participated in the first experiment but rejected the second invitation ($s_{n1} = 1$, $s_{n2} = 0$), and finally those who participated in both experiments ($s_{n1} = s_{n2} = 1$). Like the likelihood function under the heterogeneous preferences model without correction for selection and attrition in (A10), $L_n(\boldsymbol{\Theta}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \gamma, \mu)$ is a multidimensional integral that does not have a closed form expression. We compute the MSL estimates of $\boldsymbol{\Theta}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2\ \gamma$, and μ by maximizing a simulated analogue to the sample log-likelihood function $\sum_n \ln(L_n(\boldsymbol{\Theta}, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \gamma, \mu))$, where index n iterates over all 1,996 individuals who were invited to the first experiment.

Additional References

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Online Appendix B: Additional Results Assuming Rank-Dependent Utility Theory

Table B1: Correlation Coefficients with Full Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Correlation coefficients					
ϱ_{s1s2}	-0.273	0.055	<0.001	-0.382	-0.165
$\varrho_{s1\,\delta1}$	0.030	0.029	0.299	-0.011	0.102
$\varrho_{s1\,\delta2}$	0.389	0.029	<0.001	0.338	0.450
$\varrho_{s1\,\beta1}$	-0.216	0.026	<0.001	-0.273	-0.171
$\varrho_{s1\,\beta2}$	0.270	0.049	<0.001	0.147	0.341
$\varrho_{s1\,r1}$	-0.370	0.031	<0.001	-0.431	-0.309
$\varrho_{s1\,r2}$	-0.495	0.043	<0.001	-0.580	-0.410
$\varrho_{s1\,\varphi1}$	0.410	0.038	<0.001	0.336	0.484
$\varrho_{s1\,\varphi2}$	0.419	0.050	<0.001	0.321	0.516
$\varrho_{s2\,\delta1}$	-0.041	0.029	0.155	-0.113	-0.001
$\varrho_{s2\,\delta2}$	-0.196	0.062	0.001	-0.304	-0.061
$\varrho_{s2\,\beta1}$	0.179	0.031	<0.001	0.107	0.230
$\varrho_{s2\,\beta2}$	-0.143	0.067	0.032	-0.258	0.007
$\varrho_{s2\,r1}$	-0.198	0.070	0.005	-0.335	-0.060
$\varrho_{s2\,r2}$	0.194	0.090	0.031	0.018	0.371
$\varrho_{s2\,\varphi1}$	-0.346	0.051	<0.001	-0.445	-0.246
$\varrho_{s2\,\varphi2}$	-0.483	0.057	<0.001	-0.594	-0.372
$\varrho_{\delta1\,\beta1}$	-0.119	0.067	0.075	-0.229	0.030
$\varrho_{\delta1\,\beta2}$	-0.030	0.052	0.565	-0.134	0.069
$\varrho_{\delta2\,\beta1}$	-0.356	0.035	<0.001	-0.404	-0.267
$\varrho_{\delta2\,\beta2}$	0.283	0.083	0.001	0.114	0.439
$\varrho_{\delta1\,r1}$	-0.378	0.040	<0.001	-0.465	-0.306
$\varrho_{\delta1\,r2}$	-0.311	0.065	<0.001	-0.446	-0.189
$\varrho_{\delta2\,r1}$	-0.456	0.053	<0.001	-0.555	-0.346
$\varrho_{\delta2\,r2}$	-0.349	0.063	<0.001	-0.463	-0.216
$\varrho_{\delta1\,\varphi1}$	0.268	0.063	<0.001	0.158	0.405
$\varrho_{\delta1\,\varphi2}$	0.112	0.066	0.089	0.002	0.260
$\varrho_{\delta2\,\varphi1}$	0.354	0.088	<0.001	0.181	0.526
$\varrho_{\delta2\,\varphi2}$	0.342	0.115	0.003	0.124	0.567
$\varrho_{\beta1\,r1}$	0.183	0.037	<0.001	0.108	0.255
$\varrho_{\beta1\,r2}$	0.307	0.038	<0.001	0.226	0.375

$\varrho_{\beta2\,r1}$	-0.071	0.066	0.284	-0.194	0.066
$\varrho_{\beta2\,r2}$	0.005	0.074	0.949	-0.114	0.176
$\varrho_{\beta1\,\varphi1}$	-0.213	0.035	<0.001	-0.271	-0.132
$\varrho_{\beta1\,\varphi2}$	-0.341	0.088	<0.001	-0.491	-0.144
$\varrho_{\beta2\,\varphi1}$	-0.213	0.035	<0.001	-0.271	-0.132
$\varrho_{\beta2\,\varphi2}$	0.098	0.074	0.189	-0.078	0.215
$\varrho_{r1\,\varphi1}$	-0.337	0.068	<0.001	-0.470	-0.204
$\varrho_{r1\,\varphi2}$	-0.149	0.102	0.143	-0.349	0.050
$\varrho_{r2\,\varphi1}$	-0.596	0.067	<0.001	-0.727	-0.466
$\varrho_{r2\,\varphi2}$	-0.524	0.084	<0.001	-0.689	-0.360

Table B2: Correlation Coefficients without Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
			Correlation coefficients		
$\varrho_{\delta 1 \, \beta 1}$	-0.225	0.034	<0.001	-0.293	-0.159
$\varrho_{\delta 1 \, \beta 2}$	-0.366	0.101	<0.001	-0.511	-0.119
$\varrho_{\delta 2 \, \beta 1}$	-0.327	0.047	<0.001	-0.434	-0.250
$\varrho_{\delta 2 \, \beta 2}$	0.269	0.067	<0.001	0.142	0.403
$\varrho_{\delta 1 \, r1}$	-0.265	0.041	<0.001	-0.338	-0.177
$\varrho_{\delta 1 \, r2}$	-0.492	0.042	<0.001	-0.544	-0.379
$\varrho_{\delta 2 \, r1}$	-0.492	0.042	<0.001	-0.544	-0.379
$\varrho_{\delta 2 \, r2}$	-0.539	0.035	<0.001	-0.587	-0.453
$\varrho_{\delta 1 \, \varphi 1}$	0.266	0.045	<0.001	0.173	0.347
$\varrho_{\delta 1 \, \varphi 2}$	0.059	0.064	0.360	-0.085	0.168
$\varrho_{\delta 2 \, \varphi 1}$	0.663	0.057	<0.001	0.507	0.724
$\varrho_{\delta 2 \, \varphi 2}$	0.252	0.102	0.014	-0.005	0.404
$\varrho_{\beta 1 \, r1}$	0.247	0.084	0.003	0.107	0.437
$\varrho_{\beta 1 \, r2}$	0.034	0.056	0.546	-0.057	0.167
$\varrho_{\beta 2 \, r1}$	0.205	0.079	0.009	0.047	0.357
$\varrho_{\beta 2 \, r2}$	-0.265	0.066	<0.001	-0.381	-0.119
$\varrho_{\beta 1 \, \varphi 1}$	0.137	0.073	0.061	-0.044	0.239
$\varrho_{\beta 1 \, \varphi 2}$	-0.111	0.117	0.342	-0.310	0.154
$\varrho_{\beta 2 \, \varphi 1}$	0.137	0.073	0.061	-0.044	0.239
$\varrho_{\beta 2 \, \varphi 2}$	0.200	0.177	0.259	-0.149	0.525
$\varrho_{r1 \, \varphi 1}$	-0.608	0.101	<0.001	-0.805	-0.411
$\varrho_{r1 \, \varphi 2}$	-0.433	0.192	0.024	-0.810	-0.056
$\varrho_{r2 \, \varphi 1}$	-0.558	0.097	<0.001	-0.748	-0.368
$\varrho_{r2 \, \varphi 2}$	-0.467	0.125	<0.001	-0.712	-0.222

Online Appendix C: Additional Results Assuming Expected Utility Theory

Table C1: Estimates of Structural Parameters
with Full Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Selection equation: $\beta_1/\sqrt{Var(u_{n1})}$					
female	-0.133	0.067	0.046	-0.264	-0.002
young	0.109	0.123	0.372	-0.131	0.350
middle	0.205	0.113	0.071	-0.017	0.427
old	0.363	0.104	<0.001	0.160	0.566
high_fee	0.168	0.067	0.013	0.036	0.299
dist	-0.034	0.006	<0.001	-0.046	-0.021
$dist^2$	0.001	<0.001	<0.001	<0.001	0.001
constant	-0.716	0.108	<0.001	-0.928	-0.504
Attrition equation: $\beta_2/\sqrt{Var(u_{n2})}$					
female	-0.092	0.128	0.474	-0.343	0.160
young	-0.179	0.274	0.512	-0.716	0.357
middle	0.041	0.240	0.863	-0.429	0.511
old	-0.215	0.213	0.311	-0.632	0.201
IncLow	-0.167	0.183	0.361	-0.525	0.191
IncHigh	-0.287	0.152	0.059	-0.586	0.011
earnings	0.076	0.042	0.074	-0.007	0.159
constant	0.682	0.226	0.003	0.239	1.126
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.124	0.007	<0.001	0.111	0.136
δ_2	0.133	0.009	<0.001	0.118	0.152
β_1	0.993	0.002	<0.001	0.988	0.996
β_2	1.003	0.002	<0.001	0.998	1.008
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.061	0.005	<0.001	0.050	0.072
δ_2	0.075	0.007	<0.001	0.060	0.090
β_1	0.999	0.002	<0.001	0.995	1.003
β_2	1.010	0.003	<0.001	1.005	1.015

Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2

$\sigma_{\delta 1}$	0.147	0.007	<0.001	0.133	0.160
$\sigma_{\delta 2}$	0.146	0.009	<0.001	0.128	0.165
$\sigma_{\beta 1}$	0.048	0.006	<0.001	0.041	0.055
$\sigma_{\beta 2}$	0.043	0.004	<0.001	0.036	0.051
$\varrho_{\delta 1 \delta 2}$	0.295	0.034	<0.001	0.214	0.350
$\varrho_{\beta 1 \beta 2}$	0.212	0.076	0.006	0.074	0.373

Means of r parameters in wave 1 and wave 2

r_1	0.572	0.046	<0.001	0.481	0.662
r_2	0.776	0.056	<0.001	0.666	0.885

Standard deviations and correlation coefficients of r parameters in wave 1 and wave 2

σ_{r1}	0.658	0.042	<0.001	0.575	0.741
σ_{r2}	0.535	0.043	<0.001	0.450	0.619
$\varrho_{r1 r2}$	0.795	0.063	<0.001	0.671	0.919

Other correlation coefficients

ϱ_{s1s2}	-0.341	0.057	<0.001	-0.450	-0.232
$\varrho_{s1 \delta 1}$	0.074	0.024	0.002	0.052	0.144
$\varrho_{s1 \delta 2}$	-0.350	0.022	<0.001	-0.402	-0.316
$\varrho_{s1 \beta 1}$	0.073	0.019	<0.001	0.015	0.091
$\varrho_{s1 \beta 2}$	-0.434	0.038	<0.001	-0.481	-0.348
$\varrho_{s2 \delta 1}$	-0.137	0.026	<0.001	-0.189	-0.087
$\varrho_{s2 \delta 2}$	0.079	0.063	0.210	-0.038	0.209
$\varrho_{s2 \beta 1}$	0.084	0.023	<0.001	0.033	0.123
$\varrho_{s2 \beta 2}$	0.057	0.063	0.365	-0.078	0.168
$\varrho_{s1 r1}$	0.107	0.039	0.006	0.030	0.184
$\varrho_{s1 r2}$	0.329	0.041	<0.001	0.248	0.410
$\varrho_{s2 r1}$	0.071	0.039	0.070	-0.006	0.147
$\varrho_{s2 r2}$	-0.524	0.047	<0.001	-0.616	-0.431
$\varrho_{\delta 1 \beta 1}$	-0.148	0.033	<0.001	-0.209	-0.080
$\varrho_{\delta 1 \beta 2}$	-0.227	0.043	<0.001	-0.321	-0.152
$\varrho_{\delta 2 \beta 1}$	-0.310	0.038	<0.001	-0.368	-0.220
$\varrho_{\delta 2 \beta 2}$	0.493	0.042	<0.001	0.401	0.565

$\varrho_{\delta1\,r1}$	-0.154	0.030	<0.001	-0.208	-0.092
$\varrho_{\delta1\,r2}$	-0.046	0.038	0.223	-0.120	0.028
$\varrho_{\delta2\,r1}$	-0.267	0.056	<0.001	-0.378	-0.158
$\varrho_{\delta2\,r2}$	-0.242	0.056	<0.001	-0.371	-0.150
$\varrho_{\beta1\,r1}$	0.238	0.022	<0.001	0.205	0.290
$\varrho_{\beta1\,r2}$	0.269	0.060	<0.001	0.156	0.391
$\varrho_{\beta2\,r1}$	0.122	0.064	0.056	0.013	0.263
$\varrho_{\beta2\,r2}$	0.210	0.074	0.004	0.066	0.352

Table C2: Individual-level Discounting Behavior
with Full Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
A. Constant discounting					
$Pr(|\beta_1-1|<0.025)$	0.396	0.029	<0.001	0.349	0.463
$Pr(|\beta_2-1|<0.025)$	0.411	0.037	<0.001	0.346	0.489
B. Temporal stavbility					
$Pr(|\Delta\beta|<0.025)$	0.357	0.024	<0.001	0.312	0.409
$Pr(|\Delta\delta|<0.050)$	0.408	0.016	<0.001	0.370	0.434
$Pr(|\Delta\beta|<0.025,|\Delta\delta|<0.05)$	0.179	0.017	<0.001	0.150	0.216
$Pr(|\Delta\beta|<0.025,|\Delta\delta|>0.05)$	0.178	0.014	<0.001	0.150	0.205
$Pr(|\Delta\beta|>0.025,|\Delta\delta|<0.05)$	0.228	0.016	<0.001	0.188	0.252
$Pr(|\Delta\beta|>0.025,|\Delta\delta|>0.05)$	0.414	0.020	<0.001	0.383	0.460
C. Dynamic consistency					
$Pr(|\beta_2-1|<0.025,|\Delta\delta|<0.05)$	0.210	0.020	<0.001	0.171	0.250
$Pr(|\beta_2-1|<0.025,|\Delta\delta|>0.05)$	0.201	0.020	<0.001	0.167	0.246
$Pr(|\beta_2-1|>0.025,|\Delta\delta|<0.05)$	0.198	0.016	<0.001	0.162	0.224
$Pr(|\beta_2-1|>0.025,|\Delta\delta|>0.05)$	0.391	0.028	<0.001	0.337	0.447

Table C3: Estimates of Structural Parameters
without Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.131	0.008	<0.001	0.114	0.145
δ_2	0.114	0.009	<0.001	0.096	0.132
β_1	0.999	0.002	<0.001	0.995	1.003
β_2	0.988	0.004	<0.001	0.980	0.996
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.064	0.007	<0.001	0.050	0.079
δ_2	0.066	0.007	<0.001	0.053	0.079
β_1	1.006	0.003	<0.001	0.999	1.012
β_2	0.992	0.004	<0.001	0.984	0.999
Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2					
$\sigma_{\delta 1}$	0.153	0.008	<0.001	0.135	0.166
$\sigma_{\delta 2}$	0.124	0.014	<0.001	0.096	0.148
$\sigma_{\beta 1}$	0.043	0.006	<0.001	0.033	0.057
$\sigma_{\beta 2}$	0.035	0.005	<0.001	0.028	0.048
$\varrho_{\delta 1\, \delta 2}$	0.639	0.063	<0.001	0.507	0.752
$\varrho_{\beta 1\, \beta 2}$	0.250	0.119	0.036	-0.055	0.412
Means of r parameters in wave 1 and wave 2					
r_1	0.625	0.057	<0.001	0.513	0.737
r_2	0.760	0.059	<0.001	0.645	0.875
Standard deviations and correlation coefficients of r parameters in wave 1 and wave 2					
σ_{r1}	0.655	0.047	<0.001	0.562	0.747
σ_{r2}	0.514	0.057	<0.001	0.401	0.627
$\varrho_{r1\, r2}$	0.885	0.059	<0.001	0.770	1.000

$\varrho_{\delta_1 \beta_1}$	0.039	0.070	0.576	-0.129	0.147
$\varrho_{\delta_1 \beta_2}$	-0.241	0.128	0.060	-0.427	0.065
$\varrho_{\delta_2 \beta_1}$	-0.225	0.057	<0.001	-0.344	-0.120
$\varrho_{\delta_2 \beta_2}$	0.020	0.129	0.878	-0.188	0.309
$\varrho_{\delta_1 \beta_1}$	-0.225	0.034	<0.001	-0.293	-0.159
$\varrho_{\delta_1 \beta_2}$	-0.366	0.101	<0.001	-0.511	-0.119
$\varrho_{\delta_2 \beta_1}$	-0.327	0.047	<0.001	-0.434	-0.250
$\varrho_{\delta_2 \beta_2}$	0.269	0.067	<0.001	0.142	0.403
$\varrho_{\delta_1 r_1}$	-0.374	0.042	<0.001	-0.447	-0.283
$\varrho_{\delta_1 r_2}$	-0.418	0.039	<0.001	-0.491	-0.337
$\varrho_{\delta_2 r_1}$	-0.614	0.047	<0.001	-0.698	-0.514
$\varrho_{\delta_2 r_2}$	-0.364	0.087	<0.001	-0.527	-0.185
$\varrho_{\beta_1 r_1}$	0.231	0.086	0.007	0.088	0.420
$\varrho_{\beta_1 r_2}$	0.071	0.129	0.586	-0.149	0.359
$\varrho_{\beta_2 r_1}$	0.088	0.076	0.247	-0.067	0.233
$\varrho_{\beta_2 r_2}$	0.202	0.096	0.035	0.027	0.401

Table C4: Individual-level Discounting Behavior
without Controls for Sample Selection and Attrition

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
A. Constant discounting					
$\Pr(\|\beta_1 - 1\| < 0.025)$	0.430	0.062	<0.001	0.317	0.558
$\Pr(\|\beta_2 - 1\| < 0.025)$	0.528	0.059	<0.001	0.402	0.629
B. Temporal stavbility					
$\Pr(\|\Delta\beta\| < 0.025)$	0.384	0.042	<0.001	0.294	0.460
$\Pr(\|\Delta\delta\| < 0.050)$	0.534	0.036	<0.001	0.466	0.609
$\Pr(\|\Delta\beta\| < 0.025, \|\Delta\delta\| < 0.05)$	0.227	0.034	<0.001	0.160	0.295
$\Pr(\|\Delta\beta\| < 0.025, \|\Delta\delta\| > 0.05)$	0.157	0.017	<0.001	0.119	0.188
$\Pr(\|\Delta\beta\| > 0.025, \|\Delta\delta\| < 0.05)$	0.306	0.031	<0.001	0.257	0.381
$\Pr(\|\Delta\beta\| > 0.025, \|\Delta\delta\| > 0.05)$	0.309	0.032	<0.001	0.248	0.374
C. Dynamic consistency					
$\Pr(\|\beta_2 - 1\| < 0.025, \|\Delta\delta\| < 0.05)$	0.297	0.044	<0.001	0.209	0.381
$\Pr(\|\beta_2 - 1\| < 0.025, \|\Delta\delta\| > 0.05)$	0.231	0.028	<0.001	0.173	0.279
$\Pr(\|\beta_2 - 1\| > 0.025, \|\Delta\delta\| < 0.05)$	0.237	0.026	<0.001	0.194	0.297
$\Pr(\|\beta_2 - 1\| > 0.025, \|\Delta\delta\| > 0.05)$	0.235	0.040	<0.001	0.163	0.319

Figure C1: Population Distributions of Discounting Parameters

Baseline Discount Rate

Present Bias
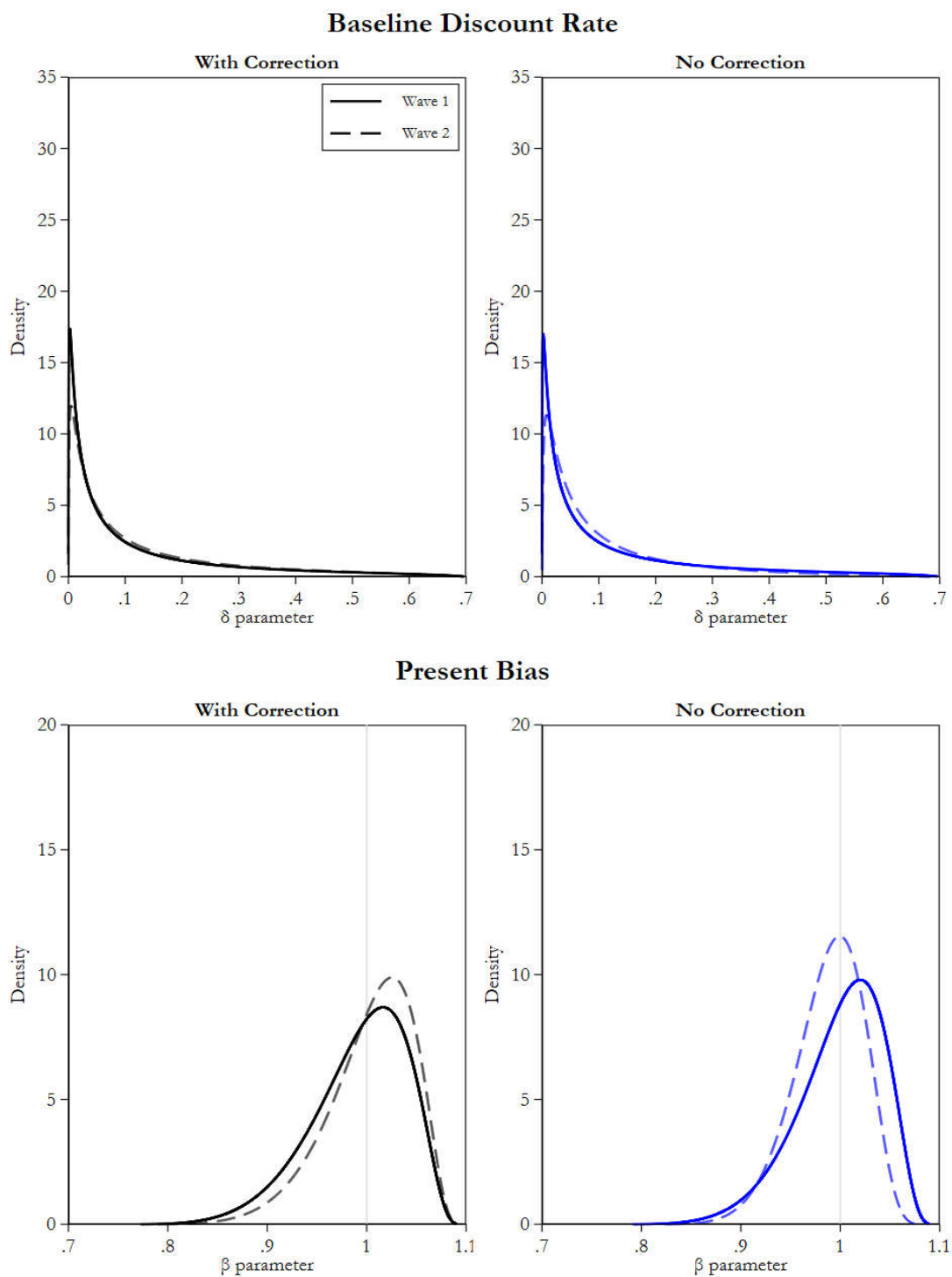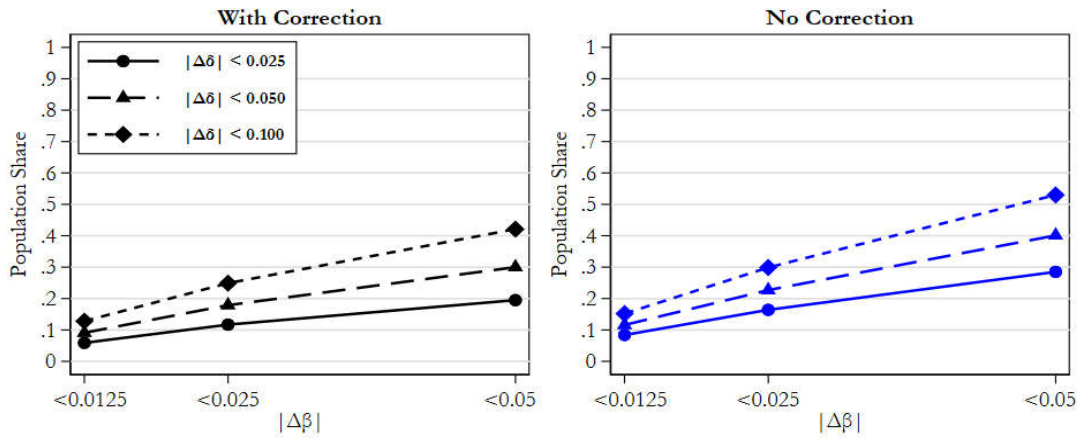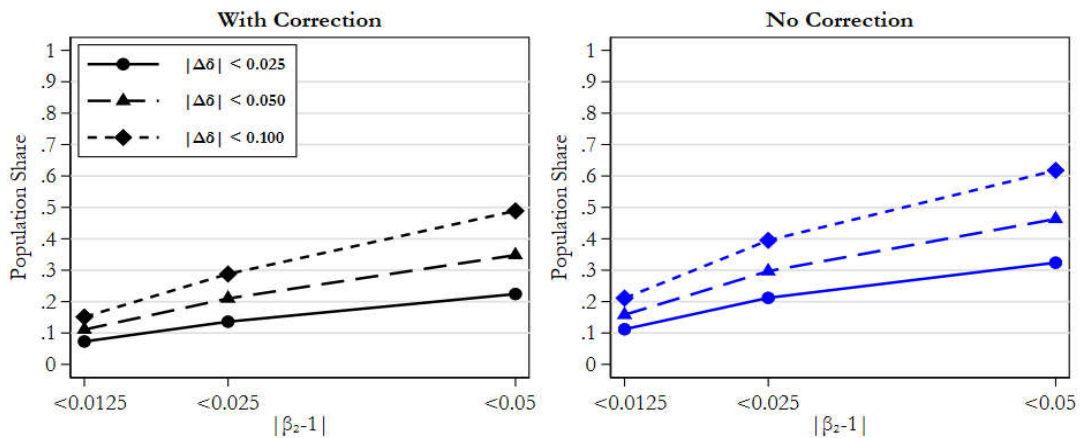
Figure C2: Cumulative Probabilities

Constant Discounting
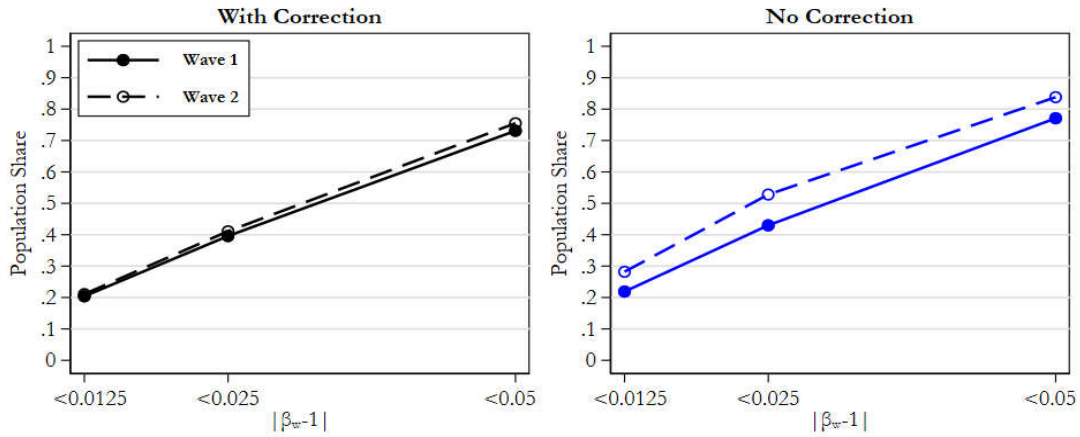
Temporal Stability

Dynamic Consistency

Online Appendix D: Alternative Control Variables in Selection and Attrition Equations

Table D1: Estimates of Structural Parameters
with Control for Recruitment Fee in Sample Selection and Attrition Equations

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Selection equation: $\beta_1/\sqrt{Var(u_{n1})}$					
female	-0.046	0.063	0.465	-0.170	0.078
young	0.188	0.117	0.109	-0.042	0.417
middle	0.326	0.111	0.003	0.109	0.544
old	0.395	0.102	<0.001	0.194	0.595
high_fee	0.192	0.065	0.003	0.064	0.320
dist	-0.033	0.006	<0.001	-0.044	-0.021
dist^2	0.001	<0.001	<0.001	0.000	0.001
constant	-0.829	0.104	<0.001	-1.033	-0.624
Attrition equation: $\beta_2/\sqrt{Var(u_{n2})}$					
female	-0.100	0.120	0.403	-0.336	0.135
young	-0.314	0.235	0.182	-0.775	0.147
middle	-0.279	0.214	0.193	-0.699	0.141
old	-0.226	0.199	0.256	-0.617	0.165
IncLow	-0.226	0.169	0.182	-0.558	0.106
IncHigh	-0.138	0.139	0.320	-0.410	0.134
earnings	0.049	0.037	0.187	-0.024	0.123
high_fee	-0.186	0.122	0.126	-0.425	0.052
constant	0.813	0.229	<0.001	0.365	1.262
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.106	0.007	<0.001	0.096	0.124
δ_2	0.075	0.009	<0.001	0.062	0.096
β_1	1.002	0.002	<0.001	0.999	1.006
β_2	0.989	0.003	<0.001	0.984	0.994
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.043	0.007	<0.001	0.029	0.058
δ_2	0.041	0.006	<0.001	0.029	0.053
β_1	1.005	0.002	<0.001	1.001	1.008
β_2	0.992	0.002	<0.001	0.987	0.996

Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2

$\sigma_{\delta 1}$	0.144	0.007	<0.001	0.131	0.159
$\sigma_{\delta 2}$	0.092	0.009	<0.001	0.077	0.113
$\sigma_{\beta 1}$	0.025	0.003	<0.001	0.019	0.032
$\sigma_{\beta 2}$	0.029	0.004	<0.001	0.023	0.037
$\varrho_{\delta 1 \delta 2}$	0.341	0.055	<0.001	0.248	0.464
$\varrho_{\beta 1 \beta 2}$	0.300	0.135	0.027	0.000	0.526

Means of r and φ parameters in wave 1 and wave 2

r_1	0.953	0.065	<0.001	0.826	1.081
r_2	1.078	0.087	<0.001	0.907	1.248
φ_1	2.172	0.196	<0.001	1.788	2.555
φ_2	2.093	0.222	<0.001	1.659	2.527

Standard deviations and correlation coefficients of r and φ parameters in wave 1 and wave 2

σ_{r1}	0.727	0.055	<0.001	0.621	0.834
σ_{r2}	0.596	0.064	<0.001	0.471	0.722
$\sigma_{\varphi 1}$	3.578	0.861	<0.001	1.891	5.265
$\sigma_{\varphi 2}$	2.869	0.844	0.001	1.215	4.523
$\varrho_{r1 r2}$	0.667	0.081	<0.001	0.509	0.825
$\varrho_{\varphi 1 \varphi 2}$	0.870	0.063	<0.001	0.747	0.993

Table D2: Estimates of Structural Parameters
with Alternative Measure of Previous Earnings in Sample Attrition Equation

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Selection equation: $\beta_1/\sqrt{Var(u_{n1})}$					
female	-0.046	0.063	0.466	-0.170	0.078
young	0.187	0.117	0.110	-0.043	0.417
middle	0.326	0.111	0.003	0.109	0.544
old	0.395	0.102	<0.001	0.196	0.595
high_fee	0.189	0.066	0.004	0.060	0.317
dist	-0.033	0.006	<0.001	-0.044	-0.021
dist^2	0.001	<0.001	<0.001	0.000	0.001
constant	-0.828	0.104	<0.001	-1.032	-0.623
Attrition equation: $\beta_2/\sqrt{Var(u_{n2})}$					
female	-0.078	0.125	0.534	-0.323	0.168
young	-0.326	0.237	0.168	-0.790	0.138
middle	-0.282	0.219	0.197	-0.711	0.147
old	-0.224	0.204	0.270	-0.623	0.175
IncLow	-0.189	0.170	0.267	-0.523	0.145
IncHigh	-0.113	0.140	0.421	-0.388	0.162
ra_income	0.071	0.072	0.320	-0.069	0.211
constant	0.697	0.226	0.002	0.254	1.141
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.109	0.007	<0.001	0.097	0.123
δ_2	0.075	0.009	<0.001	0.062	0.096
β_1	1.002	0.002	<0.001	0.999	1.006
β_2	0.989	0.003	<0.001	0.984	0.994
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.044	0.007	<0.001	0.030	0.058
δ_2	0.041	0.006	<0.001	0.029	0.053
β_1	1.005	0.002	<0.001	1.001	1.008
β_2	0.992	0.002	<0.001	0.987	0.996

Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2

$\sigma_{\delta 1}$	0.145	0.007	<0.001	0.131	0.159
$\sigma_{\delta 2}$	0.092	0.009	<0.001	0.078	0.113
$\sigma_{\beta 1}$	0.025	0.004	<0.001	0.018	0.033
$\sigma_{\beta 2}$	0.030	0.004	<0.001	0.022	0.038
$\varrho_{\delta 1 \delta 2}$	0.354	0.058	<0.001	0.247	0.474
$\varrho_{\beta 1 \beta 2}$	0.274	0.129	0.033	0.004	0.507

Means of r and φ parameters in wave 1 and wave 2

r_1	0.951	0.064	<0.001	0.827	1.076
r_2	1.076	0.086	<0.001	0.907	1.246
φ_1	2.168	0.194	<0.001	1.787	2.549
φ_2	2.091	0.232	<0.001	1.636	2.547

Standard deviations and correlation coefficients of r and φ parameters in wave 1 and wave 2

σ_{r1}	0.725	0.055	<0.001	0.617	0.833
σ_{r2}	0.597	0.066	<0.001	0.468	0.725
$\sigma_{\varphi 1}$	3.567	0.864	<0.001	1.873	5.261
$\sigma_{\varphi 2}$	2.865	0.867	0.001	1.164	4.565
$\varrho_{r1 r2}$	0.668	0.076	<0.001	0.519	0.818
$\varrho_{\varphi 1 \varphi 2}$	0.870	0.062	<0.001	0.748	0.992

Table D3: Estimates of Structural Parameters
without Socio-Demographic Variables in Sample Selection and Attrition Equations

Variable	Estimate	Standard Error	p-value	95% Confidence Interval	
Selection equation: $\beta_1/\sqrt{Var(u_{n1})}$					
high_fee	0.227	0.066	0.001	0.099	0.356
constant	-0.790	0.044	<0.001	-0.876	-0.704
Attrition equation: $\beta_2/\sqrt{Var(u_{n2})}$					
high_fee	-0.152	0.123	0.216	-0.393	0.089
ra_income	0.056	0.072	0.434	-0.085	0.198
constant	0.377	0.106	<0.001	0.168	0.585
Means of δ and β parameters in wave 1 and wave 2					
δ_1	0.109	0.006	<0.001	0.098	0.120
δ_2	0.076	0.007	<0.001	0.065	0.094
β_1	1.003	0.001	<0.001	1.000	1.006
β_2	0.993	0.001	<0.001	0.990	0.996
Medians of δ and β parameters in wave 1 and wave 2					
δ_1	0.041	0.004	<0.001	0.033	0.050
δ_2	0.039	0.005	<0.001	0.029	0.049
β_1	1.005	0.002	<0.001	1.002	1.009
β_2	0.992	0.001	<0.001	0.993	0.998
Standard deviations and correlation coefficients of δ and β parameters in wave 1 and wave 2					
$\sigma_{\delta 1}$	0.149	0.007	<0.001	0.135	0.161
$\sigma_{\delta 2}$	0.094	0.009	<0.001	0.081	0.117
$\sigma_{\beta 1}$	0.024	0.002	<0.001	0.020	0.028
$\sigma_{\beta 2}$	0.029	0.003	<0.001	0.025	0.037
$\varrho_{\delta 1\,\delta 2}$	0.388	0.105	<0.001	0.153	0.558
$\varrho_{\beta 1\,\beta 2}$	0.204	0.063	0.001	0.078	0.322

Means of r and φ parameters in wave 1 and wave 2

r_1	0.968	0.048	<0.001	0.874	1.063
r_2	1.096	0.070	<0.001	0.960	1.232
φ_1	2.123	0.181	<0.001	1.767	2.478
φ_2	2.010	0.226	<0.001	1.567	2.453

Standard deviations and correlation coefficients of r and φ parameters in wave 1 and wave 2

σ_{r1}	0.738	0.047	<0.001	0.646	0.830
σ_{r2}	0.610	0.097	<0.001	0.420	0.800
$\sigma_{\varphi1}$	3.310	0.746	<0.001	1.848	4.773
$\sigma_{\varphi2}$	2.483	0.836	0.001	0.844	4.121
$\varrho_{r1\,r2}$	0.665	0.104	<0.001	0.461	0.868
$\varrho_{\varphi1\,\varphi2}$	0.871	0.071	<0.001	0.732	1.010