

# ESTIMATING RISK PREFERENCES FOR INDIVIDUALS: A BAYESIAN APPROACH

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## Abstract

Risk preferences play a central role in many descriptive and normative inferences about economic behavior. It is natural to expect that risk preference are heterogeneous. We evaluate a series of Bayesian econometric models that allow for different levels of heterogeneity of individual risk preferences. We carefully compare inferences about risk preferences across these models using both simulated and observed data. Using simulated data, we show the effectiveness of the Bayesian approach in recovering the underlying true parameters and evaluate the cost of model mis-specification. Using observed data, we evaluate the heterogeneity present in a typical subject pool. We suggest extensions and applications of the models and illustrate the application to the evaluation of behavioral welfare gains or losses from insurance purchases.

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# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Experimental Data</b>	<b>5</b>
<b>3</b>	<b>Expected Utility Theory</b>	<b>7</b>
3.1	Model and Specifications . . . . .	7
3.1.1	Specification VI . . . . .	9
3.1.2	Specification V . . . . .	11
3.1.3	Specification IV . . . . .	11
3.1.4	Specification III . . . . .	12
3.1.5	Specification II . . . . .	12
3.1.6	Specification I . . . . .	12
3.2	Data Simulation and Parameter Recovery . . . . .	12
3.2.1	Parameter Recovery . . . . .	13
3.2.2	Model Sensitivity to Mis-specification . . . . .	15
3.3	EUT Inferences with Observed Data . . . . .	16
<b>4</b>	<b>Rank Dependent Utility Theory</b>	<b>19</b>
4.1	Model and Specifications . . . . .	19
4.2	RDU Inferences with Observed Data . . . . .	21
<b>5</b>	<b>Applications</b>	<b>23</b>
5.1	Characterizing Risk Preferences for Their Own Sake . . . . .	23
5.2	Joint Estimation of Risk Preferences and Other Preferences . . . . .	23
5.3	Inferences Based on Risk Preferences . . . . .	24
<b>6</b>	<b>Conclusions</b>	<b>26</b>
<b>A</b>	<b>Experiment Instructions and Lottery Parameters (ONLINE)</b>	<b>39</b>
A.1	Instructions in the Gain Frame . . . . .	39
A.2	Lottery Parameters in the Gain Frame . . . . .	43
<b>B</b>	<b>Additional Results (ONLINE)</b>	<b>48</b>
<b>C</b>	<b>Template Stata Codes (ONLINE)</b>	<b>54</b>
C.1	Data and Variables . . . . .	54
C.2	Likelihood Function . . . . .	54
C.3	Main Syntax . . . . .	57

# 1 Introduction

Individual risk preferences are heterogeneous, and inferring risk preferences for individuals can be problematic. Sometimes it is not possible to elicit a *large sample* of responses from every individual, due to time constraints or concerns with boredom. This problem arises often when risk preferences are not the primary focus of analysis, but are still needed to control for potential confounds. Sometimes the *variety of stimuli* needed to identify certain models of risk preferences makes it difficult to ask many questions of each type of stimulus. An important example arises when considering gain frame, mixed frame and loss frame lotteries needed to estimate risk preferences under Cumulative Prospect Theory (CPT). In this case one might have to present a limited number of lottery choices to each subject that sample from a wider battery of lottery choices, knowing *a priori* that the wider battery may have useful information for estimation.<sup>1</sup> Sometimes the *precision of estimates* of risk preferences directly affects the precision of parameters conditioned on risk preferences, placing a premium on reliable estimates of risk preferences.<sup>2</sup> Sometimes the estimation of individual risk preferences is needed in order to make *normative evaluations* of observed out-of-sample choices. We illustrate this type of application in section 5, with welfare evaluations of decisions to purchase insurance or not. And sometimes, there is simply no standard model of risk preferences that seems to characterize the observed behavior of *some* individuals well, even if standard models do characterize the observed behavior of *most* individuals.

These considerations motivate a derived demand for conditioning inferences about individual risk preferences with priors from other sources, which is what Bayesian analysis allows one to do systematically.

One natural source of priors comes from estimates of models of risk preferences that pool data from all subjects, and then conditions inferences about each parameter on a list of observable demographic characteristics. One can then generate predictions about the distributions of these

<sup>1</sup>To take an extreme example, one might have elicited responses from subjects over gain frame and loss frame lotteries, but not over mixed frame lotteries. In this case one cannot identify a key parameter of CPT, the utility loss aversion parameter  $\lambda$ . One might then return to a different sample from the same population and elicit responses to mixed frame lotteries, and seek to make inferences about loss aversion from both samples with priors over the exchangeability of the two samples.

<sup>2</sup>Section 5.2 discusses several important examples.

parameters that *condition* on the specific value of the characteristics of each individual, and use these predictions as priors for Bayesian inferences that pool the sample data for that individual. The posterior distributions that are estimated for each individual are then a reflection of the prior and the sample. When the sample is relatively uninformative, for one reason or another, the prior will play a greater role in conditioning the posterior. The advantage of this approach is that it will “always” generate priors for each individual, and these priors can be conditioned on a potentially long list of characteristics specific to each individual. We focus on the role of this class of priors, since they are generally available.

In various forms Bayesian analysis has long been applied to condition inferences from experimental data. Nilsson, Rieskamp and Wagenmakers (2011) employ hierarchical Bayesian methods to make inferences about risk preferences under CPT.<sup>3</sup> They recognized the identification problem for CPT models with certain utility specifications that allow different utility curvature for “gains” and “losses.” They simulated data, using the popular point estimates from Tversky and Kahneman (1992), to test the ability of their model to recover them. They found that their model generated biased results for several key CPT parameters, and correctly concluded (p. 89) that it “... is likely that these results are caused by a peculiarity of CPT, that is, its ability to account for loss aversion in multiple ways.” As explained by Harrison and Swarthout (2021), what they discovered is a well-known *theoretical* identification issue with CPT that requires the use of one of several dogmatic priors about the definition of loss aversion. In any event, they estimated all models with the dogmatic prior that the utility curvature for “gains” and “losses” was the same, to avoid this identification problem.

We consider a simple case in which one collects data from subjects using a binary choice task in which the subject selects one of two possible lotteries. We provide an overview of Bayesian estimation of the parameters of the utility functions and probability weighting functions characterizing popular models of risk preferences. A key focus of our analysis is to consider a number of model structures that vary in the level of subject heterogeneity, and to assess the ability of Bayesian models to account correctly for that heterogeneity. In the simplest model, *the representative agent model*, we assume all subjects share the same parameters and there is no heterogeneity in their preferences whatsoever.

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<sup>3</sup>Murphy and ten Brincke (2018) estimate hierarchical models of CPT using non-Bayesian random coefficient methods.

Somewhere in the middle, we consider a popular model in the literature where we only allow for heterogeneity at the demographic level and assume two subjects' preferences are the same as long as they share the same demographic characteristics. In the richest model, *the hierarchical model*, we assume each subject's parameters are drawn from a hierarchical distribution with the mean of the distribution conditioned on specific demographic characteristics, which allows for both demographic differences and further differences at the individual level among subjects with the same demographic characteristics.

In order to understand how to reliably apply the class of structural econometric models necessary for inferring risk preferences at the individual level with Bayesian methods, it is essential to properly understand the properties of the various types of specifications and estimation options available. These models become work-horses for the applied econometricians, and they are simply not (yet) “canned, off-the-shelf” estimation routines. Harrison and Rutström (2008) illustrated how one can develop and apply comparable structural models using classical maximum likelihood methods, as well as providing coding “templates” that allowed others to apply and adapt the methods themselves. They generally considered estimation of pooled models, in which observable characteristics characterized the heterogeneity of risk preferences.<sup>4</sup> We do the same thing for Bayesian estimation methods. We provide special attention to the demands of individual-level estimation of risk preferences, due to the derived demand for such detail in most economic applications today. Because Bayesian methods entail novel computational procedures for estimation, we devote considerable attention to documenting the properties of simpler and richer specifications of the core model using simulated data. The objective is to be able to provide the applied econometricians with confidence when applying the various templates we develop.

The obvious advantage of using simulated data is that we can compare the estimates to the values of parameters used to simulate the data and thus determine whether our estimation procedure is reliable. For each specification of risk preferences we simulate a dataset, similar to the data that would be observed by the researcher, estimate the posterior distribution of the parameters, and compare this distribution to the true parameter values used to simulate the data. We summarize the performance of

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<sup>4</sup>An extension to consider random coefficient structural models of pooled risk preferences, to better accommodate unobserved individual heterogeneity, was provided by Andersen et al. (2012).

our estimation procedures using simulated data, and conclude that the correctly-specified procedure recovers the true parameters reliably. In addition, for the simulated dataset with the highest level of heterogeneity, we investigate the consequences of using mis-specified procedures that fail to account for some of the heterogeneity present in the simulated dataset. We find that these mis-specifications lead to inaccurate estimates of risk preferences at the individual subject level. Finally, we apply the procedures to estimate risk preferences for a representative sample of subjects. Surprisingly, we find no evidence of a need to allow for demographic differences once we allow for heterogeneity at the individual subject level with the hierarchical model.

Section 2 documents the data collected from actual subjects from controlled laboratory experiments. This design is used as the basis of our simulations as well as the estimations with observed choices. In section 3 we discuss the EUT theoretical specification, our assumptions on alternative data-generating processes, and the results of our simulations and estimations designed to evaluate alternative Bayesian estimators. A key feature of our simulation approach is to use the observed and incentivized choices described in section 2, and key characteristics of these data with the exact data-generating process proposed in section 3. Section 4 extends the analysis to Rank-Dependent Utility (RDU). Section 5 briefly considers two broad types of *descriptive* applications of the Bayesian econometric methods for estimating individual risk preferences that we evaluate, and then shows how they can be used for the *normative* welfare evaluation of insurance choices. Section 6 concludes.

## 2 Experimental Data

We have  $N = 73$  subjects each making  $T = 60$  lottery choices. The subjects are recruited from the undergraduate student population of Georgia State University. The 60 pairs of lottery are each in the gain domain: for any  $k = 1, \dots, K$ , outcome  $x_k \geq 0$  and subjects receive an endowment  $e$  of \$0. Use subscript  $i = 1, 2, \dots, 73$  to represent the subject,  $t = 1, 2, \dots, 60$  to represent the  $t^{\text{th}}$  lottery pair, and  $y_{it} = 1$  to represent subject  $i$  choosing the left lottery in lottery pair  $t$ . Collect all decisions of one subject in  $\mathbf{y}_i = (y_{i1}, y_{i2}, \dots, y_{i60})$  and collect all decisions of all subjects in  $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{73})$ .

Each subject was asked to make choices for pairs of lotteries designed to provide evidence of risk aversion as well as the tendency to make decisions consistently with EUT or RDU models. The battery

of lottery choices is based on designs from Loomes and Sugden (1998) to test the Compound Independence Axiom, designs from Harrison et al. (2015) to test the Reduction of Compound Lotteries (ROCL) axiom, and a series of lotteries that are actuarially-equivalent versions of some separate index insurance choices studied by Harrison et al. (2020). Each subject faced an individually randomized sequence of choices from this battery. The typical interface used is shown in Appendix A, which contains all instructions and lottery parameters.

The key insight of the Loomes and Sugden (1998) design is to vary the “gradient” of the EUT-consistent indifference curves within a Marschak-Machina (MM) triangle.<sup>5</sup> The reason for this variation in gradient is to generate some choice patterns that are more powerful tests of EUT for any given risk attitude. Under EUT the slope of the indifference curve within a MM triangle is a measure of risk aversion, so choice pairs on the same slope should always generate the same observed choices under EUT. When these choices differ, the subject is said to have exhibited Common Ratio (CR) violations of EUT. So there always exists some risk attitude such that the subject is indifferent over CR lottery pairs, and evidence of CR violations in that case has virtually zero power on a test of EUT.<sup>6</sup>

The beauty of this design is that even if the risk attitude of the subject makes the tests of a CR violation from some sets of lottery pairs have low power, then the tests based on *other* sets of lottery pairs *must* have higher power for this subject. By presenting each subject with several such sets, varying the slope of the EUT-consistent indifference curve, one can be sure of having some tests for CR violations that have decent power for each subject, without having to know *a priori* what their risk attitude is. Harrison, Johnson, McInnes and Rutström (2007) refer to this as a “complementary slack experimental design,” since low-power tests of EUT in one set mean that there *must* be higher-power tests of EUT in another set.

A simple variant on these tests for a CR violation allows one to detect an empirically important pattern known as “boundary effects.” These effects arise when one nudges the lottery pairs in CR and

<sup>5</sup>In the MM triangle there are always one, two or three prizes in each lottery that have positive probability of occurring. The vertical axis in each panel shows the probability attached to the high prize of that triple, and the horizontal axis shows the probability attached to the low prize of that triple. So when the probability of the highest and lowest prize is zero, 100% weight falls on the middle prize. Any lotteries strictly in the interior of the MM triangle have positive weight on all three prizes, and any lottery on the boundary of the MM triangle has zero weight on one or two prizes.

<sup>6</sup>EUT does not, then, predict 50:50 choices, as some claim. It does say that the expected utility differences will not explain behavior, and that then allows a variety of psychological factors to explain behavior. In effect, EUT has no prediction in this instance, and that is not the same as predicting an even split.

related tests of EUT into the interior of the MM triangle, or moves them significantly into the interior. The striking finding is that EUT often performs better when one does this (see Camerer (1992)(1989) and Harless (1992)). Our battery includes 15 lottery pairs based on Loomes and Sugden (1998) and a corresponding 15 lottery pairs that are interior variants of those 15 that are “on the border.”

Harrison, Martínez-Correa and Swarthout (2015) designed a battery to test ROCL by posing lottery pairs that include an explicit compound lottery and a simple (non-compound) lottery. These lottery pairs have a corresponding set of pairs that replace the explicit compound lottery with the actuarially equivalent simple lottery. Thus a ROCL-consistent subject would make the same choices in the first and second set.

Subjects were paid for one of their choices. Prizes ranged from \$0 up to \$70 across the lotteries, with average expected earning of \$27.8.<sup>7</sup> They also received a \$7 participation fee.

### 3 Expected Utility Theory

In section 3.1 we present six EUT specifications that vary how we generate the simulated data we use in section 3.2. When applicable, we always use the observed *covariates* from the data we observed. But we simulate the EUT risk preference *parameters* using the specification described.

#### 3.1 Model and Specifications

In the evaluation of lottery prizes individuals are assumed to perfectly integrate the prizes with their endowments.<sup>8</sup> They are also assumed to be characterized with the Constant Relative Risk Aversion (CRRA) utility functionals  $u(e, x_k) = (e + x_k)^{(1-r)} / (1-r)$  for any  $k = 1, \dots, K$ . In our battery  $K = 4$ . A lottery is evaluated by the weighted sum of utilities of prizes, with the weights being the objective probabilities associated with the prizes:

$$EUT_{it}^l(r_i) = \sum_{k=1, \dots, K} [p_{ik}^l \times (e + x_{ik}^l)^{(1-r_i)} / (1-r_i)], \quad (1)$$

<sup>7</sup>These are the earnings that the subjects would have expected given their observed choices.

<sup>8</sup>This assumption can be relaxed: see Andersen, Cox, Harrison, Lau, Rutström and Sadiraj (2018).

where  $l = L, R$  is the index for the two lottery options within a lottery pair  $t$ . Define the latent index as the difference between the EUT of the L and R lotteries subject to a Fechner noise parameter  $\mu_i$ , and a random noise term  $\varepsilon_{it}$ :

$$y_{it}^* = \nabla EU_{it}(r_i) + \varepsilon_{it} = \frac{(EUT_{it}^L(r_i) - EUT_{it}^R(r_i))/v_{it}}{\mu_i} + \varepsilon_{it}, \quad (2)$$

where  $\varepsilon_{it}$  is an independently distributed noise that follows standard normal distribution,  $v_{it}$  is a “contextual utility” term specific to choice  $i$  to normalize utilities of prizes between 0 and 1 following Wilcox (2008)(2011), and  $r_i$  and  $\mu_i$  are the parameters characterizing the risk preferences of subject  $i$  that we want to estimate. Subject  $i$  selects the L lottery in lottery pair  $t$  whenever the latent index  $y_{it}^*$  is greater or equal to 0:

$$y_{it} = I(y_{it}^* \geq 0), \quad (3)$$

where  $I(\cdot)$  is the indicator function.

We observe the endowments, prizes and probabilities of the lottery pairs the subject faces, as well as the lottery she chooses in each lottery pair. We also observe a vector of demographic characteristics for each subject: whether a subject is female, black, a business major, whether he or she has a high GPA, and whether she owns any insurance.<sup>9</sup>

Assuming EUT for now, we estimate the two parameters for each subject,  $r_i$  and  $\mu_i$ . Six econometric specifications are presented, with different assumptions on the heterogeneity of these two parameters across the subjects. These models vary from the least general specification I, to most general specification VI. Since all models are nested in the most general model that allows for heterogeneity at both demographic and individual levels, we present it first and describe the other models as special cases.

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<sup>9</sup>We report summary statistics of demographics in our sample in Table B.1.

### 3.1.1 Specification VI

Assume that the individual CRRA coefficient  $r_i$  is independently drawn from a normal distribution with the mean conditional on a set of demographic covariates. That is, we have the following specification for  $r_i$

$$r_i \sim \mathcal{N}(m_r + \beta \cdot \mathbf{X}_i, \sigma_r^2), \quad (4)$$

where  $\mathbf{X}_i$  is a vector of observed demographic covariates, and  $m_r + \beta \cdot \mathbf{X}_i$  is the mean of the distribution within the demographic subgroup that subject  $i$  belongs to. We assume the same variance  $\sigma_r^2$  across different demographic groups for simplicity and lack of *a priori* belief that this parameter is different across demographic groups. Assume that  $\mu_i$  is independently drawn from a log-normal prior:<sup>10</sup>

$$\ln(\mu_i) \sim \mathcal{N}(m_{\ln\mu}, \sigma_{\ln\mu}^2) \quad (5)$$

In this specification we estimate  $r_i, \mu_i, m_r, \beta, \sigma_r, m_{\ln\mu}$  and  $\sigma_{\ln\mu}$  for each subject. We use a normal hyperprior  $\mathcal{N}(0, 100)$  for each of the parameters  $m_r, \beta$  and  $m_{\ln\mu}$  and an inverse Gamma hyperprior for  $\sigma_r$  and  $\sigma_{\ln\mu}$ . The same set of diffuse priors is used when applicable in subsequent specifications, unless otherwise noted.

We use a combination of the Metropolis Hastings algorithm and Gibbs sampler to estimate the model parameters from this posterior, using the Bayesian estimation package of Stata. The Stata package requires only an input of the likelihood function based on equations (1)-(3) as a user-defined function, and automatically applies the Gibbs sampler to parameters from the hierarchical structures in (4) and (5). We consider this close connection of programing syntax to the econometric formalization to be an advantage of the Stata package for novice users of Bayesian econometric methods, and provide the template for this specification in Appendix C.<sup>11</sup>

<sup>10</sup>We do not consider systematic demographic differences in the Fechner noise parameter  $\mu_i$  for three reasons. First, we assume homogeneity for simplicity. Second, the core parameter that characterizes the risk aversion of a subject under EUT is the CRRA coefficient  $r_i$ , which is also the parameter we use to predict or evaluate subjects' decisions externally in other decision environments. The Fechner noise parameter is a nuisance parameter that describes the noisiness of subjects' decision making in this specific decision environment, i.e., choosing preferred lotteries from a pair of options. Third, *a priori* we do not expect this decision noise parameter to be systematically different among different demographic subgroups.

<sup>11</sup>For all estimations the sample size is 10000 for the MCMC chain and 2500 for burn-in.

In this specification we are essentially assuming that the parameter  $r_i$  of each individual subject is specified as:

$$r_i = m_r + \beta \cdot \mathbf{X}_i + \epsilon_i, \quad (6)$$

where  $\epsilon_i \sim \mathcal{N}(0, \sigma_r^2)$ . We consider two sources of heterogeneity for  $r_i$ : the demographic differences introduced in the slope coefficients  $\beta$  and the individual differences within the demographic group introduced in the error term  $\epsilon_i$ .

At this point, and only for specification VI, we estimated the Bayesian hierarchical model using the actual data from our experiment. The mean of the posterior distribution for each parameter is then used in the simulation process described below. In this way we have “calibrated” the simulated data to generate insights that are relevant for the target data. We choose  $m_r$ ,  $\beta$ ,  $\sigma_r^2$ ,  $m_{\ln \mu}$  and  $\sigma_{\ln \mu}^2$  close to their estimated values using our observed data to allow for an efficient test of the Bayesian models in the parameter regions where they are intended to work within, rather than in some extreme or unrealistic regions.

When we simulate data for the parameter recovery exercise reported in section 3.2, we use the same covariates  $\mathbf{X}_i$  that characterize the subjects in our observed data, choose values for  $m_r$ ,  $\beta$  and  $\sigma_r^2$  that are close to their estimated values using our observed data, randomly draw  $\epsilon_i$  from the normal distribution  $\mathcal{N}(0, \sigma_r^2)$ , and calculate  $r_i$  for each simulated subject as specified in (6). We also choose values for  $m_{\ln \mu}$  and  $\sigma_{\ln \mu}^2$  that are close to their estimated values using our observed data, and draw  $\mu_i$  from the distribution in (5).<sup>12</sup> Lastly we simulate their choices for all 60 lottery pairs according to (1)-(3). We provide the specific parameters used in the simulation later in Table 1.

For the remaining specifications, we constrain different components of specification VI, starting with two specifications in which we allow only one of these two sources.

<sup>12</sup>The estimated values we refer to are listed in the “Mean” column for specification VI in Table 4. The simulated values for EUT are listed in the “True” column for the same specification VI in Table 2. The simulated values are very close to the estimated values for all parameters except those elements of  $\beta$  that were estimated to be close to zero. We deliberately simulated these elements of  $\beta$  to be non-zero and small in value, to be certain that our estimators can in fact detect demographic differences when they actually exist in the data. If we were to choose coefficients close to the estimated values, the validation process provides little insight since it can be confounded by statistical insignificance.

### 3.1.2 Specification V

In this specification, starting with (6) we impose the constraints that the demographic coefficients in  $\beta$  are 0, and only consider differences in  $r_i$  due to the error term  $\epsilon_i$ :

$$r_i = m_r + \epsilon_i \quad (7)$$

In other words, we assume that the prior means are the same across the different demographic subgroups. Formally, we assume that the distribution for  $r_i$  is

$$r_i \sim \mathcal{N}(m_r, \sigma_r^2) \quad (8)$$

The assumptions on  $\mu_i$  are the same as those in (5) in specification VI. In this specification we estimate  $r_i, \mu_i, m_r, \sigma_r, m_{\ln \mu}$  and  $\sigma_{\ln \mu}$  for each subject. We use the same diffuse priors on  $m_r, \sigma_r, m_{\ln \mu}, \sigma_{\ln \mu}$  as in specification VI.

### 3.1.3 Specification IV

In this specification, starting again with (6) we impose the constraint that the standard error of  $\epsilon_i$  is 0, so  $\sigma_r^2 = 0$ , but allow the slope coefficients in  $\beta$  to be different from 0. That is, we assume the CRRA coefficient,  $r_i$ , is the same for all subjects that belong to the same demographic group, but may be different across different demographic groups. Formally,

$$r_i = m_r + \beta \cdot \mathbf{X}_i \quad (9)$$

The assumption that a list of observable demographic characteristics *determines* a subject's CRRA coefficient is a popular approach to model heterogeneity of samples of subjects in the older literature, such as Harrison, Lau and Rutström (2007).

The assumptions on  $\mu_i$  are the same as those in (5) in specification VI. In this specification we estimate  $\mu_i, m_r, \beta, m_{\ln \mu}$  and  $\sigma_{\ln \mu}$  for each subject.

### 3.1.4 Specification III

In this specification we consider neither of the two sources of heterogeneity in specification VI, and assume that  $r_i$  is the same for all subjects. We impose the constraints that the standard error and the demographic coefficients for  $r$  are all 0:

$$r_i = r \quad (10)$$

The assumptions on  $\mu_i$  are the same as those in (5) in specification VI. In this specification we estimate  $r$ ,  $\mu_i$ ,  $m_{\ln \mu}$  and  $\sigma_{\ln \mu}$  for each subject.

The four specifications introduced so far differ in the degree of individual heterogeneity of  $r_i$ , but assume the same log-normal prior for  $\mu_i$ . We next consider two specifications used in the literature in which  $\mu_i$  is assumed to be the same for all subjects.

### 3.1.5 Specification II

In this specification we assume  $\mu_i = \mu$  for  $\mu_i$ . This specification shares the same assumption on  $r_i$  as specification IV. In this specification we only estimate  $\beta$  and  $\mu$ . We use the same diffuse normal priors for each parameter in  $\beta$  and the Jeffreys prior for  $\mu$ .

### 3.1.6 Specification I

In this specification we assume  $\mu_i = \mu$  and  $r_i = r$ . This specification assumes homogeneity of both parameters across all subjects, and is sometimes referred to as the representative agent model in which we pool observations from all subjects as if they reflect the choices of just one agent. We use the diffuse normal prior for  $r$  and the Jeffreys prior for  $\mu$ .

## 3.2 Data Simulation and Parameter Recovery

Prior to estimating these specifications of the EUT model using observed data, we conduct a simulation exercise. We simulate datasets for each specification and estimate models to provide

evidence on the performance of the correctly specified models as well as report sensitivity to models mis-specification. In the simulation we keep all the covariates  $\mathbf{X}_i$  of the actual 73 subjects. We then draw  $r_i$  and  $\mu_i$  for each subject based on each specification and use the EUT model described in (1)-(3) to simulate their decisions. Since specifications I-V are nested within specification VI, we list the parameters in simulated datasets I-V as special parameterizations of specification VI in Table 1.

### 3.2.1 Parameter Recovery

For each simulated dataset we first estimate the correctly specified model and report the results. Through this exercise we evaluate the performance of the algorithms for each specification and see whether they successfully recover the true parameters used in the simulations. We report these results in Tables 2 and B.2.<sup>13</sup>

Table 2 presents summary statistics for parameters at the population level. The MCMC chains converge successfully for all specifications, and we recover the true parameters for all six datasets. With the exception of the standard deviation of the CRRA coefficient parameter for simulated dataset VI, the true parameters are all within their 95% Highest Posterior Density (HPD) credible intervals, which is the smallest possible 95% credible interval for a given posterior distribution.<sup>14</sup> The standard deviation of the CRRA coefficient parameter for simulated dataset VI is underestimated in comparison to the true value; however, this underestimate does not affect the successful recovery of demographic differences in this specification when these differences exist.<sup>15</sup>

For specifications that allow for *some* heterogeneity of  $r_i$ , we also report the summary statistics of the posterior distributions of the  $r_i$  of each subject in Table B.2. For simulated datasets II and IV, based on the posterior sample of  $\beta$  and the demographic characteristic  $\mathbf{X}_i$ , we estimate the posterior distribution of each subject's CRRA coefficient using (9). For simulated dataset VI, we sample the posterior distributions of  $\beta$  and  $\epsilon_i$ , and estimate the CRRA coefficient of each subject through (6). For simulated dataset II, the correct CRRA coefficients of only 1 subjects is estimated to be outside of the

<sup>13</sup>For ease of interpretation, in the estimation tables we report the mean and standard deviation of  $\mu$  whenever applicable, rather than the  $m_{\ln \mu}$  and  $\sigma_{\ln \mu}^2$  in (5).

<sup>14</sup>By default, we report the 95% HPD credible intervals rather than equal-tailed credible intervals.

<sup>15</sup>It follows that the later inference with observed data under specification VI, that no systematic demographic differences are present among our subjects, remains valid and unaffected by the potential mis-identification of the standard deviation of the CRRA coefficient.

95% credible interval; for simulated dataset IV, V and VI, respectively, we only have 1, 3 and 10 such subjects.

We can also compare the posterior means to the true values of individual CRRA coefficients. In each panel of Figure 1 we show a scatter plot with the true value of the CRRA coefficient of each subject on the x-axis and the estimated posterior mean on the y-axis, for simulated datasets II, IV, V or VI. We also provide the 45° line in each scatter plot as a reference, and expect the scatter points to be more tightly aligned around this line if the estimated posterior means are closer to the true parameters.

The scatter points in Figure 1 are almost perfectly aligned around the 45° line for simulated datasets II and IV, with correlation coefficients almost equal to 1. The points are very tightly aligned around the 45° line for simulated datasets VI, with a correlation coefficient of 0.94. For simulated dataset V the scatter points in Figure 1 are less tightly distributed around the 45° line. For the true CRRA coefficients that are at the extremes, the estimated posterior means are generally pulled closer to the average value of CRRA coefficients across all subjects, as one would expect with the Bayesian Hierarchical prior. As a result, the correlation coefficient between the true values and posterior means for this model is only 0.77 in this case. Despite these effects, the 95% credible intervals of these individual CRRA coefficients contain the true parameter with the exception of only 3 subjects.

We conclude that the parameter recovery exercise is generally successful for all simulated datasets, since most true parameter values are within the 95% credible interval of the corresponding posterior distributions. For dataset V all parameters are recovered with great success, and we observe that the posterior means of CRRA coefficients for the subjects at the tail are pulled towards the population mean. We have excellent convergence for all parameters in the estimation of datasets I through V. For dataset VI, where both systematic demographic difference and individual level differences of the CRRA coefficient are present, although the quality of convergence of population parameters is slightly slow and the mixing is less than perfect, when we use them to infer individual CRRA coefficient we have excellent convergence on the CRRA coefficients at the individual level.

### 3.2.2 Model Sensitivity to Mis-specification

For dataset VI, in addition to the estimation under the correct specification, we also estimated the EUT models under *incorrect* specifications to see how sensitive the estimates are to model mis-specification. Recall that in dataset VI we have the highest level of heterogeneity of the  $r_i$  parameter: there are systematic differences across demographic groups as well as individual differences within the same demographic groups. Specification V fails to account for the demographic differences, specification IV fails to account for the individual differences, and specification III fails to account for both. Compared to specification VI or III, specifications II or I further fail to account for the individual heterogeneity of  $\mu_i$ . We report estimation results under these incorrect specifications in Tables 3 and B.4. We also include estimation results under the correct specification for ease of comparison.

In Table 3 we report the population level parameters. In specification V the standard deviation of  $r_i$  is larger, to compensate for the lack of systemic demographic differences in the prior mean. In specification II and IV the constant term and coefficients are correctly recovered despite the fact that these two specifications incorrectly assume no individual heterogeneity within the same demographic group. In specification III all subjects are assumed to have the same CRRA coefficient, and the noise in the simulated decisions are all attributed to a larger Fechner noise parameter  $\mu$ . We estimate a similarly larger  $\mu$  as a consequence of assuming away any heterogeneity of the CRRA coefficient in specification I. Despite these abnormalities, the *cost of model mis-specification appears to be small if we only look at estimates at the population level*.

However, we find *more severe effects of model mis-specification when we look at estimates at the individual level*, shown in Table B.4.<sup>16</sup> Scatter plots between posterior means and true values for these four sets of estimations are shown in Figure 2. The correlation coefficient becomes larger as we move towards the correct specification: 0.87 under specifications II and IV, 0.91 under specification V, and 0.94 under specification VI. While the correlation coefficients appear high under the incorrect

<sup>16</sup>In specifications I and III all subjects are assumed to have the same CRRA coefficient, which means we will not have any variability in individual estimates. Therefore we do not report these two specifications in Table B.4.

specifications, a closer look at the credible intervals as well as some individual estimates that are farther off the 45° line reveals more problems with these individual estimates.

Under specification V we find that for 12 subjects out of 73 the true value of their CRRA coefficients is outside the 95% credible intervals, with roughly the same set of subjects under specification VI. When we look at results under specifications II and IV, however, the number becomes 51 of 73, respectively. This is due to the fact that the standard deviations of the posterior samples are generally smaller under specifications II and IV (between 0.01 and 0.06) than under specifications V and VI (between 0.04 and 0.23). As a result, the scatter points in Figure B.4 are less tightly aligned along the 45° line with tight credible intervals, showing false confidence for inaccurate point estimates.

Inspecting some individuals that are well off the 45° line, simulated subject 54 is risk neutral with a posterior mean of  $-0.001$  under specification II, while she is actually moderately risk averse with a true value of  $r_{54} = 0.329$  (the estimated posterior mean is  $0.280$  under the correct specification). Simulated subject 31 is actually risk loving with  $r_{31} = -0.203$ , but under specification V she is considered risk averse with an estimated posterior mean of  $0.150$  (the estimated posterior mean is  $-0.168$  under the correct specification). In addition to these two examples of incorrect signs of the CRRA coefficient due to model mis-specification, we also observed inaccurate levels of estimated risk aversion among other subjects. For example, simulated subjects 9 and 10 are estimated to be more risk averse than they actually are under specifications II and IV, and less risk averse than they are under specification V.

### 3.3 EUT Inferences with Observed Data

We now report and analyze EUT estimation results using the observed choice data of the 73 subjects across the various specifications introduced above. All parameters converge successfully, some with better mixing than the others,<sup>17</sup> and we report the main estimates of each specification in Table 4.

Consider the estimation results under specification VI in Table 4, where we consider both demographic and individual differences in CRRA coefficients. We do not find systematic differences

<sup>17</sup>The demographic coefficients in specification VI converge slower than other parameters.

across demographic groups, since the posterior means of the five demographic coefficients are all close to 0 and the 95% credible intervals also include 0. We find substantial differences at the individual level as the standard deviation of the prior distribution is 0.163, and provide the histogram of the posterior mean of  $r_i$  for specification VI shown in Figure 3. We conclude that 62 of the 73 subjects are risk averse, in the sense that the estimated posterior means of their CRRA coefficients are positive, and that the lower bounds of the 95% confidence levels are greater than 0. For the other 11 subjects, the estimated posterior mean of the CRRA coefficient is negative for 3 subjects and positive for 8 subjects; however,  $r_i = 0$  is within the 95% confidence levels, so they are deemed risk neutral. The Fechner noise parameter  $\mu_i$  follows a log-normal distribution with median of 0.184 and standard deviation of 0.058, with the histogram of the posterior means of individual  $\mu_i$  shown in Figure B.2.

Next consider specification V, where we assume there is no demographic difference in the prior mean of  $r_i$ . From the estimation  $r_i$  follows a normal distribution with a mean of 0.512, close to the constant term of (6) under specification VI, with a standard deviation of 0.280. We show scatter plots to compare the individual  $r_i$  under this specification with those under specification VI in the left panel of Figure 4. The points are almost perfectly aligned along the 45° line, with a correlation coefficient of 1. The estimates of the prior of  $\mu_i$ , as well as individual  $\mu_i$ , are also very similar under these two specifications as well, as shown in the left panel of Figure B.3.

Next we consider specification IV, where we assume demographic differences are the only source of the heterogeneity of  $r_i$ . Using the estimates in specification IV, we calculate each subject's  $r_i$  based on her demographics. To compare each subject's CRRA coefficient  $r_i$  between specifications IV and VI, we show the scatterplot of  $r_i$  under these two specifications in the middle panel of Figure 4. The plots are not at all close to the 45° line, and the linear correlation coefficient is only 0.23, indicating very different estimates for each individual subject's CRRA coefficient. We provide a scatterplot of  $\mu_i$  under these two specifications in the middle panel of Figure B.3, and find  $\mu_i$  to be generally greater under specification IV. Considering that specification VI allows for heterogeneity in  $r_i$  to a finer level of granularity, it seems natural that under specification IV the differences in the choices of subjects that are unaccounted for by demographic differences would result in a larger  $\mu_i$ .

We undertook a similar exercise for specification II, where  $\mu_i$  is the same for all subjects, whereas in specification IV  $\mu_i$  is drawn from a log normal distribution. Under specification II we find that the estimates of  $r_i$  are close to those under specification IV. Consequently, we have the same conclusions when we compare the individual CRRA coefficients under this specification to specification VI as we had for the comparison between specifications IV and VI.

Under specifications I and III, where we assume there is no heterogeneity in  $r_i$ , the estimates of the Fechner noise parameter are generally larger than specifications V and VI but similar to specifications II and IV. This observation again indicates that the individual level differences in  $r_i$ , rather than the demographic differences, are more important in the dataset. We also performed pairwise Bayesian model comparisons of specifications I-V to specification VI. The log-Bayes Factors are 221.81, 207.44, 165.10 and 146.45 for specification VI when we use specifications I, II, III and IV as the base models, showing decisive support for specification VI over each of the four models.<sup>18</sup>

The log-Bayes Factor is  $-6.61$  when we use specification V as the base model, showing support for specification V over specification VI. We also compute the posterior probabilities of the six models, assuming they are equally probable *a priori*, and specification V has a posterior probability of 99.9%.<sup>19</sup> This indicates that within the population from which our sample of subjects is drawn, we do not find systematic demographic differences in their CRRA coefficients. In addition, the MCMC sampling process takes *much* longer for specification VI than specification V. Although specification VI is the most general model, considering the high cost in terms of longer computing time and low benefit in terms of model comparison, we *conclude that specification V is the better model to estimate for samples drawn from this particular subject population.*

<sup>18</sup>We assume all models are equally plausible in the prior, and the Bayes Factor is the posterior odds ratio

$$\text{BF}_{jk} = \frac{p(\mathbf{y}|M_j)}{p(\mathbf{y}|M_k)},$$

where  $\mathbf{y}$  is the observed choice data,  $M_j$  represents the estimated model under specification  $j$  and  $M_k$  the estimated model under specification  $k$ . As a rule of thumb, Jeffreys (1961) recommended that the evidence against  $M_k$  is decisive if the log of Bayes Factor is greater than 2.

<sup>19</sup>The posterior probability is 0.14% for specification VI and 0 for specifications I-IV.

## 4 Rank Dependent Utility Theory

In addition to the estimation of the EUT model, we extend our analysis of the observed data to the RDU model. We first present the RDU model and specifications. We then bypass the data simulation process, which is logically identical to the EUT case, and present results using observed data.

### 4.1 Model and Specifications

In the evaluation of lottery prizes, again assume that individuals perfectly integrate lottery prizes with their endowments and behave as if they evaluate CRRA utility functionals as assumed under EUT. A lottery is then evaluated by the weighted sum of utilities of prizes, where the weights are the associated *decision weights*. RDU departs from EUT in the manner in which decision weights depend on objective probabilities: under EUT the decision weight for each prize *was* the corresponding objective probability.

Under RDU we first rank the prizes from best to worst, such that  $x_1 \geq x_2 \dots \geq x_K$  (omitting the index for lottery and subject for now). The decision weight associated with each prize is then calculated as:

$$\pi(x_1) = \omega(p_1), \quad (11)$$

$$\pi(x_2) = \omega(p_1 + p_2) - \omega(p_1), \quad (12)$$

$$\dots, \quad (13)$$

$$\pi(x_K) = \omega(1) - \omega(p_1 + \dots + p_{K-1}), \quad (14)$$

where  $\omega(\cdot)$  is the probability weighting function (PWF): a strictly increasing and continuous function with  $\omega(0) = 0$ ,  $\omega(1) = 1$ . We use the flexible two parameter PWF from Prelec (1998):

$$\omega(p) = \exp(-\eta(-\ln p)^\phi), \quad (15)$$

where  $\eta > 0$  and  $\phi > 0$ . The RDU of a lottery is then calculated as

$$RDU_{it}^l(r_i, \eta_i, \phi_i) = \sum_{k=1, \dots, K} [\pi_{tk}^l \times (e + x_{tk}^l)^{(1-r_i)} / (1 - r_i)], \quad (16)$$

which is the same as the definition of the EUT of a lottery in (1) apart from  $p_{tk}^l$  being replaced by  $\pi_{tk}^l$ . Define the latent index as the difference on the RDU of the left and right lottery subject to a Fechner noise parameter  $\mu_i$  and a random noise term  $\varepsilon_{it}$ :

$$y_{it}^* = \nabla RDU_{it}(r_i, \eta_i, \phi_i) + \varepsilon_{it} = \frac{(RDU_{it}^L(r_i, \eta_i, \phi_i) - RDU_{it}^R(r_i, \eta_i, \phi_i)) / v_{it}}{\mu_i} + \varepsilon_{it}, \quad (17)$$

where  $v_{it}$  is again the term to normalize utilities of prizes between 0 and 1, and  $r_i$ ,  $\eta_i$ ,  $\phi_i$  and  $\mu_i$  are the parameters we want to estimate. The subject is again assumed to select the left lottery in a pair whenever the latent index  $y_{it}^*$  is greater or equal to 0.

The specifications for the estimations are very similar to those of the EUT models, therefore we skip details. In addition, we assume all parameters are independently distributed in the prior distribution. Although we specify the prior distribution separately for each parameter, the posterior distribution of each parameter is correlated with other parameters, both within a subject and across subjects. In essence, the RDU model decomposes the risk premium presumed to drive the observed choices by subject  $i$  into two components: the premium due to utility curvature governed by parameter  $r_i$ , and the premium due to probability weighting governed by parameters  $\eta_i$  and  $\phi_i$ .<sup>20</sup> There is a well-understood tradeoff between the two components explaining the risk premium, which introduces the correlation between the three parameters in the sampling of their joint posterior distribution.

For  $r_i$  and  $\mu_i$  we use exactly the same specifications from (4) to (10) as in the EUT models. The parameters  $\eta_i$  and  $\phi_i$  must be positive, by definition, so we assume they are drawn from log-normal distributions. So for specification VI we have

$$\ln(\eta_i) \sim \mathcal{N}(\gamma \cdot \mathbf{X}_i, \sigma_\eta^2) \quad (18)$$

$$\ln(\phi_i) \sim \mathcal{N}(\lambda \cdot \mathbf{X}_i, \sigma_\phi^2) \quad (19)$$

<sup>20</sup>In the extreme case of EUT the risk premium is solely determined by utility curvature. In the extreme case of “dual theory,” due to Yaari (1987), the risk premium is solely determined by the probability weighting function.

For specification V we have

$$\ln(\eta_i) \sim \mathcal{N}(m_\eta, \sigma_\eta^2) \quad (20)$$

$$\ln(\phi_i) \sim \mathcal{N}(m_\phi, \sigma_\phi^2) \quad (21)$$

For specifications II and IV we have

$$\ln(\eta_i) = \gamma \cdot \mathbf{X}_i \quad (22)$$

$$\ln(\phi_i) = \lambda \cdot \mathbf{X}_i \quad (23)$$

## 4.2 RDU Inferences with Observed Data

We report estimation results of RDU models and compare them to the estimation results of EUT models. The summary results of population parameters are reported in Table 5 for each specification.

First consider specification I, which does not allow for heterogeneity in any parameters and estimates the preference of a representative agent. The parameters converge with excellent mixing. The PWF has an inverse-S shape implied by the estimate of  $\phi = 0.841$ : the inflexion point given  $\phi = 0.841$  and  $\eta = 1.273$  is at  $p = 0.1$ , so the estimated PWF is convex when  $p > 0.1$  and concave when  $p < 0.1$ . In Figure 5 we graph the Prelec PWF, and the implied decision weights with equal-probable reference lotteries. The estimated posterior mean of the CRRA coefficient is 0.412, lower than the estimated value under the EUT model with specification I.

We also perform pairwise model comparisons using Bayes Factors, and compute posterior probabilities of the six RDU Prelec models, and *again find specification V to be the preferred model*.<sup>21</sup> Therefore, we consider in detail the results under specifications V, which allows the model parameters to be different for each individual subject.

<sup>21</sup>Using specifications I, II, III, IV and VI as the base model, the log-Bayes Factor of specification V is 300.02, 264.37, 225.48, 204.88 and 12.59, respectively, showing decisive support for specification V over each of the base models. In addition, the posterior probability of specification V is 100%, assuming equal probabilities *a priori*.

All model parameters converge with excellent mixing.<sup>22</sup> The CRRA coefficient  $r_i$  follows a normal prior with mean 0.338 and standard deviation 0.287. The PWF parameter  $\eta_i$  follows a log-normal prior with a median of 1.316 and standard deviation of 0.522. The PWF parameter  $\phi_i$  follows a log-normal prior with a median of 0.836 and standard deviation of 0.268. We show the histograms of the posterior means of  $r_i$ ,  $\eta_i$ ,  $\phi_i$  and  $\mu_i$  for the 73 subjects in Figures 6 and B.4, respectively. We observe a great deal of individual heterogeneity for all parameters. For 56 subjects we have posterior means  $\phi_i < 1$ , and for 17 subjects we have posterior means  $\phi_i > 1$ . In addition, we compare the posterior means of the individual CRRA coefficients to those from specification V of EUT model in Figure 7. We observe that the CRRA coefficients are slightly higher under EUT than under the RDU Prelec model, consistent with what we find when comparing the two representative agent models (specification I).

We consider the estimation results under other specifications briefly, since the findings are similar to when we compare different specifications of the EUT model. In specification VI we do not find much difference in the mean of priors of  $r_i$ ,  $\eta_i$  or  $\phi_i$  due to demographic differences, since 0 is within the 95% credible interval for all coefficients of each of the demographic variables. We show the scatterplot of  $r_i$ ,  $\eta_i$  and  $\phi_i$  under specifications V and II in the left panel in Figure 8. The posterior means of individual  $r_i$  and  $\phi_i$  estimates are very tightly aligned along the 45° line, with correlation coefficient of 0.95 and 0.93. The posterior means of individual  $\eta_i$  estimates are also aligned along the 45° line, although not as tight, with correlation coefficient of 0.88. Therefore, introducing demographic differences in the mean of the priors does not provide more information on the individual preferences. The estimates of the prior of  $\mu_i$  are very similar under these two specifications, as are the individual  $\mu_i$  shown in the left panel of Figure B.4. In specifications II and IV the individual estimates of CRRA coefficients are not very consistent with those under specifications V or VI, as shown in the middle and right panels of Figure 8. The individual Fechner noise parameters are estimated to be larger under specifications I through IV.

<sup>22</sup>For ease of interpretation we report the mean and standard deviation for  $\eta$ ,  $\phi$  and  $\mu$  in specification V in Table 5, rather than the mean and standard deviation of  $\ln(\eta)$ ,  $\ln(\phi)$  and  $\ln(\mu)$ .

## 5 Applications

There are three reasons to be interested in estimated risk preferences at the individual level. We briefly explain what each is, how our Bayesian analysis of risk preferences at the individual level facilitates those applications, and demonstrate one application from behavioral welfare economics.

### 5.1 Characterizing Risk Preferences for Their Own Sake

Of course, there is direct interest in knowing the risk preferences of individuals. Are there demographic effects on risk preferences? How important are each of the different pathways to the risk premium: aversion to variability of final outcomes, probability weighting, disappointment aversion, regret aversion, or loss aversion, to name some of the more popular? As the models become more complex, it becomes important to be able to harness informative priors to facilitate inferences at the individual level, and hierarchical priors allow that in a flexible manner, as we have demonstrated.

### 5.2 Joint Estimation of Risk Preferences and Other Preferences

Economic theory tells us that inferences about time preferences defined over time-dated monetary amounts depend on the curvature of the atemporal utility function defined over those monetary amounts. One of several ways to control for that dependence is to jointly estimate risk and time preferences, so that inferences about the latter can account for the effect of the former (Andersen, Harrison, Lau and Rutström (2008)). This general point has nothing to do with assuming EUT risk preferences: if the subject is characterized by RDU there are still inferences about the extent of diminishing marginal utility, and that is what is important for correct identification and estimation of time preferences. For the same reason, it is not “risk” that is correlated somehow with time preferences over non-stochastic, temporally-dated monetary amounts, it is the curvature of the atemporal utility function. Hence good estimates of risk preferences, as one way to get good estimates of that curvature, are fundamental to generating good estimates of time preferences.

The need for good estimates of risk preferences is particularly important at the level of the individual. Andersen, Harrison, Lau and Rutström (2014; p.25) report attempts to estimate time

preferences at the individual level, and find that they cannot obtain Maximum Likelihood (ML) estimates for 238 of their 413 subjects, which is 42% of the sample. The reason is simple: the ML approach rests on numerical methods finding a set of estimates that characterizes a *maximum* log-likelihood for the observed binary choices. If the likelihood function has some “flatness” around the maxima, standard methods, particularly derivative-based methods, can fail to converge. Critically, there is no difficulty *evaluating* the log-likelihood for a wide range of possible estimates, just a difficulty finding the one best set of estimates. A Bayesian is not bothered by this latter difficulty at all, and just needs the likelihood function evaluations in order to derive the posterior distribution.<sup>23</sup>

The same general point applies with even greater force when making inferences about intertemporal risk aversion, which derives from the non-additivity of the intertemporal utility function and depends on *both* the curvature of the atemporal utility function *and* time preferences over non-stochastic outcomes. Now there are two “nuisance parameters” from economic theory to attend to in order to make the inferences of interest, and well-defined non-linearities connecting them. Moreover, there can be a “cascading” effect, since one set of nuisance parameters (time preferences) depends on the other nuisance parameters (utility curvature), generating an even greater derived demand for good estimates of risk preferences. Andersen, Harrison, Lau and Rutström (2018) generate estimates of the intertemporal risk aversion of the adult Danish population, but do not even consider individual heterogeneity beyond including observable demographic characteristics in the pooled model.

### 5.3 Inferences Based on Risk Preferences

There is a final class of applications of our approach which uses estimates of the posterior distributions of individual risk preferences to make an inference over “different data” than were used to estimate the posterior. This is distinct from joint estimation in the sense that the inferences over different data do not entail estimation of core parameters of preference models. The usual application in Bayesian modeling is to additional out-of-sample instances of the same sample data used to estimate the posterior. Hence these are referred to as posterior *predictive* distributions. A typical example would

<sup>23</sup>Of course, if the likelihood function is globally flat, the posterior will just be a replica of the prior, and the data from the subject non-informative, but that is a separate matter: there will still be a posterior, albeit derived solely from the prior.

be to predict choices by one of our subjects if she had been offered a new, different battery of choices over risky lotteries.

A more interesting example from behavioral welfare economics arises when making inferences about the consumer surplus generated by observed choices of a subject over insurance. In the simplest case, considered by Harrison and Ng (2016), subjects made a binary choice to purchase a full indemnity insurance product or not. The actuarial characteristics of the insurance product were controlled over 24 choices: the loss probability, the premium, the absence of a deductible, and the absence of non-performance risk. In effect, these insurance purchase decisions are just re-framed choices over risky lotteries. The risky lottery here is to not purchase insurance and run the risk of the loss probability reducing income from some known endowment, and the (very) safe lottery is to purchase insurance and deduct the known premium from the known endowment.

The same subjects that made these insurance choices also made choices over a battery of risky lotteries. So one immediate application of our Bayesian approach to estimating individual risk parameters is to infer the posterior predictive distribution of *welfare* for each insurance choice of an individual. The predictive distribution is just a distribution of unobserved data (the expected insurance choice given the actuarial parameters offered) conditional on estimated risk preferences based on observed data (the actual choices in the risk lottery task). All that is involved is marginalizing the likelihood function for the insurance choices with respect to the posterior distribution of estimated model parameters from the risk lottery choices. The upshot is that we predict a distribution of welfare for a given choice by a given individual, rather than a scalar (which is what one would do if just using point estimates from an ML approach). We can then report that distribution as a kernel density, or select some measure of central tendency such as the mean or median.

Figure 9 illustrates for one subject and four insurance purchase decisions by that subject. For decision #1 the posterior predictive density shows a clear gain in consumer surplus, and for decision #4 a clear loss in consumer surplus. In each case, of course, there is a distribution for the inferred consumer surplus from the observed purchase decisions, with a standard deviation of \$0.76. The prediction posterior distributions for decision #13 and decision #17 illustrate an important case, where

we can only say that there has been a consumer surplus gain with some probability. More extensive results of this application are provided in Gao, Harrison and Tchernis (2020).

## **6 Conclusions**

We carefully examine the properties of a popular class of Bayesian models for the estimation of individual risk preferences. Using hierarchical priors, information from the complete sample is used to generate informative priors for inferences about each individual. Given the importance of models of individual risk preferences for a wide range of inferences in economics, there is value in knowing the properties of alternative specifications. Using simulated data, from experimental tasks that are widely used, we consider the reliability of alternative specifications at characterizing what we know to be the true, underlying risk preferences. The results show that a hierarchical model that assumes unconditional exchangeability of subjects provides an excellent basis for inferences about individual risk preferences.

**Table 1:** Parameters in Simulated Datasets

Parameters		Simulation Parameters					
		I	II	III	IV	V	VI
$r$	Constant	<i>0.5</i>	<i>0.4</i>	<i>0.5</i>	<i>0.4</i>	<i>0.5</i>	<i>0.4</i>
	Female	-	<i>0.2</i>	-	<i>0.2</i>	-	<i>0.2</i>
	Black	-	<i>-0.2</i>	-	<i>-0.2</i>	-	<i>-0.2</i>
	BusinessMajor	-	<i>-0.5</i>	-	<i>-0.5</i>	-	<i>-0.5</i>
	HighGPA	-	<i>0.1</i>	-	<i>0.1</i>	-	<i>0.1</i>
	Insured	-	<i>0.3</i>	-	<i>0.3</i>	-	<i>0.3</i>
	Std. Dev.	-	-	-	-	<i>0.2</i>	<i>0.2</i>
$\mu$	Mean	<i>0.25</i>	<i>0.25</i>	<i>0.23</i>	<i>0.23</i>	<i>0.23</i>	<i>0.23</i>
	Std. Dev.	-	-	<i>0.07</i>	<i>0.07</i>	<i>0.07</i>	<i>0.07</i>

- (1) The first six rows correspond to  $\beta$  from the distribution of  $r_i$ , and the seventh row corresponds to  $\sigma_r^2$ . For ease of interpretation we list the mean and standard deviation of  $\mu_i$  rather than  $m_{\ln\mu}$  and  $\sigma_{\ln\mu}^2$  as the last two rows here and in the presentation of estimation results.
- (2) Compared to specification VI, specifications I - IV assume the standard deviation of the normal distribution of  $r_i$  is 0, and specifications I, III and V assume the demographic coefficients of the mean are 0. In addition, specifications I and II assume the standard deviation of the log-normal distribution of  $\mu_i$  is 0.
- (3) For ease of reading, we display the simulation parameters in italics to differentiate them from estimated values in all tables.

**Table 2: Expected Utility Models Using Simulated Choices under Correct Specifications**

Parameters	Specification I				Specification II				Specification III				Specification IV				Specification V				Specification VI			
	True	Mean	Std. Dev.	95% C.I.	True	Mean	Std. Dev.	95% C.I.	True	Mean	Std. Dev.	95% C.I.	True	Mean	Std. Dev.	95% C.I.	True	Mean	Std. Dev.	95% C.I.	True	Mean	Std. Dev.	95% C.I.
$\tau$	0.5	0.503	0.017	0.471 0.535	0.4	0.351	0.055	0.244 0.449	0.5	0.496	0.015	0.469 0.526	0.4	0.377	0.043	0.293 0.459	0.5	0.493	0.025	0.443 0.543	0.4	0.359	0.071	0.223 0.494
Constant	-	-	-	-	0.2	0.214	0.036	0.148 0.284	-	-	-	-	0.2	0.193	0.032	0.129 0.250	-	-	-	-	0.2	0.191	0.056	0.084 0.298
Female	-	-	-	-	-0.2	-0.206	0.031	-0.264 -0.142	-	-	-	-	-0.2	-0.205	0.027	-0.259 -0.156	-	-	-	-	-0.2	-0.181	0.058	-0.281 -0.064
Black	-	-	-	-	-0.5	-0.457	0.043	-0.540 -0.374	-	-	-	-	-0.5	-0.457	0.042	-0.561 -0.398	-	-	-	-	-0.5	-0.488	0.064	-0.606 -0.358
BusinessMajor	-	-	-	-	0.1	0.093	0.034	0.031 0.162	-	-	-	-	0.1	0.088	0.031	0.029 0.147	-	-	-	-	0.1	0.117	0.059	0.007 0.237
HighGPA	-	-	-	-	0.3	0.327	0.033	0.264 0.389	-	-	-	-	0.3	0.324	0.028	0.267 0.374	-	-	-	-	0.3	0.304	0.061	0.185 0.421
Insured	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.2	0.169	0.022	0.127 0.211	0.2	0.098	0.014	0.073 0.125
$\mu$	0.25	0.260	0.014	0.234 0.286	0.25	0.242	0.011	0.220 0.263	0.23	0.240	0.018	0.207 0.276	0.23	0.221	0.014	0.197 0.249	0.23	0.230	0.017	0.201 0.264	0.23	0.210	0.016	0.180 0.242
Mean	-	-	-	-	-	-	-	-	0.07	0.082	0.026	0.030 0.137	0.07	0.061	0.019	0.027 0.097	0.07	0.078	0.023	0.034 0.123	0.07	0.082	0.021	0.045 0.122
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

\* indicates the true parameter is outside of the 95% credible interval of the posterior sample.

**Table 3: Expected Utility Models Using Simulated Dataset VI under Misspecified Specifications**

Parameters	True Value	Specification I			Specification II			Specification III			Specification IV			Specification V			Specification VI						
		Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.				
$\tau$	Constant	0.432	0.019	0.393	0.467	0.419	0.042	0.342	0.504	0.417	0.025	0.368	0.467	0.395	0.043	0.315	0.474	0.354	0.047	0.263	0.445		
	Female	-	-	-	-	0.170	0.030	0.112	0.227	-	-	-	-	0.179	0.031	0.121	0.242	-	-	-	-		
	Black	-	-	-	-	-0.207	0.027	-0.261	-0.154	-	-	-	-	-0.196	0.027	-0.246	-0.142	-	-	-	-		
	BusinessMajor	-	-	-	-	-0.509	0.038	-0.586	-0.436	-	-	-	-	-0.503	0.043	-0.585	-0.417	-	-	-	-		
	HighGPA	-	-	-	-	0.128	0.031	0.067	0.186	-	-	-	-	0.144	0.029	0.096	0.206	-	-	-	-		
	Insured	-	-	-	-	0.269	0.028	0.211	0.318	-	-	-	-	0.271	0.028	0.212	0.325	-	-	-	-		
	Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
$\mu$	0.23	0.271	*	0.014	0.245	0.301	0.216	0.009	0.198	0.234	0.304	*	0.031	0.247	0.366	0.232	0.019	0.198	0.269	0.208	0.015	0.179	0.238
Std. Dev.	0.07	-	-	-	-	-	-	-	-	-	0.145	0.046	0.068	0.238	0.104	0.025	0.059	0.153	0.083	0.020	0.049	0.122	

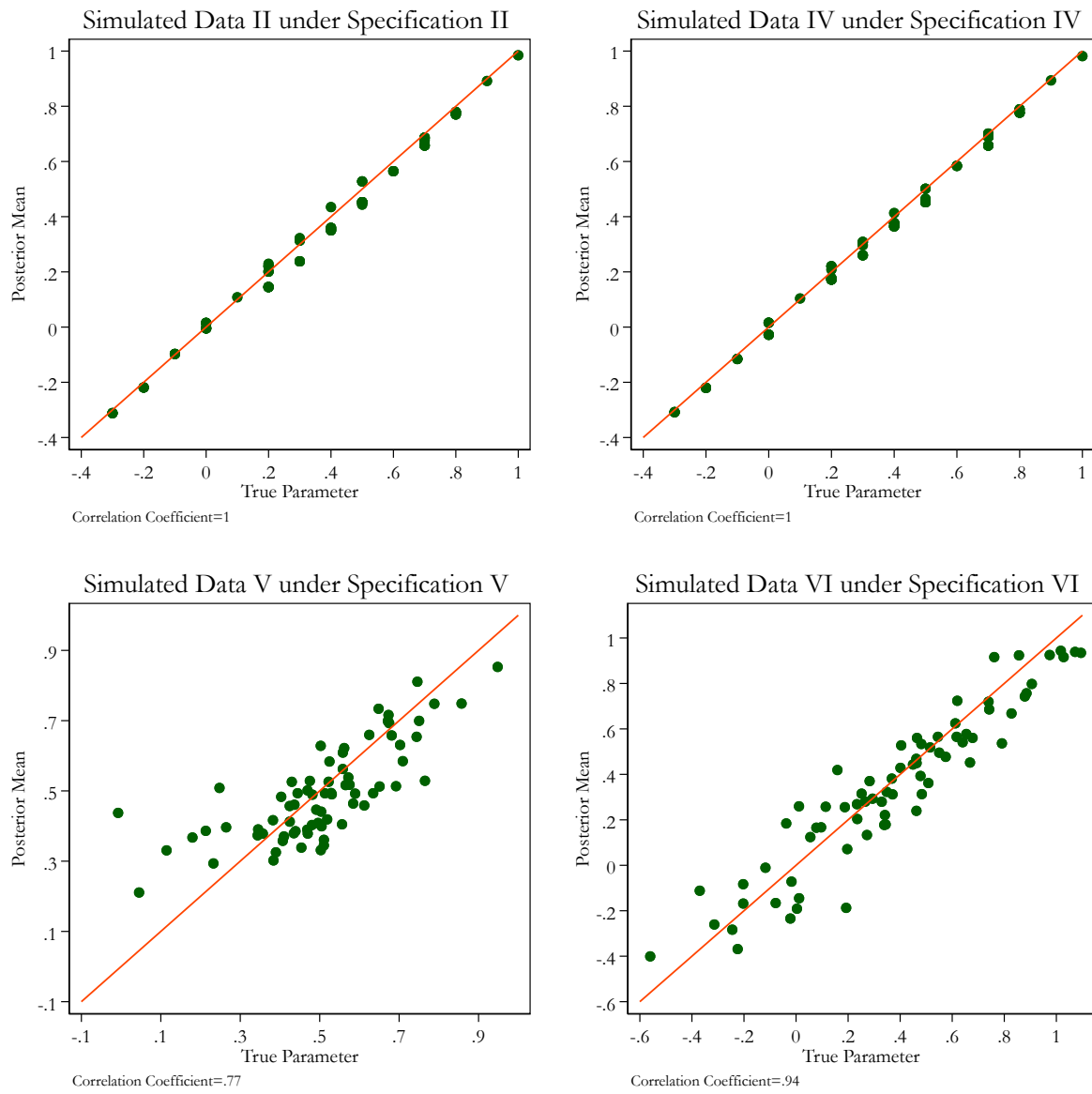
\* indicates the true parameter is outside of the 95% credible interval of the posterior sample.

**Table 4: Expected Utility Estimates with Observed Choices**

Parameters	Specification I			Specification II			Specification III			Specification IV			Specification V			Specification VI					
	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.			
$\tau$																					
Constant	0.575	0.014	0.546	0.602	0.495	0.040	0.415	0.570	0.688	0.631	0.037	0.562	0.703	0.512	0.035	0.446	0.582	0.454	0.077	0.301	0.595
Female	-	-	-	-	0.080	0.028	0.023	0.132	-	0.083	0.031	0.029	0.145	-	-	-	-	0.067	0.084	-0.105	0.230
Black	-	-	-	-	0.055	0.030	-0.003	0.114	-	0.006	0.026	-0.045	0.057	-	-	-	-	0.058	0.072	-0.084	0.207
BusinessMajor	-	-	-	-	-0.077	0.028	-0.128	-0.015	-	-0.104	0.032	-0.169	-0.041	-	-	-	-	-0.013	0.083	-0.192	0.144
HighGPA	-	-	-	-	0.074	0.029	0.016	0.130	-	0.018	0.031	-0.041	0.078	-	-	-	-	0.033	0.075	-0.107	0.181
Insured	-	-	-	-	-0.048	0.028	-0.100	0.006	-	-0.068	0.028	-0.124	-0.017	-	-	-	-	-0.079	0.075	-0.217	0.072
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.280	0.030	0.224	0.338
$\mu$	0.232	0.011	0.211	0.255	0.230	0.012	0.208	0.253	0.441	0.303	0.042	0.232	0.385	0.183	0.011	0.161	0.205	0.184	0.012	0.161	0.207
Mean	-	-	-	-	-	-	-	-	-	0.279	0.093	0.127	0.458	0.227	0.070	0.121	0.367	0.058	0.015	0.032	0.088
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.058	0.016	0.028	0.093

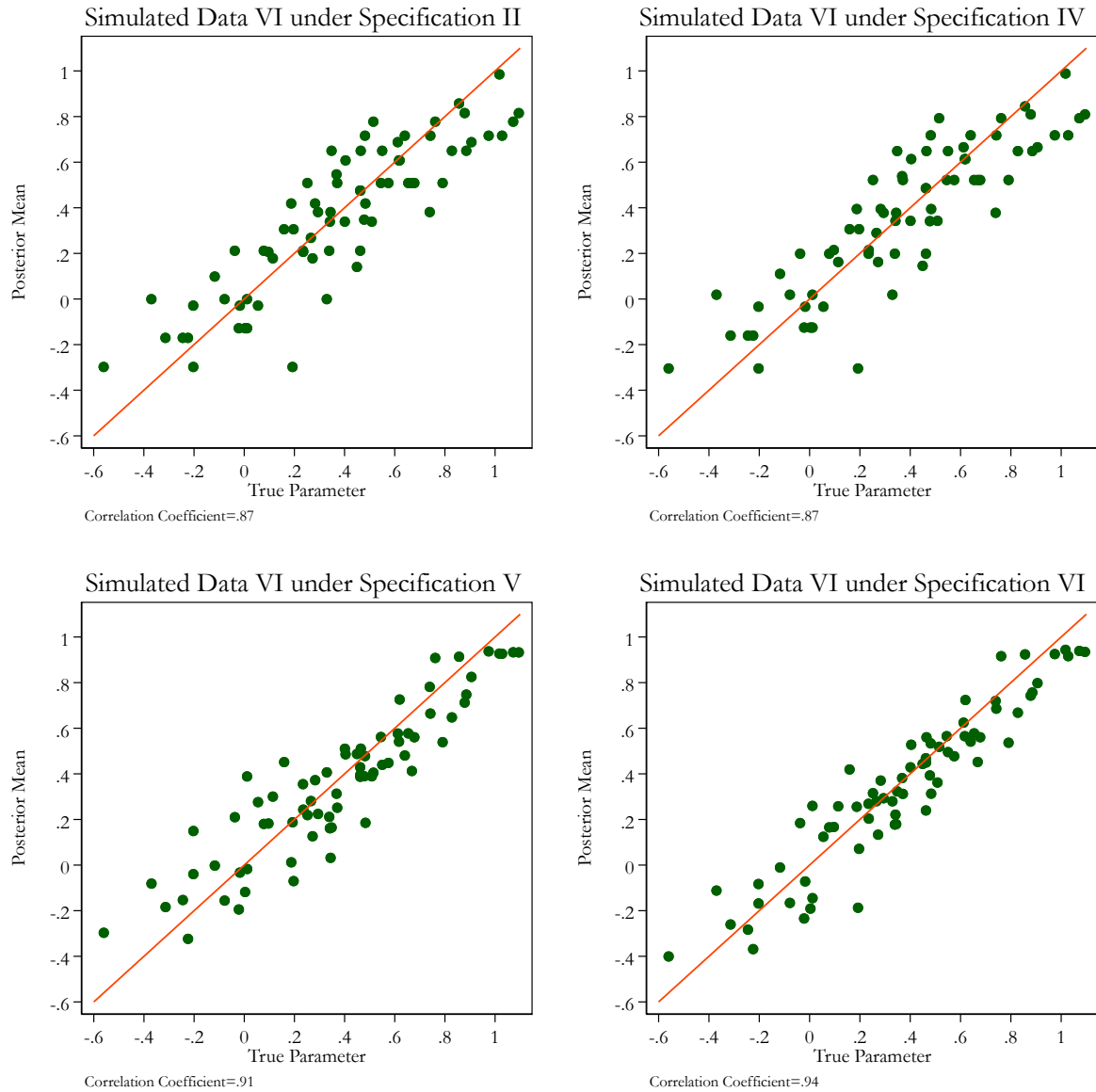
Table 5: Rank Dependent Utility Estimates with Observed Choices

Parameters	Specification I			Specification II			Specification III			Specification IV			Specification V			Specification VI				
	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.	Mean	Std. Dev.	95% C.I.		
r																				
Constant	0.412	0.050	0.315	0.509	0.560	0.137	0.248	0.770	0.495	0.038	0.418	0.566	0.463	0.110	0.268	0.671	0.290	0.098	0.091	0.466
Female	-	-	-	-	-0.300	0.067	-0.431	-0.172	-	-	-	-	-0.070	0.161	-0.301	0.259	-0.034	0.128	-0.316	0.214
Black	-	-	-	-	0.065	0.096	-0.105	0.269	-	-	-	-	0.066	0.146	-0.234	0.292	0.105	0.105	-0.082	0.307
BusinessMajor	-	-	-	-	0.234	0.065	0.103	0.355	-	-	-	-	0.076	0.158	-0.251	0.317	-0.002	0.114	-0.227	0.231
HighGPA	-	-	-	-	-0.179	0.076	-0.325	-0.036	-	-	-	-	-0.069	0.071	-0.213	0.075	-0.064	0.107	-0.275	0.138
Insured	-	-	-	-	-0.033	0.062	-0.159	0.083	-	-	-	-	0.009	0.073	-0.121	0.165	0.003	0.109	-0.213	0.220
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.177	0.034	0.112	0.239
η																				
Constant	1.273	0.111	1.047	1.484	0.667	0.197	0.371	1.118	1.346	0.112	1.134	1.557	1.117	0.240	0.721	1.578	1.123	0.153	0.867	1.432
Female	-	-	-	-	0.822	0.129	0.558	1.062	-	-	-	-	0.497	0.280	-0.086	0.917	0.385	0.147	0.117	0.684
Black	-	-	-	-	0.066	0.166	-0.241	0.385	-	-	-	-	-0.048	0.266	-0.497	0.521	-0.035	0.133	-0.309	0.204
BusinessMajor	-	-	-	-	-0.497	0.134	-0.787	-0.257	-	-	-	-	-0.343	0.321	-0.872	0.291	-0.102	0.150	-0.417	0.163
HighGPA	-	-	-	-	0.419	0.163	0.092	0.730	-	-	-	-	0.087	0.156	-0.224	0.388	0.122	0.131	-0.146	0.382
Insured	-	-	-	-	0.080	0.123	-0.151	0.316	-	-	-	-	-0.039	0.150	-0.333	0.235	0.001	0.157	-0.323	0.320
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.287	0.046	0.200	0.377
ψ																				
Constant	0.841	0.034	0.777	0.907	0.701	0.074	0.560	0.842	0.818	0.033	0.734	0.885	0.656	0.062	0.340	0.777	0.671	0.073	0.528	0.799
Female	-	-	-	-	0.089	0.086	-0.073	0.261	-	-	-	-	0.173	0.101	-0.016	0.371	0.147	0.107	-0.072	0.356
Black	-	-	-	-	0.106	0.079	-0.057	0.256	-	-	-	-	0.089	0.088	-0.088	0.254	0.095	0.113	-0.137	0.314
BusinessMajor	-	-	-	-	0.254	0.080	0.097	0.405	-	-	-	-	0.156	0.084	-0.003	0.324	0.095	0.101	-0.099	0.290
HighGPA	-	-	-	-	-0.133	0.080	-0.287	0.023	-	-	-	-	-0.124	0.082	-0.278	0.040	-0.057	0.114	-0.278	0.155
Insured	-	-	-	-	0.165	0.077	0.011	0.312	-	-	-	-	0.184	0.079	0.031	0.347	0.204	0.100	-0.006	0.392
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.268	0.051	0.171	0.370
μ																				
Constant	0.235	0.013	0.210	0.261	0.218	0.012	0.195	0.242	0.382	0.070	0.265	0.523	0.314	0.049	0.250	0.411	0.177	0.011	0.158	0.200
Female	-	-	-	-	-	-	-	-	0.349	0.129	0.145	0.589	0.244	0.081	0.124	0.402	0.048	0.017	0.017	0.083
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.043	0.019	0.003	0.075

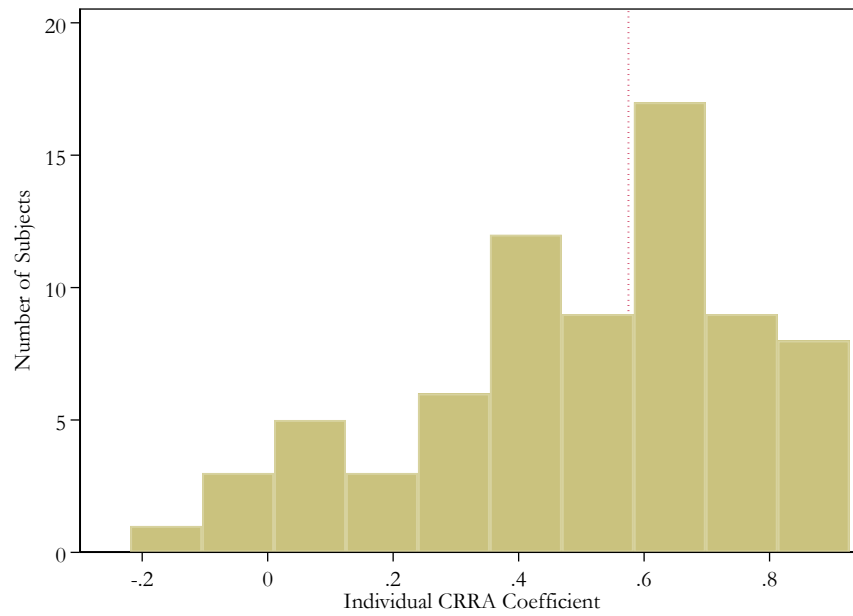
**Figure 1:** Individual CRRA Coefficients in Simulated Data Estimation

**Figure 2:** Individual CRRA Coefficients Using Simulated Dataset VI

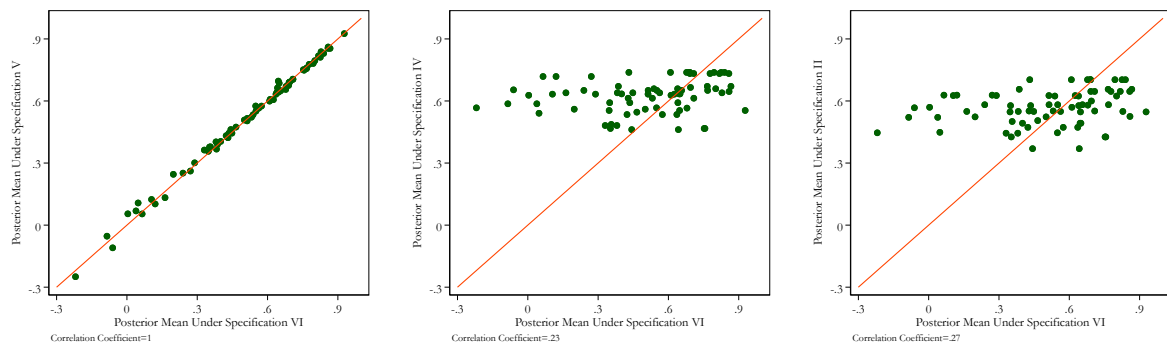
Assuming Incorrect Specifications



**Figure 3:** Individual CRRA Coefficients  $r_i$  from Observed Choices  
under EUT Specification VI

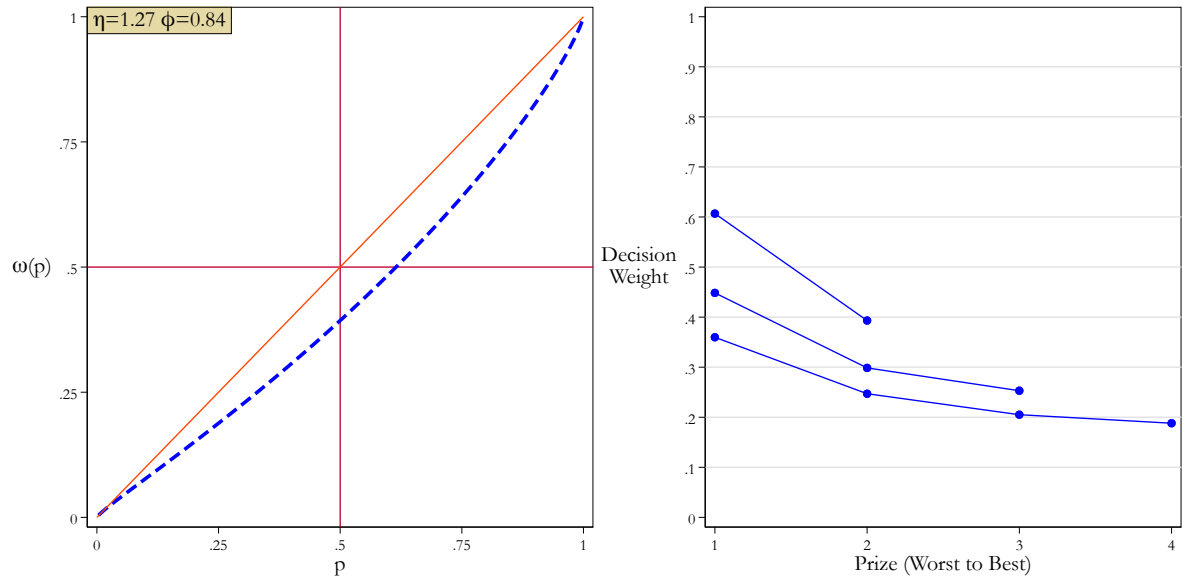


**Figure 4:** Individual CRRA Coefficients  $r_i$  from Observed Choices  
under Different EUT Specifications



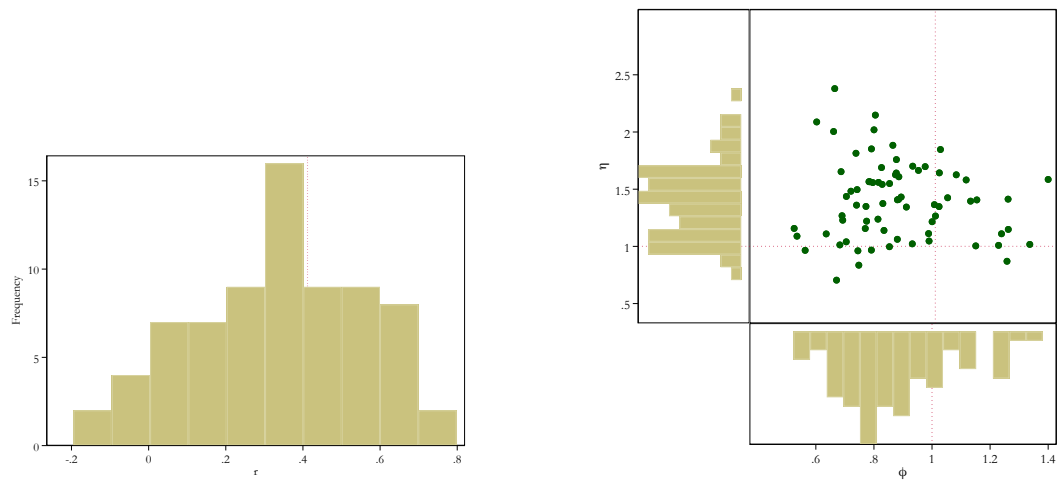
**Figure 5:** Prelec Probability Weighting and Implied Decision Weights  
under Specification I

Based on equi-probable reference lotteries



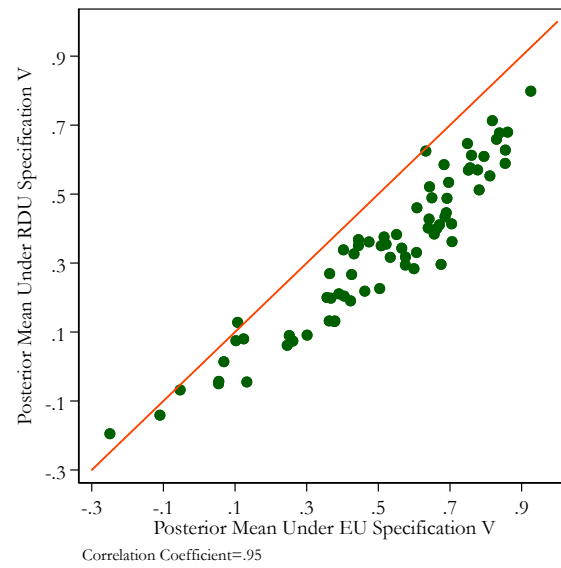
**Figure 6:** Posterior Means of Individual Estimates

Assuming Specification V of the RDU Prelec Model

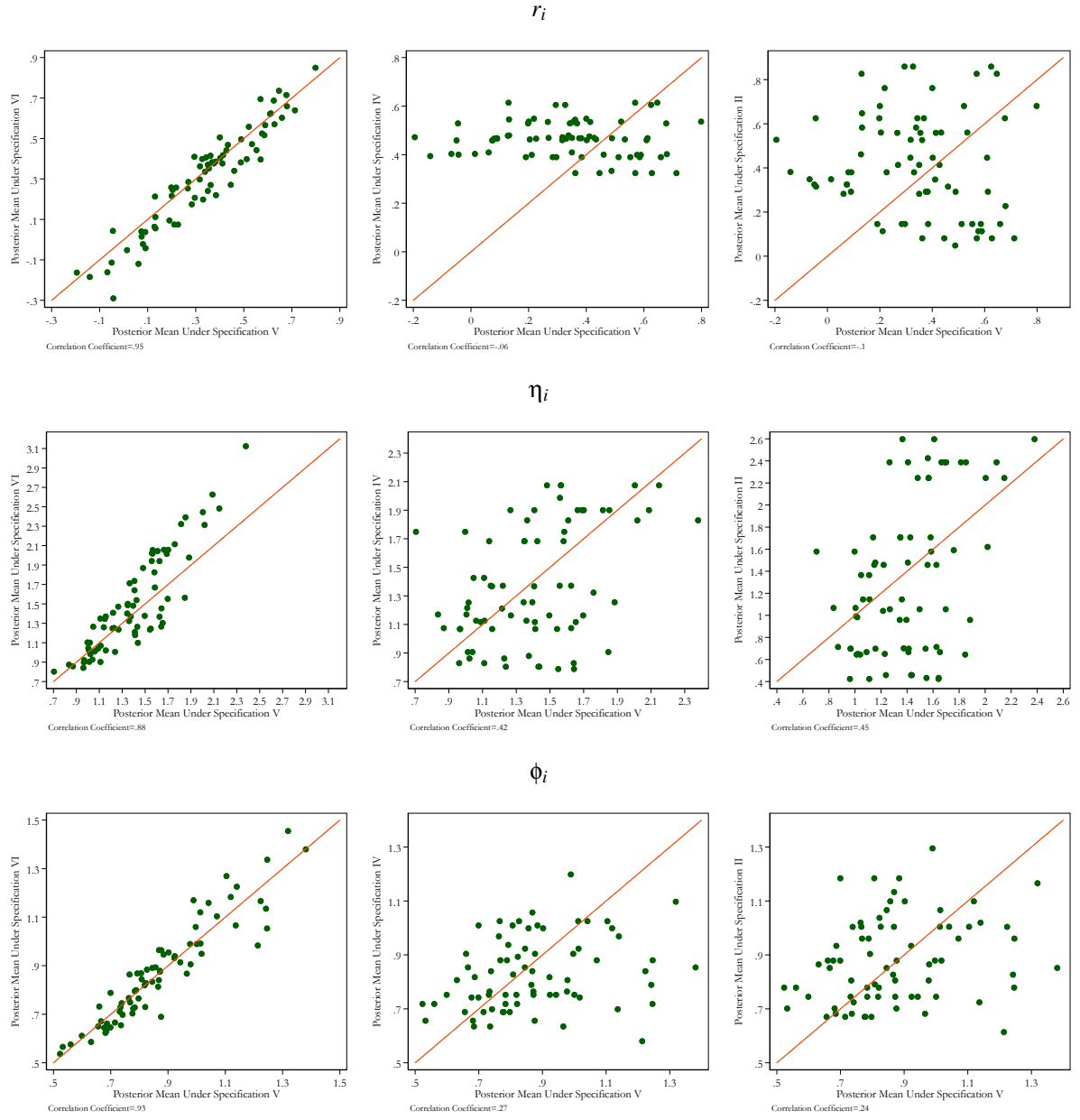


**Figure 7:** Posterior Means of Individual CRRA Coefficients

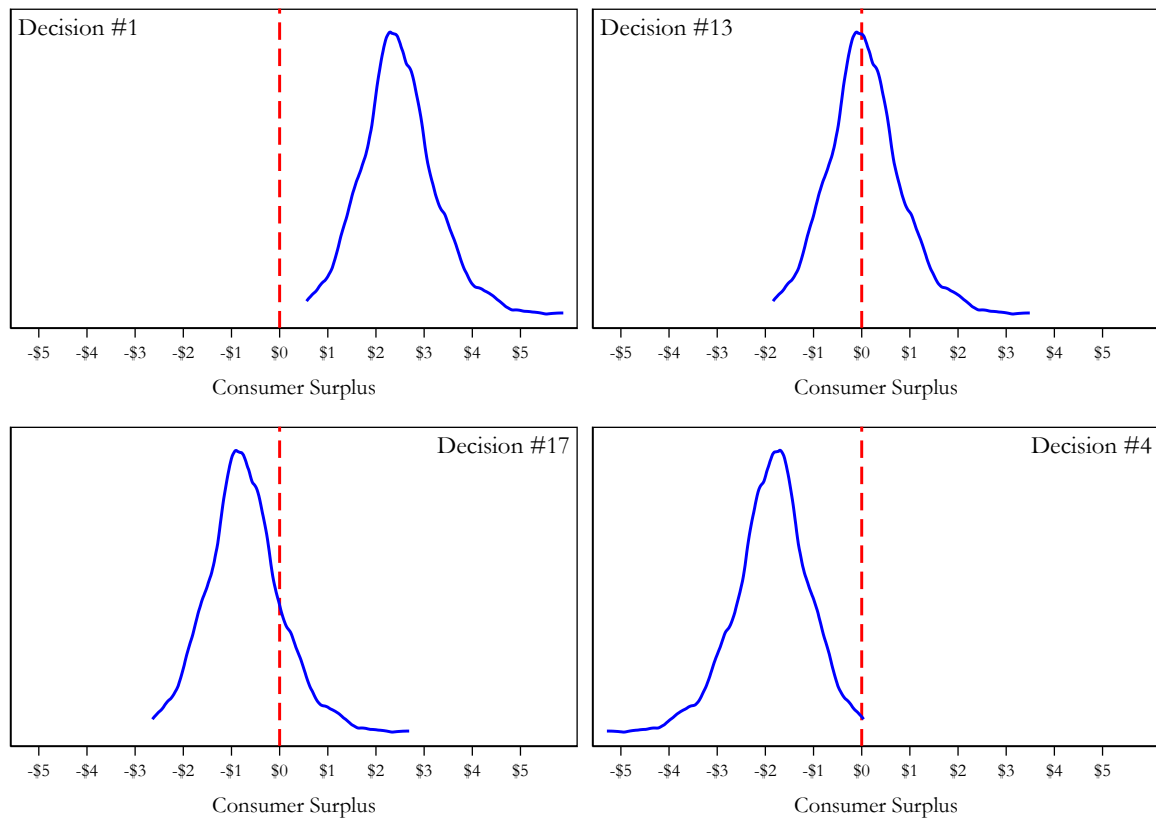
Assuming Specification V of EUT and RDU Prelec Model



**Figure 8:** Individual Preferences from Observed Choices Assuming Specifications II, IV, V and VI of RDU Prelec Model



**Figure 9:** Posterior Predictive Consumer Surplus Distribution  
for Each of Four Insurance Purchase Decisions by One Subject



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## Appendix A Experiment Instructions and Lottery Parameters (ONLINE)

### A.1 Instructions in the Gain Frame

#### Choices Over Risky Prospects

This is a task where you will choose between prospects with varying prizes and chances of winning each prize. You will be presented with a series of pairs of prospects where you will choose one of them. For each pair of prospects, you should choose the prospect you prefer. You will actually get the chance to play one of these prospects for earnings, and you will be paid according to the outcome of that prospect, so you should think carefully about which prospect you prefer on each decision screen.

Here is an example of what the computer display of such a pair of prospects will look like.



The outcome of the prospects will be determined by the draw of a random number between 1 and 100. Each number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

You might be told your cash endowment for each decision at the top of the screen. In this example it is \$35, so any earnings would be added to or subtracted from this endowment. The endowment may

change from choice to choice, so be sure to pay attention to it. The endowment you are shown only applies for that choice.

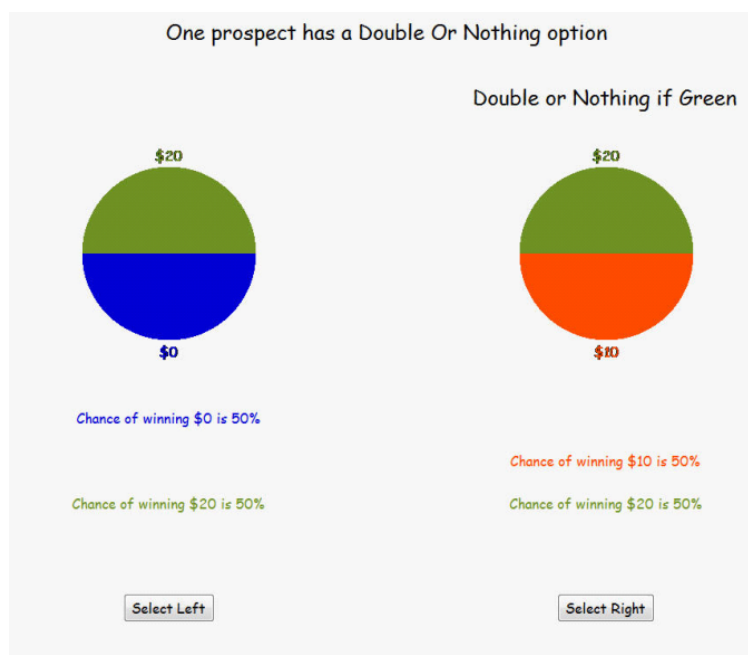
In this example the left prospect pays twenty-five dollars (\$25) if the number drawn is between 1 and 5, pays negative five dollars (\$-5) if the number is between 6 and 55, and pays negative thirty-five dollars (\$-35) if the number is between 56 and 100. The blue color in the pie chart corresponds to 5% of the area and illustrates the chances that the number drawn will be between 1 and 5 and your prize will be \$25. The orange area in the pie chart corresponds to 50% of the area and illustrates the chances that the number drawn will be between 6 and 55 and your prize will be \$-5. The green area in the pie chart corresponds to 45% of the area and illustrates the chances that the number drawn will be between 56 and 100. When you select the decision screen to be played out the computer will confirm the die rolls that correspond to the different prizes.

Now look at the pie on the right. It pays twenty-five dollars (\$25) if the number drawn is between 1 and 15, negative five dollars (\$-5) if the number is between 16 and 25, and negative thirty-five dollars (\$-35) if the number is between 26 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$25 pie slice is 15% of the total pie.

Even though the screen says that you might win a negative amount, this is actually a loss to be deducted from your endowment. So if you win \$-5, your earnings would be  $\$30 = \$35 - \$5$ .

Each pair of prospects is shown on a separate screen on the computer. On each screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

Some decision screens could also have a pair of prospects in which one of the prospects will give you the chance for “Double or Nothing.” For instance, the right prospect in this screen image pays “Double or Nothing” if the Green area is selected, which happens if the number drawn is between 51 and 100. The right pie chart indicates that if the number is between 1 and 50 you get \$10. However, if the number is between 51 and 100 we will flip a coin with you to determine if you get either double the amount or \$0. In this example, if it comes up Heads you get \$40, otherwise you get nothing. The prizes listed underneath each pie refer to the amounts before any “Double or Nothing” coin toss.

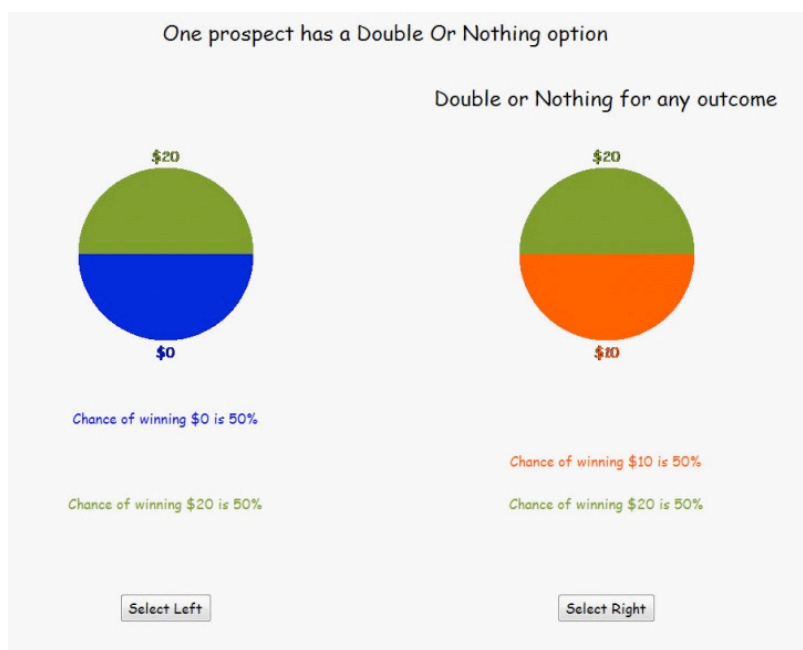


After you have worked through all of the pairs of prospects, please wait quietly until further instructions. When it is time to play this task out for earnings, you will then roll two 10-sided dice until a number comes up to determine which pair of prospects will be played out. If there are 40 pairs we will roll the dice until a number between 1 and 40 comes up, if there are 80 pairs we will roll until a number between 1 and 80 comes up, and so on. Since there is a chance that any of your choices could be played out for real, you should approach each pair of prospects as if it is the one that you will play out. Finally, you will roll the two ten-sided dice to determine the outcome of the prospect you chose, and if necessary we will then toss a coin to determine if you get “Double or Nothing.”

Here is an example: suppose your first roll was 81. We would then pull up the 81st decision that you made and look at which prospect you chose – either the left one or the right one. Let’s say that the 81st lottery was the same as the last example, and you chose the left prospect. If the random number from your second roll was 37, you would win \$0; if it was 93, you would get \$20.

If you picked the prospect on the right and drew the number 37, you would get \$10; if it was 93, we would have to toss a coin to determine if you get “Double or Nothing.” If the coin comes up Heads then you would get \$40. However, if it comes up Tails you would get nothing from your chosen prospect.

It is also possible that you will be given a prospect in which there is a “Double or Nothing” option no matter what the outcome of the random number. This screen image illustrates this possibility.



In summary, your payoff is determined by five things:

- by your endowment, if there is one, shown at the top of the screen;
- by which prospect you selected, the left or the right, for each of these pairs;
- by which prospect pair is chosen to be played out in the series of pairs using the two
- 10-sided dice;
- by the outcome of that prospect when you roll the two 10-sided dice; and
- by the outcome of a coin toss if the chosen prospect outcome is of the “Double or Nothing” type.

Which prospects you prefer is a matter of personal choice. The people next to you may be presented with different prospects, and may have different preferences, so their responses should not matter to you or influence your decisions. Please work silently, and make your choices by thinking carefully about each prospect.

All payoffs are in cash, and are in addition to the \$5 show-up fee that you receive just for being here, as well as any other earnings in other tasks from the session today.

## **A.2 Lottery Parameters in the Gain Frame**

**Table A.1:** Parameters for the Lotteries Based on Loomes and Sugden [1998]

Lottery ID	Left Lottery			Right Lottery		
	Prize 1	Prob 1	Prize 2	Prob 2	Prize 3	Prob 3
ls2_lr	\$10	0.3	\$30	0	\$50	0.7
ls6_lr	\$10	0.6	\$30	0	\$50	0.4
ls7_lr	\$10	0.6	\$30	0	\$50	0.4
ls10_lr	\$10	0.5	\$30	0	\$50	0.5
ls13_rl	\$10	0.5	\$30	0.4	\$50	0.1
ls15_rl	\$10	0.4	\$30	0.6	\$50	0
ls17_lr	\$10	0.1	\$30	0	\$50	0.9
ls18_rl	\$10	0.1	\$30	0.75	\$50	0.15
ls21_lr	\$10	0.7	\$30	0	\$50	0.3
ls26_rl	\$10	0.2	\$30	0.6	\$50	0.2
ls29_rl	\$10	0.5	\$30	0.3	\$50	0.2
ls32_rl	\$10	0.7	\$30	0.3	\$50	0
ls34_rl	\$10	0.1	\$30	0.6	\$50	0.3
ls35_rl	\$10	0	\$30	1	\$50	0
ls39_rl	\$10	0.5	\$30	0.2	\$50	0.3
ls11_lr	\$10	0.12	\$30	0.05	\$50	0.83
ls31_lr	\$10	0.27	\$30	0.05	\$50	0.68
ls7i_lr	\$10	0.54	\$30	0.1	\$50	0.36
ls9i_lr	\$10	0.08	\$30	0.04	\$50	0.88
ls13i_lr	\$10	0.65	\$30	0.1	\$50	0.25
ls16i_lr	\$10	0.88	\$30	0.04	\$50	0.08
ls17i_rl	\$10	0.04	\$30	0.15	\$50	0.81
ls18i_rl	\$10	0.14	\$30	0.65	\$50	0.21
ls22i_lr	\$10	0.66	\$30	0.1	\$50	0.24
ls28i_rl	\$10	0.12	\$30	0.84	\$50	0.04
ls30i_rl	\$10	0.45	\$30	0.45	\$50	0.1
ls31i_lr	\$10	0.48	\$30	0.36	\$50	0.16
ls35i_lr	\$10	0.2	\$30	0.2	\$50	0.6
ls36i_rl	\$10	0.02	\$30	0.92	\$50	0.06
ls37i_lr	\$10	0.48	\$30	0.28	\$50	0.24
	Prize 1	Prob 1	Prize 2	Prob 2	Prize 3	Prob 3
	\$10	0.15	\$30	0.25	\$50	0.6
	\$10	0	\$30	1	\$50	0
	\$10	0.15	\$30	0.75	\$50	0.1
	\$10	0.1	\$30	0.8	\$50	0.1
	\$10	0.7	\$30	0	\$50	0.3
	\$10	0.5	\$30	0.4	\$50	0.1
	\$10	0	\$30	0.25	\$50	0.75
	\$10	0.4	\$30	0	\$50	0.6
	\$10	0.6	\$30	0.25	\$50	0.15
	\$10	0.4	\$30	0	\$50	0.6
	\$10	0.6	\$30	0	\$50	0.4
	\$10	0.8	\$30	0	\$50	0.2
	\$10	0.25	\$30	0	\$50	0.75
	\$10	0.25	\$30	0	\$50	0.75
	\$10	0.55	\$30	0	\$50	0.45
	\$10	0.03	\$30	0.2	\$50	0.77
	\$10	0.03	\$30	0.45	\$50	0.52
	\$10	0.18	\$30	0.7	\$50	0.12
	\$10	0.05	\$30	0.1	\$50	0.85
	\$10	0.55	\$30	0.3	\$50	0.15
	\$10	0.83	\$30	0.14	\$50	0.03
	\$10	0.08	\$30	0.05	\$50	0.87
	\$10	0.38	\$30	0.05	\$50	0.57
	\$10	0.54	\$30	0.4	\$50	0.06
	\$10	0.18	\$30	0.66	\$50	0.16
	\$10	0.55	\$30	0.15	\$50	0.3
	\$10	0.42	\$30	0.54	\$50	0.04
	\$10	0.1	\$30	0.6	\$50	0.3
	\$10	0.08	\$30	0.68	\$50	0.24
	\$10	0.44	\$30	0.44	\$50	0.12

### Table A.2: Parameters for Double or Nothing Lotteries

Lottery ID	Left Lottery				Right Lottery							
	Prize 1	Prob 1	Prize 2	Prob 2	Prize 3	Prob 3	Prize 1	Prob 1	Prize 2	Prob 2	Prize 3	Prob 3
rdon1	\$0	0.5	\$10	0.5	\$20	0	\$0	0.5	\$10	0.5	\$20	0
rdon2	\$0	0	\$10	1	\$20	0	\$0	0.5	\$10	0.5	\$20	0
rdon3	\$0	0	\$10	1	\$35	0	\$0	0	\$5	0.5	\$18	0.5
rdon4	\$0	0.25	\$10	0.75	\$70	0	\$0	0	\$35	1	\$70	0
rdon5	\$0	0	\$10	1	\$70	0	\$0	0	\$35	1	\$70	0
rdon6	\$0	0	\$20	1	\$35	0	\$0	0	\$10	0.5	\$35	0.5
rdon7	\$0	0	\$20	0.5	\$70	0.5	\$0	0	\$35	0.5	\$70	0.5
rdon8	\$0	0	\$35	1	\$70	0	\$0	0	\$35	0.5	\$70	0.5
rdon9	\$0	0	\$20	0.5	\$35	0.5	\$0	0.5	\$20	0	\$70	0.5
rdon10	\$0	0	\$35	0.75	\$70	0.25	\$0	0	\$35	1	\$70	0
rdon11	\$0	0	\$20	1	\$70	0	\$0	0	\$20	0.5	\$35	0.5
rdon12	\$0	0	\$35	0.75	\$70	0.25	\$0	0	\$35	0.5	\$70	0.5
rdon13	\$0	0.25	\$10	0.75	\$35	0	\$0	0.5	\$18	0.5	\$35	0
rdon14	\$0	0	\$20	0.75	\$35	0.25	\$0	0	\$18	0.5	\$35	0.5
rdon15	\$0	0	\$20	0.75	\$70	0.25	\$0	0	\$35	0.5	\$70	0.5

### Table A.3: Parameters for the Actuarially-Equivalent Lotteries

Lottery ID	Left Lottery						Right Lottery					
	Prize 1	Prob 1	Prize 2	Prob 2	Prize 3	Prob 3	Prize 1	Prob 1	Prize 2	Prob 2	Prize 3	Prob 3
rae1	\$0	0.5	\$10	0.5	\$20	0	\$0	0.75	\$10	0	\$20	0.25
rae2	\$0	0	\$10	1	\$20	0	\$0	0.75	\$10	0	\$20	0.25
rae3	\$0	0	\$10	1	\$35	0	\$0	0.5	\$10	0.25	\$35	0.25
rae4	\$0	0.25	\$10	0.75	\$70	0	\$0	0.5	\$10	0	\$70	0.5
rae5	\$0	0	\$10	1	\$70	0	\$0	0.5	\$10	0	\$70	0.5
rae6	\$0	0	\$20	1	\$35	0	\$0	0.25	\$20	0.25	\$35	0.5
rae7	\$0	0	\$20	0.5	\$70	0.5	\$0	0.25	\$20	0	\$70	0.75
rae8	\$0	0	\$35	1	\$70	0	\$0	0.25	\$35	0	\$70	0.75
rae9	\$0	0.25	\$20	0.5	\$70	0.25	\$0	0.5	\$20	0	\$70	0.5
rae10	\$0	0	\$35	0.75	\$70	0.25	\$0	0.5	\$35	0	\$70	0.5
rae11	\$0	0	\$20	1	\$70	0	\$0	0.25	\$20	0.5	\$70	0.25
rae12	\$0	0	\$35	0.75	\$70	0.25	\$0	0.25	\$35	0	\$70	0.75
rae13	\$0	0.25	\$10	0.75	\$35	0	\$0	0.75	\$10	0	\$35	0.25
rae14	\$0	0	\$20	0.75	\$35	0.25	\$0	0.25	\$20	0	\$35	0.75
rae15	\$0	0	\$20	0.75	\$70	0.25	\$0	0.25	\$20	0	\$70	0.75

**Table A.4:** Text for Double or Nothing Lotteries

Also see parameters for the Right Lottery in Table A.2

<b>Lottery ID</b>	<b>Double or Nothing Text</b>
rdon1	Double or Nothing if outcome 2 in right lottery
rdon2	Double or Nothing if outcome 2 in right lottery
rdon3	Double or Nothing for any outcome in right lottery
rdon4	Double or Nothing for any outcome in right lottery
rdon5	Double or Nothing for any outcome in right lottery
rdon6	Double or Nothing if outcome 2 in right lottery
rdon7	Double or Nothing if outcome 2 in right lottery
rdon8	Double or Nothing if outcome 2 in right lottery
rdon9	Double or Nothing if outcome 3 in left lottery
rdon10	Double or Nothing for any outcome in right lottery
rdon11	Double or Nothing if outcome 3 in right lottery
rdon12	Double or Nothing if outcome 2 in right lottery
rdon13	Double or Nothing if outcome 2 in right lottery
rdon14	Double or Nothing if outcome 2 in right lottery
rdon15	Double or Nothing if outcome 2 in right lottery

## Appendix B Additional Results (ONLINE)

**Table B.1:** Summary of Demographics

Variable	Obs	Mean	Std. Dev.	Min	Max
young	73	.9726027	.1643677	0	1
female	73	.5479452	.5011403	0	1
black	73	.6712329	.4730162	0	1
asian	73	.1506849	.3602173	0	1
business	73	.3835616	.4896182	0	1
freshman	73	.1780822	.3852296	0	1
senior	73	.3013699	.4620285	0	1
gpaHI	73	.5616438	.4996193	0	1
christian	73	.7123288	.4558098	0	1
insured	73	.4109589	.4954127	0	1

**Table B.2:** Expected Utility Models Using Simulated Choices under Correct specifications, Individual CRRA Coefficients

Subject	Specification II					Specification IV					Specification V					Specification VI					
	True	Mean	Std. Dev.	95% Cred. Int.		True	Mean	Std. Dev.	95% Cred. Int.		True	Mean	Std. Dev.	95% Cred. Int.		True	Mean	Std. Dev.	95% Cred. Int.		
1	0.50	0.444	0.047	0.357	0.543	0.50	0.465	0.038	0.392	0.537	0.65	0.513	0.096	0.327	0.702	0.37	0.382	0.088	0.203	0.545	
2	-0.20	-0.219	0.054	-0.325	-0.113	-0.20	-0.220	0.050	-0.326	-0.127	0.48	0.528	0.126	0.276	0.767	-0.24	-0.283	0.181	-0.629	0.076	
3	0.30	0.322	0.050	0.222	0.419	0.30	0.296	0.046	0.206	0.383	0.67	0.699	0.083	0.517	0.852	0.27	0.280	0.113	0.052	0.489	
4	0.20	0.201	0.052	0.101	0.301	0.20	0.177	0.049	0.077	0.270	0.18	0.367	0.105	0.154	0.568	0.10	0.167	0.135	-0.094	0.427	
5	0.50	0.452	0.032	0.392	0.518	0.50	0.453	0.029	0.398	0.512	0.56	0.405	0.093	0.221	0.589	0.37	0.312	0.101	0.114	0.507	
6	0.80	0.770	0.036	0.705	0.846	0.80	0.789	0.032	0.726	0.854	0.50	0.628	0.087	0.468	0.811	0.88	0.743	0.078	0.599	0.902	
7	0.20	0.145	0.044	0.059	0.228	0.20	0.172	0.038	0.098	0.246	0.41	0.370	0.117	0.130	0.585	0.34	0.178	0.126	-0.054	0.436	
8	0.70	0.658	0.033	0.595	0.723	0.70	0.658	0.028	0.599	0.707	0.43	0.526	0.108	0.310	0.737	0.74	0.686	0.085	0.523	0.860	
9	0.80	0.779	0.031	0.720	0.841	0.80	0.777	0.027	0.722	0.828	0.41	0.358	0.116	0.101	0.562	0.52	0.519	0.109	0.299	0.735	
10	0.70	0.686	0.036	0.613	0.755	0.70	0.689	0.033	0.624	0.753	0.48	0.403	0.109	0.201	0.620	0.35	0.323	0.114	0.100	0.548	
11	0.20	0.145	0.044	0.059	0.228	0.20	0.172	0.038	0.098	0.246	0.11	0.331	0.124	0.079	0.557	0.08	0.165	0.124	-0.089	0.396	
12	0.60	0.565	0.042	0.486	0.651	0.60	0.584	0.038	0.506	0.654	0.35	0.390	0.102	0.191	0.594	0.62	0.724	0.080	0.562	0.879	
13	0.20	0.229	0.049	0.130	0.324	0.20	0.208	0.046	0.119	0.298	0.42	0.457	0.090	0.267	0.622	0.45	0.443	0.100	0.241	0.639	
14	-0.30	-0.312	0.050	-0.408	-0.215	-0.30	-0.308	0.047	-0.402	-0.220	0.56	0.622	0.078	0.467	0.775	-0.56	-0.401	0.157	-0.731	-0.106	
15	0.00	0.015	0.049	-0.082	0.110	0.00	0.016	0.046	-0.076	0.105	0.50	0.441	0.107	0.218	0.642	-0.20	-0.083	0.163	-0.408	0.224	
16	0.40	0.351	0.055	0.244	0.449	0.40	0.377	0.043	0.293	0.459	0.42	0.413	0.091	0.238	0.590	0.28	0.371	0.080	0.213	0.525	
17	0.30	0.238	0.043	0.156	0.324	0.30	0.260	0.038	0.187	0.332	0.62	0.660	0.095	0.471	0.845	0.40	0.429	0.129	0.173	0.674	
18	0.50	0.452	0.032	0.392	0.518	0.50	0.453	0.029	0.398	0.512	0.57	0.517	0.090	0.327	0.683	0.65	0.578	0.089	0.399	0.751	
19	0.00	-0.004	0.051	-0.103	0.094	0.00	-0.027	0.047	-0.119	0.065	-0.01	0.437	*	0.100	0.234	0.621	0.01	0.260	0.147	-0.036	0.529
20	0.40	0.351	0.055	0.244	0.449	0.40	0.377	0.043	0.293	0.459	0.50	0.331	0.114	0.103	0.547	0.48	0.313	0.096	0.123	0.504	
21	0.70	0.658	0.033	0.595	0.723	0.70	0.658	0.028	0.599	0.707	0.68	0.658	0.101	0.472	0.861	0.97	0.925	0.053	0.825	1.000	
22	-0.10	-0.097	0.050	-0.193	0.002	-0.10	-0.115	0.047	-0.209	-0.024	0.36	0.378	0.107	0.157	0.578	0.00	-0.191	0.149	-0.500	0.086	
23	0.00	0.015	0.049	-0.082	0.110	0.00	0.016	0.046	-0.076	0.105	0.45	0.339	0.122	0.104	0.590	-0.02	-0.072	0.146	-0.372	0.199	
24	0.80	0.779	0.031	0.720	0.841	0.80	0.777	0.027	0.722	0.828	0.58	0.464	0.100	0.272	0.659	1.07	0.939	*	0.047	0.847	
25	0.10	0.108	0.054	0.004	0.216	0.10	0.104	0.050	0.005	0.201	0.52	0.419	0.103	0.212	0.615	-0.12	-0.011	0.159	-0.339	0.280	
26	0.00	-0.004	0.051	-0.103	0.094	0.00	-0.027	0.047	-0.119	0.065	0.51	0.345	0.106	0.139	0.548	-0.37	-0.112	0.154	-0.431	0.177	
27	-0.20	-0.219	0.054	-0.325	-0.113	-0.20	-0.220	0.050	-0.326	-0.127	0.95	0.853	0.073	0.732	1.000	-0.31	-0.260	0.146	-0.557	0.020	
28	0.50	0.528	0.043	0.440	0.607	0.50	0.501	0.043	0.413	0.581	0.21	0.386	0.121	0.148	0.622	0.46	0.469	0.130	0.198	0.709	
29	-0.10	-0.097	0.050	-0.193	0.002	-0.10	-0.115	0.047	-0.209	-0.024	0.65	0.734	0.095	0.554	0.923	-0.02	-0.234	0.157	-0.546	0.057	
30	0.10	-0.097	0.050	-0.193	0.002	-0.10	-0.115	0.047	-0.209	-0.024	0.69	0.513	0.122	0.265	0.743	0.01	-0.145	0.170	-0.477	0.178	
31	-0.30	-0.312	0.050	-0.408	-0.215	-0.30	-0.308	0.047	-0.402	-0.220	0.50	0.409	0.094	0.228	0.601	-0.20	-0.168	0.174	-0.501	0.166	
32	0.50	0.452	0.032	0.392	0.518	0.50	0.453	0.029	0.398	0.512	0.38	0.302	0.111	0.085	0.513	0.79	0.537	*	0.105	0.328	
33	0.40	0.435	0.050	0.333	0.527	0.40	0.413	0.048	0.319	0.503	0.49	0.447	0.112	0.221	0.662	0.48	0.393	0.130	0.132	0.642	
34	0.20	0.145	0.044	0.059	0.228	0.20	0.172	0.038	0.098	0.246	0.59	0.492	0.107	0.277	0.700	0.23	0.269	0.116	0.040	0.498	
35	0.70	0.686	0.036	0.613	0.755	0.70	0.689	0.033	0.624	0.753	0.44	0.461	0.091	0.269	0.633	0.55	0.495	0.088	0.323	0.671	
36	0.70	0.658	0.033	0.595	0.723	0.70	0.658	0.028	0.599	0.707	0.51	0.493	0.092	0.309	0.671	0.48	0.534	0.083	0.378	0.702	
37	0.30	0.314	0.050	0.216	0.411	0.30	0.309	0.049	0.210	0.406	0.51	0.361	0.131	0.091	0.599	0.20	0.071	0.160	-0.249	0.382	
38	0.70	0.677	0.045	0.591	0.762	0.70	0.701	0.038	0.630	0.777	0.38	0.417	0.115	0.197	0.642	0.61	0.625	0.083	0.457	0.780	
39	0.40	0.359	0.038	0.288	0.435	0.40	0.365	0.035	0.302	0.440	0.53	0.490	0.118	0.251	0.719	0.34	0.179	0.123	-0.055	0.426	
40	0.20	0.145	0.044	0.059	0.228	0.20	0.172	0.038	0.098	0.246	0.70	0.631	0.094	0.447	0.811	-0.04	0.184	*	0.107	-0.021	
41	0.20	0.201	0.052	0.101	0.301	0.20	0.177	0.049	0.077	0.270	0.77	0.529	*	0.109	0.315	0.742	0.24	0.204	0.160	-0.132	
42	0.00	0.015	0.049	-0.082	0.110	0.00	0.016	0.046	-0.076	0.105	0.34	0.374	0.107	0.172	0.591	0.05	0.124	0.160	-0.210	0.417	
43	0.80	0.779	0.031	0.720	0.841	0.80	0.777	0.027	0.722	0.828	0.75	0.811	0.083	0.651	0.976	0.76	0.916	*	0.062	0.800	
44	0.70	0.686	0.036	0.613	0.755	0.70	0.689	0.033	0.624	0.753	0.56	0.609	0.086	0.444	0.784	0.83	0.668	0.082	0.511	0.837	
45	0.50	0.452	0.032	0.392	0.518	0.50	0.453	0.029	0.398	0.512	0.56	0.562	0.079	0.405	0.718	0.55	0.565	0.075	0.407	0.708	
46	0.70	0.677	0.045	0.591	0.762	0.70	0.701	0.038	0.630	0.777	0.52	0.584	0.107	0.375	0.794	0.91	0.798	0.084	0.644	0.968	
47	0.60	0.565	0.042	0.486	0.651	0.60	0.584	0.038	0.506	0.654	0.74	0.654	0.079	0.495	0.806	0.62	0.565	0.087	0.396	0.739	
48	0.30	0.238	0.043	0.156	0.324	0.30	0.260	0.038	0.187	0.332	0.53	0.494	0.100	0.299	0.696	0.51	0.362	0.112	0.138	0.577	
49	0.80	0.770	0.036	0.705	0.846	0.80	0.789	0.032	0.726	0.854	0.86	0.749	0.089	0.570	0.921	1.09	0.935	*	0.049	0.842	
50	0.20	0.221	0.053	0.120	0.325	0.20	0.221	0.049	0.127	0.320	0.23	0.294	0.125	0.039	0.527	0.27	0.133	0.141	-0.154	0.401	
51	0.50	0.452	0.032	0.392	0.518	0.50	0.453	0.029	0.398	0.512											

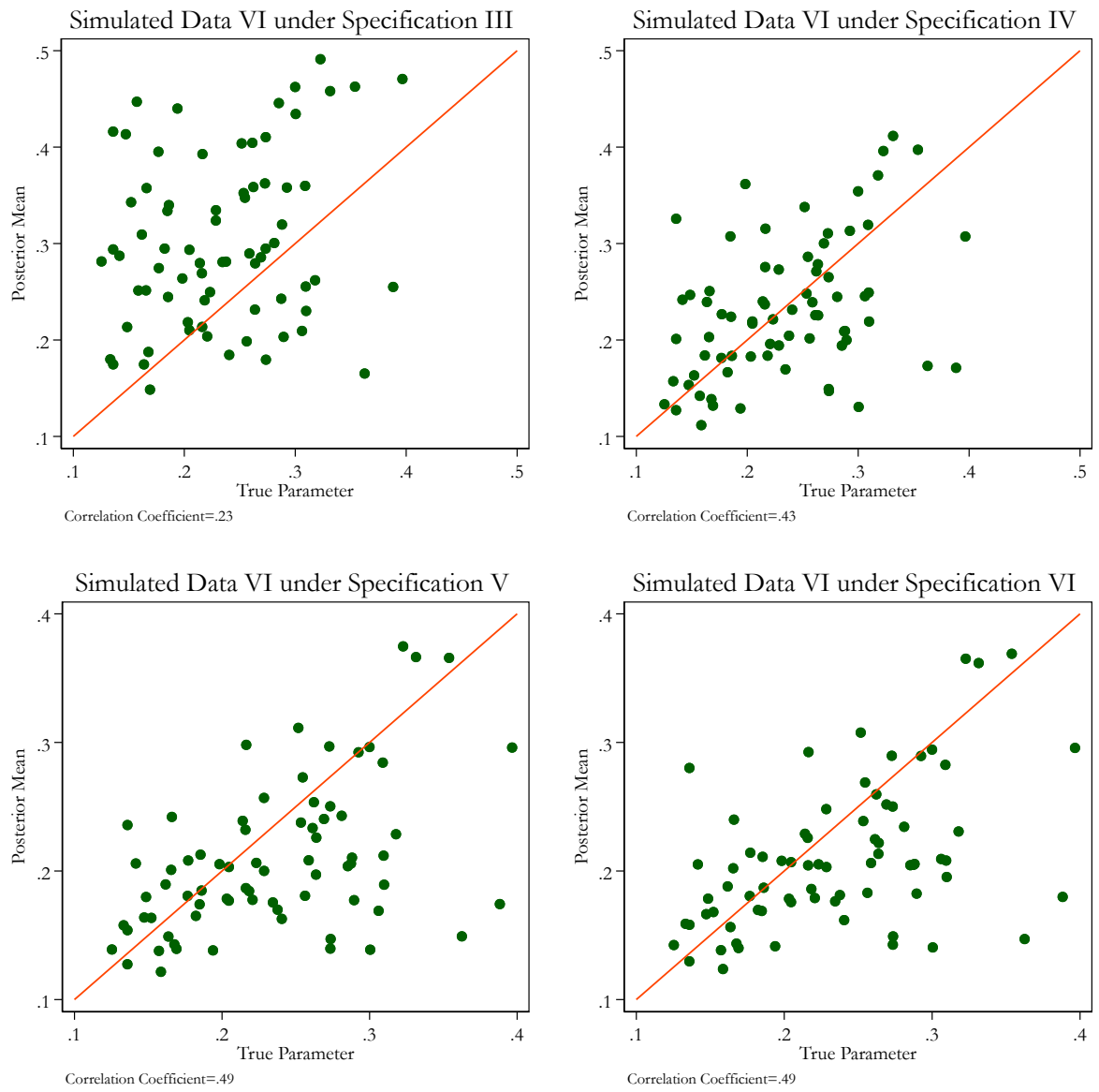
**Table B.3:** Expected Utility Models Using Simulated Choices under Correct specifications, Individual Fechner Noise Parameters

Subject	Specification III					Specification IV					Specification V					Specification VI				
	True	Mean	Std. Dev.	95% Cred. Int.		True	Mean	Std. Dev.	95% Cred. Int.		True	Mean	Std. Dev.	95% Cred. Int.		True	Mean	Std. Dev.	95% Cred. Int.	
1	0.36	0.209 *	0.062	0.106	0.328	0.36	0.199 *	0.048	0.112	0.290	0.36	0.219 *	0.061	0.114	0.342	0.37	0.147 *	0.046	0.070	0.236
2	0.40	0.318	0.090	0.179	0.501	0.40	0.271	0.059	0.181	0.401	0.40	0.319	0.094	0.179	0.499	-0.24	0.296 *	0.080	0.169	0.450
3	0.13	0.209	0.062	0.108	0.330	0.13	0.197	0.047	0.114	0.294	0.13	0.192	0.052	0.104	0.297	0.27	0.159 *	0.049	0.081	0.250
4	0.17	0.231	0.067	0.126	0.356	0.17	0.209	0.050	0.126	0.317	0.17	0.202	0.055	0.111	0.314	0.10	0.202 *	0.059	0.103	0.315
5	0.20	0.198	0.056	0.100	0.306	0.20	0.188	0.046	0.108	0.278	0.20	0.193	0.056	0.102	0.302	0.37	0.176 *	0.052	0.091	0.282
6	0.24	0.200	0.058	0.104	0.309	0.24	0.199	0.044	0.115	0.283	0.24	0.189	0.052	0.094	0.288	0.88	0.181 *	0.054	0.095	0.296
7	0.25	0.302	0.086	0.171	0.467	0.25	0.259	0.060	0.157	0.372	0.25	0.288	0.084	0.156	0.452	0.34	0.308	0.092	0.169	0.486
8	0.20	0.278	0.078	0.152	0.435	0.20	0.239	0.053	0.150	0.349	0.20	0.278	0.079	0.154	0.434	0.74	0.207 *	0.059	0.112	0.328
9	0.32	0.231	0.065	0.120	0.354	0.32	0.242	0.055	0.146	0.345	0.32	0.258	0.073	0.142	0.393	0.52	0.231 *	0.075	0.109	0.372
10	0.20	0.247	0.071	0.130	0.384	0.20	0.218	0.051	0.131	0.324	0.20	0.240	0.067	0.133	0.375	0.35	0.208 *	0.063	0.107	0.336
11	0.25	0.269	0.082	0.149	0.415	0.25	0.259	0.061	0.162	0.379	0.25	0.274	0.078	0.151	0.427	0.08	0.269 *	0.078	0.149	0.426
12	0.16	0.227	0.064	0.121	0.354	0.16	0.198	0.047	0.112	0.292	0.16	0.209	0.059	0.110	0.322	0.62	0.188 *	0.054	0.098	0.295
13	0.16	0.161	0.050	0.073	0.256	0.16	0.197	0.048	0.117	0.293	0.16	0.171	0.051	0.085	0.276	0.45	0.156 *	0.051	0.073	0.256
14	0.15	0.186	0.054	0.088	0.286	0.15	0.212	0.045	0.131	0.301	0.15	0.176	0.049	0.089	0.273	-0.56	0.168 *	0.042	0.096	0.252
15	0.17	0.235	0.065	0.119	0.360	0.17	0.232	0.053	0.141	0.343	0.17	0.230	0.067	0.126	0.359	-0.20	0.240 *	0.065	0.137	0.369
16	0.17	0.194	0.053	0.100	0.294	0.17	0.165	0.044	0.082	0.249	0.17	0.158	0.049	0.071	0.249	0.28	0.140 *	0.047	0.066	0.226
17	0.31	0.279	0.081	0.156	0.439	0.31	0.279	0.066	0.162	0.411	0.31	0.255	0.069	0.144	0.395	0.40	0.283	0.087	0.153	0.452
18	0.22	0.189	0.056	0.094	0.292	0.22	0.187	0.046	0.098	0.275	0.22	0.187	0.054	0.098	0.296	0.65	0.186 *	0.056	0.093	0.295
19	0.26	0.219	0.061	0.120	0.339	0.26	0.257	0.058	0.166	0.378	0.26	0.209	0.059	0.110	0.318	0.01	0.213 *	0.068	0.104	0.344
20	0.23	0.285	0.082	0.157	0.444	0.23	0.240	0.057	0.141	0.349	0.23	0.259	0.072	0.147	0.405	0.48	0.248 *	0.071	0.130	0.385
21	0.16	0.283	0.080	0.153	0.434	0.16	0.241	0.054	0.145	0.347	0.16	0.260	0.071	0.141	0.398	0.97	0.138 *	0.037	0.072	0.210
22	0.18	0.210	0.056	0.115	0.321	0.18	0.181	0.041	0.107	0.263	0.18	0.216	0.059	0.110	0.326	0.00	0.170 *	0.046	0.091	0.263
23	0.29	0.307	0.093	0.174	0.492	0.29	0.209	0.046	0.127	0.299	0.29	0.279	0.077	0.159	0.439	-0.02	0.205 *	0.054	0.114	0.313
24	0.30	0.247	0.070	0.132	0.385	0.30	0.222	0.052	0.129	0.326	0.30	0.243	0.068	0.135	0.383	1.07	0.141 *	0.036	0.075	0.209
25	0.25	0.223	0.062	0.121	0.352	0.25	0.207	0.048	0.118	0.296	0.25	0.219	0.062	0.116	0.343	-0.12	0.239 *	0.065	0.136	0.368
26	0.23	0.222	0.061	0.116	0.339	0.23	0.205	0.046	0.121	0.295	0.23	0.201	0.056	0.110	0.315	-0.37	0.176 *	0.048	0.096	0.272
27	0.13	0.162	0.050	0.079	0.257	0.13	0.163	0.040	0.090	0.240	0.13	0.167	0.044	0.092	0.254	-0.31	0.142 *	0.038	0.081	0.220
28	0.27	0.287	0.082	0.159	0.457	0.27	0.266	0.065	0.160	0.396	0.27	0.296	0.089	0.162	0.463	0.46	0.290	0.086	0.157	0.468
29	0.19	0.294	0.084	0.160	0.458	0.19	0.187	0.042	0.108	0.265	0.19	0.234	0.062	0.125	0.357	-0.02	0.187 *	0.050	0.106	0.284
30	0.26	0.295	0.087	0.163	0.452	0.26	0.246	0.051	0.160	0.351	0.26	0.308	0.091	0.179	0.487	0.01	0.260 *	0.068	0.149	0.395
31	0.31	0.192 *	0.056	0.103	0.300	0.31	0.198 *	0.044	0.125	0.291	0.31	0.183 *	0.052	0.090	0.283	-0.20	0.209 *	0.061	0.109	0.326
32	0.26	0.238	0.069	0.127	0.382	0.26	0.217	0.054	0.131	0.328	0.26	0.207	0.055	0.115	0.313	0.79	0.222 *	0.067	0.108	0.350
33	0.27	0.268	0.076	0.140	0.412	0.27	0.237	0.058	0.142	0.360	0.27	0.254	0.074	0.136	0.398	0.48	0.250 *	0.079	0.135	0.405
34	0.26	0.232	0.065	0.128	0.362	0.26	0.212	0.049	0.122	0.302	0.26	0.233	0.065	0.126	0.365	0.23	0.183	0.053	0.094	0.287
35	0.14	0.172	0.052	0.078	0.270	0.14	0.188	0.045	0.108	0.278	0.14	0.172	0.052	0.090	0.278	0.55	0.158 *	0.049	0.076	0.252
36	0.24	0.175	0.051	0.089	0.273	0.24	0.190	0.045	0.107	0.276	0.24	0.177	0.054	0.083	0.278	0.48	0.162 *	0.050	0.079	0.260
37	0.30	0.354	0.104	0.197	0.567	0.30	0.285	0.073	0.163	0.420	0.30	0.329	0.095	0.188	0.522	0.20	0.294	0.084	0.159	0.457
38	0.18	0.217	0.062	0.110	0.338	0.18	0.218	0.049	0.131	0.318	0.18	0.263	0.074	0.147	0.431	0.61	0.214 *	0.069	0.113	0.342
39	0.26	0.298	0.085	0.171	0.474	0.26	0.268	0.064	0.166	0.398	0.26	0.285	0.079	0.156	0.442	0.34	0.206 *	0.062	0.102	0.322
40	0.27	0.254	0.073	0.136	0.393	0.27	0.238	0.054	0.145	0.350	0.27	0.225	0.065	0.127	0.354	-0.04	0.149 *	0.044	0.072	0.236
41	0.29	0.292	0.085	0.162	0.460	0.29	0.243	0.057	0.143	0.355	0.29	0.259	0.077	0.137	0.409	0.24	0.289	0.085	0.155	0.464
42	0.27	0.239	0.068	0.122	0.369	0.27	0.256	0.057	0.152	0.364	0.27	0.229	0.062	0.128	0.351	0.05	0.252 *	0.075	0.133	0.390
43	0.29	0.274	0.078	0.154	0.432	0.29	0.201 *	0.046	0.119	0.284	0.29	0.205	0.052	0.116	0.307	0.76	0.205 *	0.047	0.124	0.298
44	0.39	0.204 *	0.059	0.111	0.322	0.39	0.199 *	0.048	0.108	0.288	0.39	0.195 *	0.055	0.105	0.308	0.83	0.180 *	0.058	0.092	0.293
45	0.17	0.152	0.049	0.067	0.246	0.17	0.163	0.043	0.081	0.241	0.17	0.163	0.050	0.076	0.259	0.55	0.144 *	0.044	0.075	0.236
46	0.26	0.280	0.081	0.154	0.440	0.26	0.256	0.061	0.153	0.374	0.26	0.277	0.086	0.146	0.429	0.91	0.225 *	0.060	0.126	0.344
47	0.20	0.249	0.071	0.130	0.393	0.20	0.192	0.047	0.104	0.282	0.20	0.190	0.053	0.103	0.301	0.62	0.178 *	0.057	0.087	0.283
48	0.19	0.239	0.068	0.127	0.371	0.19	0.215	0.051	0.131	0.319	0.19	0.229	0.067	0.119	0.358	0.51	0.211 *	0.061	0.114	0.334
49	0.19	0.246	0.074	0.129	0.393	0.19	0.239	0.055	0.149	0.343	0.19	0.220	0.060	0.120	0.332	1.09	0.141 *	0.037	0.079	0.214
50	0.22	0.247	0.068	0.129	0.374	0.22	0.262	0.064	0.157	0.382	0.22	0.291	0.087	0.165	0.466	0.27	0.293	0.085	0.154	0.453
51	0.29	0.216	0.059	0.115	0.333	0.29	0.207	0.050	0.116	0.303	0.29	0.216	0.059	0.116	0.331	0.57	0.205 *	0.062	0.109	0.320
52	0.21	0.244	0.068	0.124	0.371	0.21	0.224	0.055	0.137	0.338	0.21	0.171	0.049	0.090	0.269	0.29	0.229	0.067	0.122	0.354
53	0.15	0.196	0.059	0.095	0.305	0.15	0.220	0.050	0.133	0.321	0.15	0.186	0.053	0.104	0.298	0.64	0.178 *	0.056	0.090	0.289
54	0.22	0.210	0.062	0.111	0.329	0.22	0.195	0.045	0.114	0.281	0.22	0.192	0.053	0.097	0.302	0.33	0.204	0.065	0.093	0.330
55	0.22	0.256	0.071	0.147	0.409	0.22	0.235	0.059	0.132	0.349	0.22	0.233	0.065	0.118	0.359	0.67	0.226 *	0.067	0.118	0.364
56	0.23	0.214	0.062	0.111	0.334	0.23	0.262	0.068	0.152	0.387	0.23	0.208	0.057	0.111	0.318	0.89	0.203 *	0.059	0.104	0.317
57	0.14	0.205	0.058	0.104	0.318	0.14	0.203	0.045	0.123	0.290	0.14	0.208	0.058	0.109	0.324	0.19	0.280	0.080	0.152	0.44

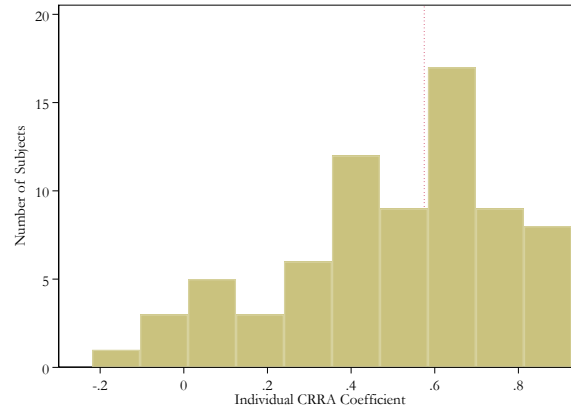
**Table B.4:** Expected Utility Models Using Simulated Dataset VI under Misspecified specifications, Individual CRRA Coefficients

Subject	True Value	Specification II				Specification IV				Specification V				Specification VI							
		Mean	Std. Dev.	95% Cred. Int.		Mean	Std. Dev.	95% Cred. Int.		Mean	Std. Dev.	95% Cred. Int.		Mean	Std. Dev.	95% Cred. Int.					
1	0.37	0.546	*	0.037	0.478	0.622	0.539	*	0.038	0.470	0.613	0.313	0.118	0.067	0.525	0.382	0.088	0.203	0.545		
2	-0.24	-0.170		0.047	-0.266	-0.080	-0.160		0.047	-0.253	-0.070	-0.153	0.229	-0.582	0.284	-0.283	0.181	-0.629	0.076		
3	0.27	0.268		0.044	0.183	0.356	0.290		0.047	0.196	0.382	0.281	0.115	0.045	0.499	0.280	0.113	0.052	0.489		
4	0.10	0.207	*	0.047	0.116	0.299	0.215	*	0.049	0.121	0.313	0.182	0.153	-0.127	0.465	0.167	0.135	-0.094	0.427		
5	0.37	0.509	*	0.029	0.455	0.565	0.522	*	0.030	0.464	0.580	0.251	0.127	-0.012	0.496	0.312	0.101	0.114	0.507		
6	0.88	0.815	*	0.031	0.757	0.875	0.809	*	0.031	0.749	0.868	0.712	*	0.081	0.550	0.874	0.743	0.078	0.599	0.902	
7	0.34	0.212	*	0.035	0.147	0.286	0.199	*	0.038	0.121	0.267	0.211	0.172	-0.118	0.548	0.178	0.126	-0.054	0.436		
8	0.74	0.716		0.028	0.663	0.774	0.718		0.028	0.662	0.774	0.664	0.093	0.474	0.842	0.686	0.085	0.523	0.860		
9	0.52	0.778	*	0.027	0.724	0.831	0.793	*	0.027	0.738	0.843	0.405	0.137	0.138	0.661	0.519	0.109	0.299	0.735		
10	0.35	0.650	*	0.032	0.587	0.714	0.649	*	0.035	0.581	0.718	0.165	0.145	-0.113	0.443	0.323	0.114	0.100	0.548		
11	0.08	0.212	*	0.035	0.147	0.286	0.199	*	0.038	0.121	0.267	0.181	0.187	-0.201	0.520	0.165	0.124	-0.089	0.396		
12	0.62	0.608		0.039	0.531	0.681	0.614		0.036	0.541	0.683	0.726	0.084	0.566	0.900	0.724	0.080	0.562	0.879		
13	0.45	0.141	*	0.045	0.049	0.225	0.146	*	0.050	0.045	0.243	0.487	0.093	0.304	0.673	0.443	0.100	0.241	0.639		
14	-0.56	-0.298	*	0.044	-0.385	-0.212	-0.304	*	0.047	-0.395	-0.210	-0.297	0.198	-0.694	0.065	-0.401	0.157	-0.731	-0.106		
15	-0.20	-0.029	*	0.045	-0.115	0.057	-0.034	*	0.051	-0.135	0.065	-0.040	0.201	-0.443	0.342	-0.083	0.163	-0.408	0.224		
16	0.28	0.419	*	0.042	0.342	0.504	0.395	*	0.043	0.315	0.474	0.372	0.107	0.164	0.579	0.371	0.080	0.213	0.525		
17	0.40	0.339		0.037	0.265	0.412	0.343		0.036	0.274	0.414	0.510	0.144	0.219	0.770	0.429	0.129	0.173	0.674		
18	0.65	0.509	*	0.029	0.455	0.565	0.522	*	0.030	0.464	0.580	0.578	0.098	0.382	0.761	0.578	0.089	0.399	0.751		
19	0.01	-0.001		0.045	-0.094	0.085	0.019		0.046	-0.072	0.110	0.389	*	0.125	0.127	0.615	0.260	0.147	-0.036	0.529	
20	0.48	0.419		0.042	0.342	0.504	0.395	*	0.043	0.315	0.474	0.185	0.176	-0.149	0.532	0.313	0.096	0.123	0.504		
21	0.97	0.716	*	0.028	0.663	0.774	0.718	*	0.028	0.662	0.774	0.937	0.050	0.839	1.000	0.925	0.053	0.825	1.000		
22	0.00	-0.128	*	0.047	-0.218	-0.035	-0.125	*	0.048	-0.222	-0.035	-0.118	0.169	-0.473	0.178	-0.191	0.149	-0.500	0.086		
23	-0.02	-0.029		0.045	-0.115	0.057	-0.034		0.051	-0.135	0.065	-0.033	0.175	-0.382	0.295	-0.072	0.146	-0.372	0.199		
24	1.07	0.778	*	0.027	0.724	0.831	0.793	*	0.027	0.738	0.843	0.933	*	0.052	0.833	1.000	0.939	*	0.047	0.847	1.000
25	-0.12	0.099	*	0.049	0.005	0.194	0.110	*	0.050	0.015	0.204	-0.002	0.179	-0.370	0.322	-0.011	0.159	-0.339	0.280		
26	-0.37	-0.001	*	0.045	-0.094	0.085	0.019	*	0.046	-0.072	0.110	-0.081	0.184	-0.447	0.250	-0.112	0.154	-0.431	0.177		
27	-0.31	-0.170	*	0.047	-0.266	-0.080	-0.160	*	0.047	-0.253	-0.070	-0.184	0.163	-0.527	0.108	-0.260	0.146	-0.557	0.020		
28	0.46	0.476		0.039	0.396	0.550	0.486		0.044	0.398	0.569	0.429	0.161	0.097	0.726	0.469	0.130	0.198	0.709		
29	-0.02	-0.128	*	0.047	-0.218	-0.035	-0.125	*	0.048	-0.222	-0.035	-0.195	0.192	-0.596	0.154	-0.234	0.157	-0.546	0.057		
30	0.01	-0.128	*	0.047	-0.218	-0.035	-0.125	*	0.048	-0.222	-0.035	-0.017	0.204	-0.439	0.347	-0.145	0.170	-0.477	0.178		
31	-0.20	-0.298	*	0.044	-0.385	-0.212	-0.304	*	0.047	-0.395	-0.210	0.150	*	0.147	-0.150	0.414	-0.168	0.174	-0.501	0.166	
32	0.79	0.509	*	0.029	0.455	0.565	0.522	*	0.030	0.464	0.580	0.539	*	0.118	0.313	0.777	0.537	*	0.105	0.328	0.743
33	0.48	0.348	*	0.047	0.256	0.437	0.342	*	0.050	0.246	0.443	0.389	0.163	0.060	0.695	0.393	0.130	0.132	0.642		
34	0.23	0.212		0.035	0.147	0.286	0.199		0.038	0.121	0.267	0.355	0.129	0.113	0.607	0.269	0.116	0.040	0.498		
35	0.55	0.650	*	0.032	0.587	0.714	0.649	*	0.035	0.581	0.718	0.440	0.102	0.236	0.634	0.495	0.088	0.323	0.671		
36	0.48	0.716	*	0.028	0.663	0.774	0.718	*	0.028	0.662	0.774	0.478	0.101	0.281	0.671	0.534	0.083	0.378	0.702		
37	0.20	0.306	*	0.045	0.216	0.388	0.306	*	0.050	0.211	0.405	-0.070	0.212	-0.532	0.301	0.071	0.160	-0.249	0.382		
38	0.61	0.688	*	0.036	0.616	0.756	0.665		0.039	0.585	0.737	0.577	0.102	0.389	0.788	0.625	0.083	0.457	0.780		
39	0.34	0.381		0.035	0.312	0.447	0.378		0.035	0.306	0.442	0.032	0.181	-0.311	0.382	0.179	0.123	-0.055	0.426		
40	-0.04	0.212	*	0.035	0.147	0.286	0.199	*	0.038	0.121	0.267	0.210	*	0.128	-0.037	0.461	0.184	*	0.107	-0.021	0.386
41	0.24	0.207		0.047	0.116	0.299	0.215		0.049	0.121	0.313	0.243	0.196	-0.164	0.603	0.204	0.160	-0.132	0.499		
42	0.05	-0.029		0.045	-0.115	0.057	-0.034		0.051	-0.135	0.065	0.276	0.157	-0.034	0.572	0.124	0.160	-0.210	0.417		
43	0.76	0.778		0.027	0.724	0.831	0.793		0.027	0.738	0.843	0.908	*	0.064	0.782	1.000	0.916	*	0.062	0.800	1.000
44	0.83	0.650	*	0.032	0.587	0.714	0.649	*	0.035	0.581	0.718	0.648	*	0.084	0.487	0.820	0.668	0.082	0.511	0.837	
45	0.55	0.509		0.029	0.455	0.565	0.522		0.030	0.464	0.580	0.562	0.079	0.411	0.721	0.565	0.075	0.407	0.708		
46	0.91	0.688	*	0.036	0.616	0.756	0.665	*	0.039	0.585	0.737	0.825	0.091	0.671	1.000	0.798	0.084	0.644	0.968		
47	0.62	0.608		0.039	0.531	0.681	0.614		0.036	0.541	0.683	0.541	0.098	0.360	0.735	0.565	0.087	0.396	0.739		
48	0.51	0.339	*	0.037	0.265	0.412	0.343	*	0.036	0.274	0.414	0.389	0.130	0.127	0.643	0.362	0.112	0.138	0.577		
49	1.09	0.815	*	0.031	0.757	0.875	0.809	*	0.031	0.749	0.868	0.933	*	0.052	0.832	1.000	0.935	*	0.049	0.842	1.000
50	0.27	0.178	*	0.047	0.088	0.268	0.162	*	0.053	0.055	0.268	0.126	0.200	-0.265	0.514	0.133	0.141	-0.154	0.401		
51	0.57	0.509	*	0.029	0.455	0.565	0.522		0.030	0.464	0.580	0.448	0.125	0.205	0.691	0.477	0.105	0.265	0.677		
52	0.29	0.381	*	0.035	0.312	0.447	0.378	*	0.035	0.306	0.442	0.224	0.168	-0.106	0.550	0.293	0.121	0.051	0.530		
53	0.64	0.716	*	0.028	0.663	0.774	0.718	*	0.028	0.662	0.774	0.481	0.110	0.262	0.696	0.541	0.088	0.369	0.716		
54	0.33	-0.001	*	0.045	-0.094	0.085	0.019	*	0.046	-0.072	0.110	0.407	0.118	0.185	0.643	0.280	0.149	-0.021	0.544		
55	0.67	0.509	*	0.029	0.455	0.565	0.522	*	0.030	0.464	0.580	0.413	0.145	0.125	0.683	0.452	*	0.115	0.207	0.657	
56	0.89	0.650	*	0.032	0.587	0.714	0.649	*	0.035	0.581	0.718	0									

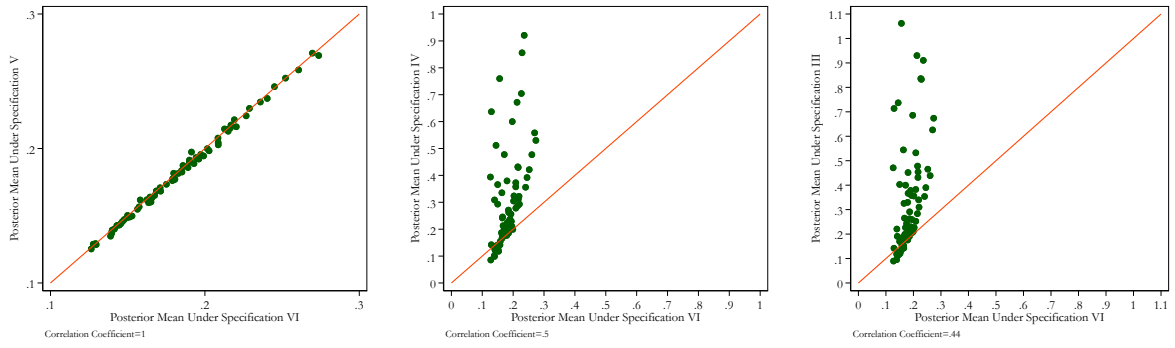
**Figure B.1:** Individual Fechner Noises Using Simulated Dataset VI under Misspecified specifications



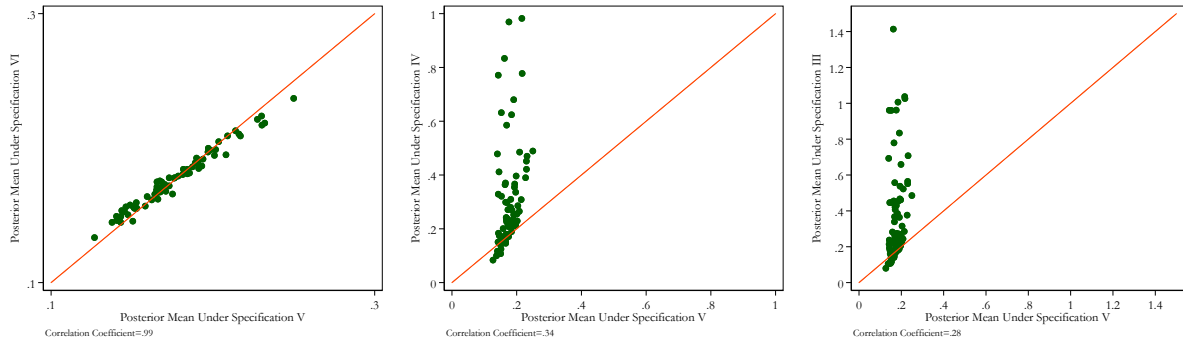
**Figure B.2:** Individual Fechner Noise  $\mu_i$  from Observed Choices under  
EUT specification VI



**Figure B.3:** Individual Fechner Noise  $\mu_i$  from Observed Choices under  
Different EUT Specifications



**Figure B.4:** Individual Fechner Noises from Observed Choices under  
Specifications III, IV, V and VI of RDU Prelec Model



## Appendix C Template Stata Codes (ONLINE)

In this section we provide the Stata codes for the estimation of specification VI.

### C.1 Data and Variables

The data are saved in a Stata dataset with each row of observation recording all the observables of subject  $i$  in lottery pair  $t$ , including a subject id, his or her demographic information, the prizes and probabilities of the lottery pair, and the subject's choice of left or right lottery from the pair. Table C.1 contains for a detailed explanation of each variable and its correspondence with the mathematical notations in the text.

**Table C.1:** Variables in Stata Dataset

Variable	Notes
female	Whether a subject is female
black	Whether a subject is African American
business	Whether a subject is business major
gpaHI	Whether a subject has a high GPA
insured	Whether a subject owns any insurance
probkL	The probabilities of the left lottery, $p_{ik}^L$ where $k = 1, \dots, 4$
prizekL	The prizes of the left lottery, $x_{ik}^L$ where $k = 1, \dots, 4$
probkR	The probabilities of the right lottery, $p_{ik}^R$ where $k = 1, \dots, 4$
prizekR	The prizes of the right lottery, $x_{ik}^R$ where $k = 1, \dots, 4$
endowment	Endowment subjects receives, $e$
sid	Continuously coded subject ID, $i = 1, 2, \dots, 73$
choiceL	Subjects' choices in lottery pairs, $y_{it}$ , equal to 1 if subject chooses the left lottery

We collect relevant variables in globals “demog\_r” and “Rdata” to allow for a more succinct presentation of the main syntax:

```
global demog_r "female black business gpaHI insured"
global Rdata "prob1L prob2L prob3L prob4L prob1R prob2R prob3R prob4R prize1L
  ↪ prize2L prize3L prize4L prize1R prize2R prize3R prize4R endowment"
```

The use of a small red arrow indicates a continuation of the previous line.

### C.2 Likelihood Function

The likelihood function in our model needs to be written in a user-defined function referred to as “user-defined likelihood evaluator” in Stata. To allow for flexibility in the specifications, we use

several globals: the “utype” global specifies the specific form of the CRRA utility function in (1), the “contextual” global specifies whether to use contextual utilities in (2), and the “cdf” global specifies whether to use a Probit or Logit link between the latent index and the observed choice in (3). To replicate specification VI in its exact form in the main text, we define these globals as follows:

```
global utype "1-r"
global contextual "y"
global cdf "normal"
```

The user-defined likelihood evaluator is then

```
program probiteUfLN

    args lnf r LNmu

    tokenize $MH_extravars

    local h = 0
    foreach par in prob prize {
        forvalues i=1/4 {
            local h = `h'+1
            local `par' `i' L `h''
        }
        forvalues i=1/4 {
            local h = `h'+1
            local `par' `i' R `h''
        }
    }
    local h = `h'+1
    local endowment ``h''
    tempvar lnfj
    tempvar euL euR eudiff m1L m2L m3L m4L m1R m2R m3R m4R u1L u2L u3L u4L u1R
    ↪ u2R u3R u4R mu
    tempvar low high

    quietly {
        generate double `mu' = exp(`LNmu')

        * add in endowments
        foreach x in L R {
            forvalues i=1/4 {
                generate double `m'i' `x'' = `endowment' + `prize'i' `x''
            }
        }
    }
```

```

* generate the utility function
foreach x in L R {
  forvalues i=1/4 {
    if "$utype" == "2" {
      generate double `u'i'`x'' = (`m'i'`x''^(1-`r'))/(1-`
        ↪ `r')
    }
    else {
      generate double `u'i'`x'' = `m'i'`x''^`r'
    }
  }
}

* evaluate the EU of each lottery
generate double `euL' = 0
generate double `euR' = 0
foreach x in L R {
  forvalues i=1/4 {
    replace `eu'x'' = `eu'x'' + `prob'i'`x''*`u'i'`x''
  }
}

* get the Fechner index
if "$contextual" == "y" {
  generate double `low' = `u1L'
  generate double `high' = `u1L'
  forvalues i=1/4 {
    foreach s in L R {
      replace `low' = `u'i'`s'' if `u'i'`s'' < `low' & `
        ↪ `prob'i'`s'' > 0
      replace `high' = `u'i'`s'' if `u'i'`s'' > `high' & `
        ↪ `prob'i'`s'' > 0
    }
  }
  generate double `eudiff' = ((`euL' - `euR')/(`high'-`low'))
    ↪ /`mu'
}
else {
  generate double `eudiff' = (`euL' - `euR')/`mu'
}

* construct the likelihood contribution
generate double `lnfj' = ln($cdf(`eudiff')) if $MH_y1 == 1 &
  ↪ $MH_touse
replace `lnfj' = ln($cdf(-`eudiff')) if $MH_y1 == 0 & $MH_touse
summarize `lnfj', meanonly

```

```

    }

    * check that the required evaluations are done
    if r(N) < $MH_n {
        scalar `lnf' = .
        exit
    }
    scalar `lnf' = r(sum)

end

```

### C.3 Main Syntax

The main Stata command for Metropolis-Hastings algorithm that allows for the use user-defined likelihood function is “bayesmh”. The template listing is provided below. In this command we first specify all the model parameters, with parameters related to CRRA coefficients in line 1 and those related to the Fechner noise parameters in line 2.

In line 3 we specify the user-defined likelihood evaluator in option “llevuator( )”, within which we can tell Stata to input the variables in “extravars( )”. The user-defined likelihood evaluator parses the variables into temporary variables when evaluating the likelihood.

For specification VI the Gibbs sampler can only be applied to the distribution of  $\mu_i$ , which is specified in lines 13-15. We use the Metropolis Hastings algorithm to sample parameters related to CRRA coefficients, specified in lines 4-12.

In lines 16-18 we specify options for the size of the MCMC sample, display of results, saving of the MCMC samples for later use, adaptation parameters for the adjustment of proposal steps in the MH algorithm, etc. For documentation one can reference the Stata manual for the “bayesmh” command.

```

1 bayesmh (r:choiceL i.sid i.sid#i.($demog_r)) ///
2   (mu: choiceL i.sid), ///
3   llevuator(probitEUfLN, extravars($Rdata)) ///
4   prior({r:i.sid }, normal({rMean:constant},{rVar})) ///
5   prior({r:i.sid#i.female}, normal({rMean:female},{rVar})) ///
6   prior({r:i.sid#i.black}, normal({rMean:black},{rVar})) ///
7   prior({r:i.sid#i.business}, normal({rMean:business},{rVar})) ///
8   prior({r:i.sid#i.gpaHI}, normal({rMean:gpaHI},{rVar})) ///
9   prior({r:i.sid#i.insured}, normal({rMean:insured},{rVar})) ///

```

```
10 block({r:},split) ///
11 prior({rMean:}, normal(0, 100)) block({rMean:}, split) ///
12 prior({rVar}, igamma(0.001, 0.001)) block({rVar}) ///
13 prior({mu:i.sid}, normal({muMean},{muVar})) block({mu:i.sid}, split) ///
14 prior({muMean}, normal(0, 100)) block({muMean},gibbs) ///
15 prior({muVar}, igamma(0.001, 0.001)) block({muVar},gibbs) ///
16 rseed(54321) mcmc(10000) burnin(2500) adapt( every(10) alpha(0.75) beta(0.8)
  ↪ gamma(0.0001)) ///
17 nomleinitial nocons initial({r:} {rMean:} 0 {mu:i.sid} 1 {rMean:} 0 {rVar} 1
  ↪ {muMean} 0 {muVar} 1) initsummary ///
18 saving(ChoiceEU6_SpecEU6, replace) dots(1,every(10))
```