

Reinsurance Versus Securitization of Catastrophe Risk *

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Abstract

We provide a novel explanation for the low volume of securitization in catastrophe risk transfer using a signaling model. Relative to securitization, reinsurance features lower adverse selection costs because reinsurers possess superior underwriting resources than ordinary capital market investors. Reinsurance premia, however, reflect markups over actuarially fair premia due to the additional costs of underwriting. Insurers' risk transfer choices trade off the costs and benefits of reinsurance relative to securitization. In equilibrium, low risks are transferred via reinsurance, while intermediate and high risks are transferred via partial and full securitization, respectively. An increase in the loss size increases the trigger risk level above which securitization is chosen. Hence, catastrophe exposures, which are characterized by lower probabilities and higher severities, are more likely to be retained or reinsured rather than securitized.

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1 Introduction

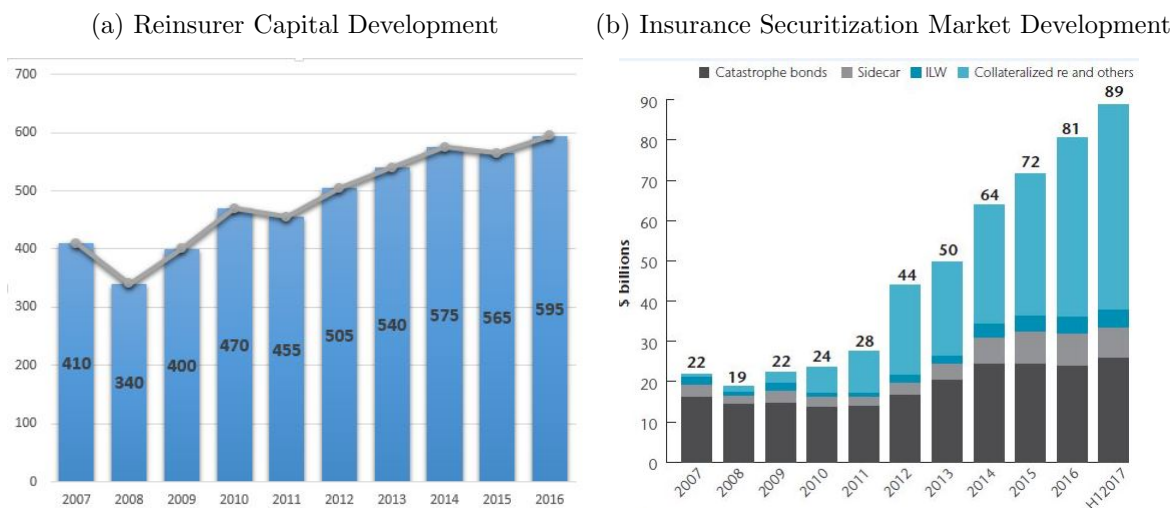
Insurers with limited capital to completely cover the risks in their portfolios often exploit external risk transfer mechanisms such as reinsurance and securitization. Although these risk-sharing mechanisms are used for all types of insurable risks, they are especially important in the case of catastrophe (CAT) risks because of the large potential losses involved. A strand of literature argues that securitization has a significant advantage over reinsurance and retention because of the substantially higher available capital and risk-bearing capacity of capital markets (Durbin (2001)). Nevertheless, an enduring puzzle is that reinsurance is still the dominant risk transfer mechanism for catastrophe risks (see http://www.naic.org/capital_markets_archive/140930.htm).

According to the Aon Benfield Research Report (2017), total reinsurance capital grew steadily to \$595 billion at the end of 2016 (Figure 1a). In contrast, the total outstanding capital of insurance-linked securities (ILS) increased slowly since their inception in 1992 to \$81 billion in 2016 (Figure 1b). Most of the increase in the volume of insurance-linked securities is, however, due to collateralized reinsurance that protects smaller losses. Hence, large losses are still primarily either retained or transferred through traditional reinsurance. To explain the low volume of securitization relative to reinsurance and retention, it is often argued that CAT bonds are too expensive, thereby suggesting that they are somehow “mispriced” relative to the risks they protect. Given that prices are endogenous equilibrium *outcomes*, however, it is problematic to argue that CAT bond prices are the *cause* of the low volume of securitization.

We show that the above puzzles can be reconciled using a signaling model of insurers’ risk transfer choices. When an insurer with private information about its portfolio faces a choice between reinsurance and securitization, its choice represents a signal of the nature of risks in its portfolio and, specifically, its actual exposure to catastrophe risk. Relative to securitization, reinsurance is associated with lower adverse selection costs because of the superior underwriting resources of reinsurers. On the flip side, however, reinsurance premia reflect markups over actuarially fair premia due to reinsurers’ underwriting costs (Froot (2001)). The insurer’s risk transfer choice reflects the tradeoff between the benefits and costs of reinsurance relative to securitization.

Perfect Bayesian Equilibria (PBE) of the signaling game have a partition form. An insurer chooses reinsurance if the “risk” of its portfolio—the probability of incurring a distress-triggering loss—is below a low threshold; partial securitization if the risk lies in an intermediate interval; and full securitization

Figure 1: Reinsurance v.s Securitization Capital Development



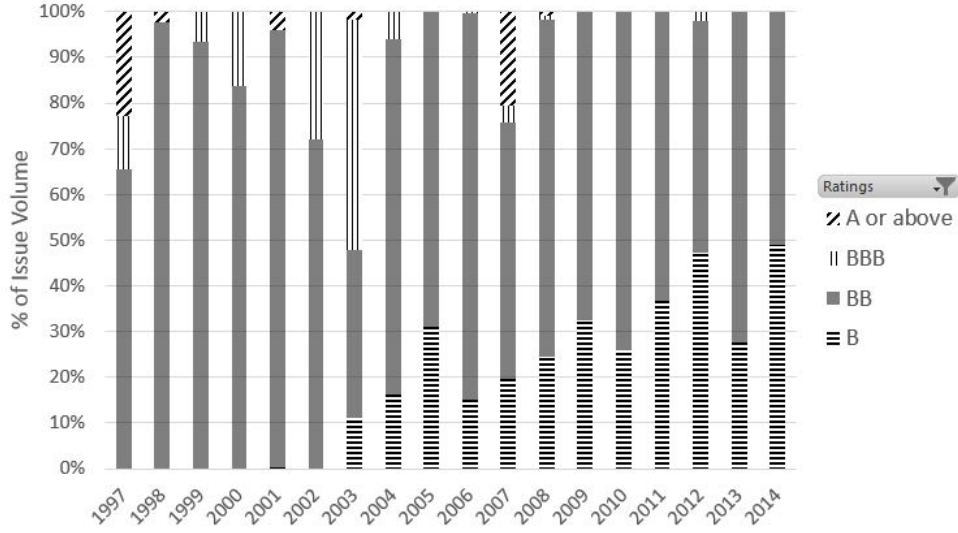
Source: Aon Securities Inc

if the risk is above a high threshold. The threshold risk level above which the insurer chooses securitization *increases* with the magnitude of potential losses in its portfolio. Given that catastrophe risk is typically associated with “low probability-high severity” losses, an insurer is more likely to choose retention or reinsurance to transfer catastrophe risk. Further, because an insurable risk is only transferred via securitization if the probability of potential distress-triggering losses is high, catastrophe bonds have high premia (relative to the ex ante expected losses) and a majority of them have ratings below investment grade (see Figure 2).¹ Our results suggest that the high costs of catastrophe securities reflect the rational incorporation of their risks by capital markets based on insurers’ observed risk transfer choices.

In our signaling model, a representative insurer with a limited amount of capital holds a portfolio of insurable risks. If the insurer incurs losses that exceed its available capital, it must raise additional capital to meet its liabilities due to which the insurer incurs external financing costs as in Froot et al. (1993). The presence of financial distress costs provides incentives for the insurer to transfer its risks. The insurer can choose to retain its risks or transfer them either partially or wholly through reinsurance or securitization. The insurer has private information about its portfolio so there is adverse selection regarding its “type.” Note that an insurer’s “type” is the ex ante probability that the insurable risks

¹Because CAT bonds are fully collateralized, the rating of a CAT bond is determined by the probability that the principal will be hit by a triggering event. Hence, the CAT bond rating captures the “risk” that is securitized.

Figure 2: CAT Bond Ratings



Source: Data are collected from Artemis Website: http://www.artemis.bm/deal_directory/ and Aon Benfield 2017 Report about Insurance-Linked Securitization.

in its portfolio incur a distress-triggering loss, and is *not* the probability of a *particular* loss event.²

Relative to capital market investors, reinsurers possess the resources to more effectively underwrite insurers' risk. For simplicity, we assume that reinsurers know an insurer's risk type and, therefore, do not face any adverse selection. (We can allow for adverse selection in reinsurance as long as its degree is less than that in securitization.) However, competitive reinsurers invest in costly underwriting technologies if and only if reinsurance premia incorporate a markup over actuarially fair premia that compensate reinsurers for their underwriting costs. The insurer's risk transfer choice reflects the tradeoff between the lower adverse selection costs associated with reinsurance and the costs stemming from the reinsurance markup.

We first analyze a framework where an insurer incurs *fixed* financial distress costs if it is unable to meet its liabilities. Perfect Bayesian Equilibria (PBE) of the signaling game (under stability restrictions on off-equilibrium beliefs along the lines of the D1 refinement (see Banks and Sobel (1987))) have a "partition form" that is characterized by two thresholds. The insurer chooses reinsurance if its risk—the probability of incurring a distress-triggering loss in its overall portfolio—is below the low threshold, retention if its risk lies between the thresholds, and securitization if its risk is above the high threshold.

²In general, insurers have differing liability portfolios that differ in the degrees of diversification. Hence, the probabilities of incurring distress-triggering losses vary across insurers. Even though there might be publicly available information on the probabilities of *particular* loss events (e.g., earthquakes in California or hurricanes in Florida), the probability that an insurer incurs a distress-triggering loss depends on its portfolio of exposures to different insurable risks.

With fixed financial distress costs, the costs the insurer incurs are independent of the magnitude of its shortfall in meeting its liabilities. Consequently, it is optimal for the insurer to either retain all its risks or completely transfer them. The presence of the reinsurance markup implies that the cost of reinsurance—the difference between the reinsurance premium and the actuarially fair premium—increases disproportionately with the insurer’s risk, while the cost of retention increases linearly. The costs of securitization, which stem from the cross-subsidization of higher risk types are, however, *decreasing* in the insurer’s risk (relative to the costs of retention). Consequently, if the insurer’s risk is below a low threshold, it prefers reinsurance. If the insurer’s risk lies in an intermediate interval, the increasing and convex cost of reinsurance dominates the cost of retention so that the insurer prefers retention. If the insurer’s risk is above a high threshold, securitization dominates because the cost is decreasing in the insurer’s risk type relative to the cost of retention.

An increase in the loss size increases the marginal cost of subsidizing higher risks as well as the marginal cost associated with the reinsurance markup. Consequently, the trigger risk level above which insurers choose securitization *increases*. In other words, the interval of risks that are securitized *shrinks* as the loss size increases. As catastrophe exposures are characterized by *low probabilities* and *large magnitudes* of potential losses, they are more likely to fall in the intervals of risks described above where retention or reinsurance rather than securitization are chosen. Hence, the volume of securitization is low relative to retention and reinsurance in catastrophe risk transfer.

To examine the robustness of our results, we next analyze a model with *variable* financial distress costs that are proportional to the magnitude of the insurer’s shortfall in meeting its liabilities. In this scenario, it may be optimal for an insurer to choose partial securitization and, thereby, signal its type. The possibility of partial securitization makes the analysis significantly more complicated, but we show the PBE of the risk transfer signaling game again have a partition structure characterized by three intervals of risk types. The lowest risks are fully reinsured. To avoid the costs associated with reinsurance, and the costs of pooling with higher risks, intermediate risks are transferred via separating securitization contracts that fully reveal the risks, and are characterized by retention levels that decline with the risk. For high risks, the signaling costs are too high so that they are transferred via full pooling securitization contracts. As catastrophic losses have lower probabilities, our results suggest that CAT risks are more likely to be reinsured or *partially* securitized. In other words, only losses above a threshold are securitized (if at all), which is consistent with evidence that CAT bonds

typically protect top layers of the loss distribution (e.g., Cummins and Trainar (2009)).

The implication of our theory that only high risks are securitized is consistent with a noticeable increase in catastrophe securitization after Hurricane Katrina. Anecdotal evidence suggests that actuaries significantly increased their estimates of catastrophe loss probabilities following Katrina (see Ahrens et al. (2009)). The spike in securitization transactions is, therefore, consistent with the higher perceived levels of risk. Our basic story is also consistent with the observation that more sophisticated investors such as dedicated hedge funds have entered the catastrophe securitization market in recent years, and this has been followed by an increase in the volume of securitization. The entry of more sophisticated and informed investors has likely reduced the level of adverse selection, thereby lowering securitization costs relative to reinsurance. Finally, even more recently, there has been a spike in the volume of securitization because of the advent of blockchain technology.³ This phenomenon is also consistent with our basic argument as blockchain technology has greatly reduced the administrative costs of securitization relative to reinsurance by lowering the costs associated with claim settlement.

A significant proportion of catastrophe securities have payoffs that are tied to an index rather than a particular insurer’s losses, that is, they employ *index-based* rather than *indemnity-based* triggers. The use of index-linked securities introduces *basis risk* because the distribution of the index is not, in general, perfectly correlated with that of an insurer’s losses. In online Appendix B, we extend our basic model to allow insurers to have access to index-linked securities. Our main implication that securitization—index-based or indemnity-based—only protects risks above a threshold is unaffected.

Our analysis and results can be directly extended to the scenario in which an insurer is exposed to multiple “classes” or “tranches” of insurable risks with differing distress-triggering probabilities. For example, this would be the case if the insurer sells multiple lines of insurance. In such a scenario, the probabilities of incurring distress-triggering losses differ across insurance lines. By risk-neutrality, an insurer chooses the optimal form of risk transfer for each class of risk in its portfolio. In this setting, our results suggest that an insurer choose reinsurance for the lowest risk exposures in its portfolio, partial securitization for the intermediate risks, and full securitization for the highest risks. Consequently, our results are also consistent with the observation that insurers often choose both reinsurance and securitization to transfer their *portfolios* of different types of risks with varying distress-triggering probabilities.

³see <https://www.bloomberg.com/view/articles/2017-08-11/blockchains-get-into-the-catastrophe-business>

2 Related Literature

Our study relates to two branches of the literature that investigate insurers' choice between reinsurance and securitization. The first branch examines the factors that affect the *demand* for insurance-linked securities (e.g., Bantwal and Kunreuther (2000), Barrieu and Louberge (2009)). The second branch examines the factors that affect the *supply* of insurance-linked securities that is closer to our perspective. Cummins and Trainar (2009) argue that the benefits of securitization relative to reinsurance increase when the magnitude of potential losses and the correlation of risks increase. Finken and Laux (2009) argue that, given low basis risk, catastrophe bonds with parametric triggers are insensitive to adverse selection, and can be attractive to low risk insurers who suffer from adverse selection with reinsurance.⁴ Lakdawalla and Zanjani (2012) argue that catastrophe bonds can improve the welfare of insureds when reinsurers face contracting constraints on the distribution of assets in bankruptcy. Gibson *et al.* (2014) analyze the tradeoff between the costs and benefits of loss information aggregation procedures to determine the prevalent risk transfer form. We complement the above literature by providing an explanation based on signaling considerations for the dominance of retention and reinsurance in the market for catastrophe risk transfer.

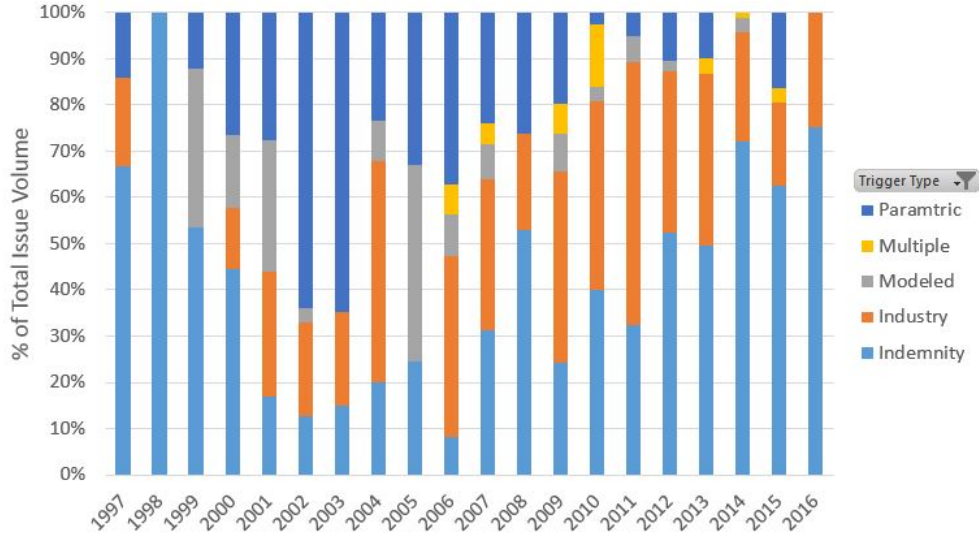
It is often argued that a significant deterrent to the growth in the market for insurance-linked securities is the presence of basis risk, which is present when security payouts are based on an index not directly tied to the sponsoring insurer's losses. It is, however, unclear what the quantitative impact of basis risk is on the securitization decision given that insurers can choose the volume of securities to hedge their exposure to the catastrophe underlying the index (Cummins, Lalonde and Phillips (2004)). Moreover, a substantial percentage of CAT bonds also have indemnity-based triggers that are tied to the insurer's losses (Braun (2016)). In fact, as shown in Figure 3, the volume of CAT bonds with indemnity triggers is higher than those with index triggers in recent years.⁵ More importantly, because the choice of index-linked securitization is *endogenous*, it is problematic to argue that index-linked securities are the cause of the low volume of securitization. Our extended model in online Appendix B endogenizes the choice of the type of securitization by incorporating both index-based and indemnity-based securitization. Our main implication that only risks above a threshold are

⁴CAT bonds are *not issued in equilibrium* in Finken and Laux's (2009) model; they play a role as an *off-equilibrium* alternative to reinsurance for low risk insurers. The purpose of parametric CAT bonds is to constrain inside reinsurers' power (arising from superior information) relative to outside reinsurers.

⁵According to Artemis website http://www.artemis.bm/deal_directory/cat_bonds_ils_by_trigger.html, the proportion of outstanding CAT bonds with indemnity triggers is 57.6%.

securitized is unaffected.

Figure 3: Trigger Types of CAT Bonds



Source: Data are collected from Artemis website: http://www.artemis.bm/deal_directory/ and Aon Benfield Research Report (2017) about Insurance-Linked Securitization.

Another related argument that is proffered for the low volume of securitization is the presence of transaction costs. A major component of these costs are *endogenous* costs due to adverse selection that play a central role in our analysis. Further, CAT bond issuers annualize the fixed costs over multiple periods, thereby reducing annual transaction costs. In addition, the favorable tax treatment of CAT bonds allow insurers to reduce tax costs associated with equity financing (Niehaus (2002), Harrington and Niehaus (2003)). Moreover, CAT bond interest paid offshore is also deducted for tax purposes in the same way as reinsurance premia. Consequently, it is not clear that transaction costs associated with securitization, *apart from adverse selection costs* that we already incorporate, are high enough to significantly deter securitization. Further, even if transaction costs were significant, it is not clear whether they explain why securitization is typically used to provide high layers of protection.

3 The Model

The economy consists of a continuum of insurers. The representative insurer has a limited amount of capital W and a risky portfolio of insurable risks. The insurer is faced with the choice between retaining the risk or transferring the risk through reinsurance or securitization. The insurer's portfolio

has two possible realizations. In the “good” state, which occurs with probability $1 - p$, the portfolio suffers no loss and the insurer obtains the premium A . However in the “bad” state, which occurs with probability p , the portfolio suffers a loss and the insurer has to make the net payment B (total indemnity net of the premium). We assume that $W - B < 0$ so that the insurer’s existing capital is not enough to cover the net loss payment in the bad state. In this scenario, the insurer raises additional external capital to meet its liability, but incurs financial distress costs.

Consistent with standard terminology in insurance markets, an insurer’s *risk* is the *loss probability* p , which is the probability that the insurer’s overall portfolio incurs a distress-triggering loss. Importantly, it is *not* the probability of a particular loss event, and is also not the severity or size of the loss. A typical insurer has a portfolio of insurance liabilities and is, therefore, exposed to different insurable risks. Further, different insurers may have distinct portfolios of liabilities and may, therefore, have contrasting exposures to different insurable risks. Moreover, insurers could differ in the degrees of diversification of their portfolios so that the loss probability p differs, in general, across insurers. In particular, even though there might be publicly available information about the probabilities of particular loss events, the probability that an insurer incurs a distress-triggering loss depends on its specific *portfolio* of exposures to different loss events.

It is also worth emphasizing here that the loss probability p is the *ex ante* probability of incurring a distress-triggering loss, that is, the probability *before* it makes its risk transfer decision. *After* it makes its risk transfer decision—reinsurance or securitization—its probability of distress is, of course, altered. Our interest, however, is on how the ex ante loss probability affects the insurer’s risk transfer decision.

The insurer has superior information about its portfolio of exposures to different insurable risks, which manifests in the insurer possessing *private information* about the probability p . Hence, there is adverse selection regarding the type p of the insurer. Note that the terminology, “insurer type” really refers to the characteristics of the insurer’s risk exposure as represented by the loss probability p , which is the same regardless of whether it is retained by the insurer or transferred via reinsurance or securitization. The loss probability p is drawn from the cumulative distribution F with support in $[0, 1]$. The insurer incurs an *additional* deadweight financial distress cost C in the bad state if its available capital is insufficient to fully cover the loss so it must raise additional external capital $B - W$ to cover it. Note that the financial distress cost C is in addition to the net external capital $B - W$ that

the insurer must raise. It is worth emphasizing here that, because p is the probability of incurring a distress-triggering loss for the insurer’s overall portfolio, the probabilities of individual loss events or subsets of loss events say little about the overall distress-triggering probability p .

We assume a fixed financial distress cost C in this section. In Section 4, we alter the model to consider variable financial distress costs that increase with the magnitude of the shortfall in the insurer’s existing capital in meeting its liabilities. Consistent with the evidence in Hoerger *et al.*(1990), the presence of financial distress costs creates an incentive for the insurer to hedge its underwriting losses through reinsurance or securitization. As mentioned earlier, the probabilities of individual loss events or subsets of events in the insurer’s portfolio are distinct from the overall distress-triggering probability p of the insurer’s entire portfolio. Hence, the insurer cannot reveal its type by choosing reinsurance for a portion of its risks.

Given its linear objective function, it is optimal for the insurer to choose either reinsurance or securitization for its entire portfolio provided it chooses to transfer its risk. As we discuss in Section 3.4, however, our results extend naturally to the scenario in which an insurer is exposed to differing risks and chooses distinct risk transfer mechanisms for different types of risks. We next analyze the insurer’s choice among retention, reinsurance and securitization.

3.1 Reinsurance

Insurers have access to a competitive reinsurance market. Reinsurers have an information advantage over ordinary investors in capital markets due to their specialized expertise in the insurance industry (e.g. Jean-Baptiste *et al.*(2000), Plantin (2006), Boyer and Dupont-Courtade (2015)). To simplify matters, and to focus attention on the information advantage of reinsurers relative to capital markets, we assume that reinsurers have the underwriting technology to know the risk type of the insurer perfectly so that they do not face any adverse selection. (Our results are robust to allowing for adverse selection in reinsurance as long as its degree is less than that in securitization.)

On the flip side, reinsurers’ underwriting technology is costly, and they charge a proportional markup $\delta > 0$ over the actuarially fair insurance premium that compensates them for their underwriting costs. For simplicity, we assume that reinsurers have sufficient capital to fully insure the insurance company so that they do not face default risk.⁶ Reinsurers usually have better diversification oppor-

⁶According to the Guy Carpenter report (2012), the total losses of the global property/casualty sector in 2011 exceeded \$100 billion, but shareholder funds exceeded \$160 billion. Consequently, the reinsurance sector continued to function

tunities that may lower their default risks (*e.g.* Jean-Baptiste *et al.*(2000)). The main objective of our study is to compare the trade-off between the information advantage of reinsurers against the lower costs of risk-sharing with capital markets. Consequently, we avoid further complicating the analysis and the intuition for our results by also introducing default risk for reinsurers.

Because reinsurance companies know the insurer's type, they offer distinguishing contracts $(A_r(p), B_r(p))$ that are contingent on the insurer's type, where $A_r(p)$ is the reinsurance premium and $B_r(p)$ is the total payment made by the reinsurer to the insurer in the bad state. The optimal contract for each insurer type, p , maximizes its expected utility subject to the reinsurance premium being at least a proportion δ above the actuarially fair premium.

Define

$$\tilde{B} = B - W, \quad (1)$$

which is the shortfall in the insurer's existing capital in meeting its liabilities in the bad state in the scenario where it retains all its risk, that is, the insurer must raise additional capital, \tilde{B} , to meet its liabilities. Given the fixed financial distress cost C , we can show that no insurer type chooses reinsurance if $\delta \geq \frac{C}{\tilde{B}}$, because it is too expensive. Consequently, we consider the case where $\delta < \frac{C}{\tilde{B}}$. If an insurer chooses reinsurance, the optimal reinsurance contract solves

$$\max_{(A_r(p), B_r(p))} (W + A - A_r(p))(1 - p) + (W - B - A_r(p) + B_r(p))p - Cp \cdot \mathbf{1}_{\{B_r(p) < B + A_r(p) - W\}} \quad (2)$$

such that

$$A_r(p) \geq (1 + \delta) \overbrace{pB_r(p)}^{\text{actuarially fair premium}} \quad (3)$$

In the objective function (2), note that the total capital of the insurer in the bad state after receiving the payment, $B_r(p)$, from the reinsurer is $W - B - A_r(p) + B_r(p)$, that is, $W - B$ less the reinsurance premium, $A_r(p)$ plus the payment, $B_r(p)$. If $W - B < 0$, the insurer must raise additional capital, $B - W$ to meet its liabilities if it does not transfer its risks due to which it incurs an additional financial distress cost, C . The constraint (3) expresses the fact that the reinsurance premium must be at least, $(1 + \delta)$ times the actuarially fair premium, $pB_r(p)$. As the insurer is risk-neutral, it is suboptimal for the insurer to overinsure as it pays a markup for this overinsurance.

normally despite the heavy losses in 2011.

Proposition 1 (Reinsurance Contract). *Define*

$$p_1^F = \frac{C - \tilde{B}\delta}{C(1 + \delta)} < 1. \quad (4)$$

If $p > p_1^F$, the insurer chooses retention. If $p < p_1^F$, the insurer chooses reinsurance. The optimal reinsurance contract, $(A_r^(p), B_r^*(p))$,⁷ is*

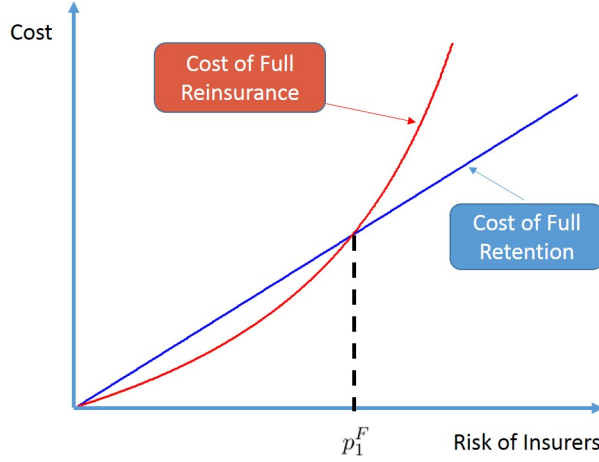
$$A_r^*(p) = \frac{\tilde{B}p(1 + \delta)}{1 - p(1 + \delta)}, \quad B_r^*(p) = A_r^*(p) + \tilde{B} = \frac{\tilde{B}}{1 - p(1 + \delta)}$$

Because the financial distress cost is fixed, the insurer chooses full reinsurance if it opts to transfer its risks. The total insurance payment, $B_r^*(p)$ equals the loss, \tilde{B} , the insurer incurs if it were to purchase no reinsurance *plus* the reinsurance premium, $A_r^*(p)$. Because the insurer is risk-neutral, it is optimal for the insurer to buy just enough reinsurance to cover its shortfall, $A_r^*(p) + \tilde{B}$, in the bad state and, thereby, avoid financial distress. In other words, it is suboptimal for the insurer to *overinsure* because it pays a markup over the actuarially fair premium for reinsurance. Hence, reinsurance takes the form of “excess of loss” that covers losses exceeding those that are not covered by the insurer’s existing capital. Consequently, the actuarially fair premium is $pB_r^*(p)$ so that the reinsurance premium, $A_r^*(p) = (1 + \delta)pB_r^*(p)$. As one would expect, a higher loss probability, p , raises the reinsurance premium, $A_r^*(p)$.

Interestingly, Proposition 1 shows that the insurer chooses retention if its risk type is above a threshold p_1^F , and reinsurance below the threshold. In other words, lower risk insurers choose reinsurance, while higher risk insurers choose retention. The intuition for this result is that the cost of reinsurance—the difference between the reinsurance premium and the actuarially fair premium—is $\frac{\tilde{B}p\delta}{1 - p(1 + \delta)}$, which is *increasing and convex* in the insurer’s risk type, p . The cost of retention, which is the expected financial distress cost pC , is *increasing and linear* in the insurer’s risk type. Consequently, in general, the cost of retention is lower (higher) than the cost of reinsurance if the insurer’s risk type is above (below) a threshold.

⁷Profit-maximizing reinsurers will not offer contracts for insurers with risk type above $\frac{1}{1 + \delta}$ because the expected payoff is negative so that insurers must choose retention.

Figure 4: Cost of Full Retention and Full Reinsurance



3.2 Securitization

We now examine the case where insurers only have access to capital markets. An insurer's cost of transferring its risks is potentially reduced by the fact that capital market investors do not charge a markup, that is, the premium equals the expected indemnity *conditional on the information possessed by capital market investors*. On the flip side, however, capital markets are marred by adverse selection since they cannot obtain the information about an insurer's risk type *ex ante*, that is, before it issues securities.

We model the securitization game as a signaling game whose timing is as follows. An insurer offers a contract, (A_s, B_s) , where A_s is the premium received by the investors, and B_s is the payment made by investors if a loss occurs. To fix ideas, we consider indemnity-based securitization in the baseline model, that is, the loss payment, B_s is contingent on the insurer's loss. In online Appendix B, we extend our baseline model to incorporate index-linked securitization where insurers can issue securities whose payouts are tied to an index that is not directly contingent on the issuing insurers' losses.

We restrict consideration to equilibria in pure strategies for the insurer. Investors update their prior beliefs based on the offered contract and then either accept or reject it. In all our subsequent results, we employ reasonable stability restrictions on off-equilibrium beliefs along the lines of Banks and Sobel's (1987) D1 refinement for signaling games to address the potential multiplicity of Perfect Bayesian Equilibria (PBE). The relevant statement of the D1 refinement in our context is as follows (see also Page 452 of Fudenberg and Tirole (1991)).

Remark (D1 Refinement) *Consider an equilibrium of the signaling game. Suppose that the set of in-*

investor beliefs under which an off-equilibrium deviation to a particular contract, $(A_s^{off-equil}, B_s^{off-equil})$, is weakly preferable to the equilibrium payoff for an insurer risk type, p_1 , is a proper subset of the set of investor beliefs under which the off-equilibrium deviation is strongly preferable for a higher risk type $p_2 > p_1$. Off-equilibrium beliefs on observing the contract offer, $(A_s^{off-equil}, B_s^{off-equil})$, should then assign probability zero that the insurer's risk type is p_1 .

Because the financial distress cost is fixed and does not depend on the magnitude of the insurer's shortfall in the bad state, separating securitization contracts are not incentive compatible. In other words, it is better for an insurer to self-insure rather than choose a securitization contract with a nonzero retention level that reveals its type because it incurs the same financial distress cost in either case so that its expected payoff is the same. (Recall that the financial distress cost is in addition to the loss payment.)

We conjecture that there exists a trigger level such that insurers with types above the trigger choose full securitization, while those with types below the trigger choose full retention. Consider a candidate equilibrium defined by a trigger level, p . Let $\mu_p(\cdot)$ denote the posterior beliefs of capital markets regarding an insurer's type given that it has chosen securitization, where the subscript indicates the dependence of the posterior beliefs on the trigger p . Given that insurers with types above p choose full securitization in the conjectured equilibrium, investors' posterior beliefs about the insurer's type are given by

$$d\mu_p(p') = \frac{dF(p')}{1 - F(p)} \quad (5)$$

The equilibrium is determined by a function, $R(\cdot)$ —the *subsidization ratio function*—that is defined as follows:

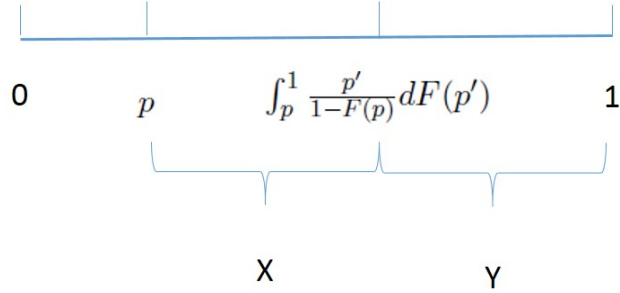
$$R(p) = \frac{\int_p^1 p' d\mu_p(p') - p}{1 - \int_p^1 p' d\mu_p(p')} = \frac{\int_p^1 \frac{p'}{1-F(p)} dF(p') - p}{1 - \int_p^1 \frac{p'}{1-F(p)} dF(p')} \quad (6)$$

The subsidization ratio function depends on the distribution of insurers' risk types and the threshold level p that defines the conjectured equilibrium. It determines the costs incurred by an insurer with risk type p if it were to pool with higher risk insurers and, thereby, subsidize them. More precisely, if insurers with types greater than p pool together by offering a single contract, then the premium must reflect the average risk of the pool, $\int_p^1 p' d\mu_p(p')$. The cost of pooling securitization for the insurer of type p is, therefore, the difference between the premium of the pooling contract and the actuarially

fair premium, that is, the premium it would pay if its type were fully observable by capital markets. This cost is given by $\tilde{B}R(p)$.

We see that $R(p)$ is the ratio of the distance X to the distance Y in Figure 5. $R(p)$ measures the degree of subsidization, and depends on the shape of the truncated distribution of insurer's risk types to the right of the threshold risk type, p . $R(p)$ is greater than 1 if the truncated distribution of insurer types is right skewed, less than 1 if the truncated distribution of insurer's type is left skewed, and equal to 1 if the truncated distribution of insurer's type is symmetrically distributed.

Figure 5: Illustration of *Subsidization Ratio Function*



If p is the *equilibrium* threshold, then the insurer with risk type p should be indifferent between full retention and full securitization. In other words, the expected cost associated with full retention should be the same as the cross-subsidization cost associated with full pooling securitization for an insurer of type p . We now characterize the equilibrium choice between retention and securitization and the optimal securitization contracts.

Proposition 2 (Securitization Contract). *Suppose there is a unique p_2^F satisfying the following equation:*

$$Cp_2^F = \tilde{B}R(p_2^F). \quad (7)$$

In the unique PBE of the securitization game (under the D1 refinement), insurers with types p in the interval $[p_2^F, 1]$ fully transfer their risks and offer the same contract (A_s^, B_s^*) , where*

$$A_s^* = \frac{\tilde{B} \int_{p_2^F}^1 \frac{p'}{1-F(p_2^F)} dF(p')}{1 - \int_{p_2^F}^1 \frac{p'}{1-F(p_2^F)} dF(p')}, \quad B_s^* = \tilde{B} + A_s^*$$

Insurers with types p below p_2^F choose full self-insurance.

The threshold, p_2^F , is the point of indifference between the cross-subsidization costs from pooling

with higher types, $\tilde{B}R(p)$, and the expected costs from retaining risk, Cp . In general, (7) could have multiple solutions so that there could be multiple PBEs each determined by the threshold risk type that is indifferent between retention and pooling securitization. As is common in the signaling literature, we add a “single crossing” assumption, which ensures that the above equation has a unique solution, that is, the curves Cp and $\tilde{B}R(p)$ intersect at exactly one point p_2^F . A sufficient condition that ensures this is

$$R'(p) < \frac{C}{\tilde{B}}^8 \quad (8)$$

Condition (8) expresses that a marginal increase in the degree of subsidization due to an increase in the lowest risk type in the pooling contract is less than the marginal increase in the cost of risk retention of the lowest risk type. Because the subsidization costs incurred by insurer types greater than p_2^F decline with the type, it is optimal for all such insurers to pool by offering full securitization contracts. Given that p_2^F satisfies (7), the expected retention cost incurred by an insurer with type less than p_2^F is less than the subsidization costs incurred by choosing securitization so that p_2^F determines the unique equilibrium.

The D1 refinement ensures that the threshold p_2^F , indeed, characterizes the equilibrium by ensuring that it is unprofitable for an insurer with type $p \geq p_2^F$ to deviate to any other contract. Intuitively, if such a deviation is profitable for the insurer of type p , then it is also profitable for higher risk types. The D1 criterion then ensures that, if such a deviation is observed, investor beliefs assign probability zero that the insurer is of type p , thereby making the deviation unprofitable for the type p insurer.

3.3 Risk Transfer Equilibria

We now show that the PBE of the risk transfer game have the conjectured “partition” form as shown in Figure 6.

Proposition 3 (Partition Equilibrium). *There exist two thresholds, p_1^F and p_2^F that determine the unique PBE (under the D1 refinement) as follows. Insurers with types in the interval $[0, p_1^F]$ choose full reinsurance, insurers with types in the interval $[p_1^F, p_2^F]$ choose full self-insurance, and insurers with types $[p_2^F, 1]$ choose full pooling securitization.*

⁸Let the function $g(p) = Cp - \tilde{B}R(p)$. Since $g(0) = -\tilde{B}R(0) < 0$, and $g(1) = C > 0$, we can show that $g(p) = 0$ has a unique solution p_2^F as long as $g(p)$ is increasing over the interval $[0, 1]$; that is, $R'(p) < \frac{C}{\tilde{B}}$.

Figure 6: Partition Equilibrium

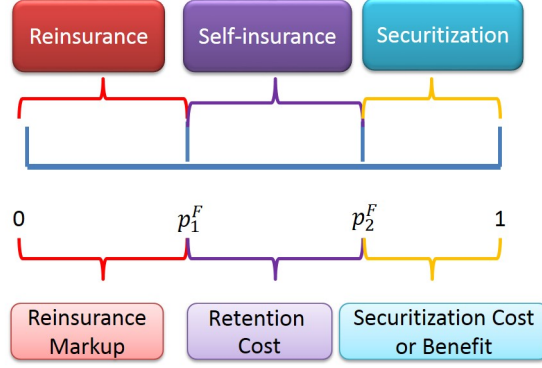


Figure 7: The Cost of Different Risk Transfer Mechanisms

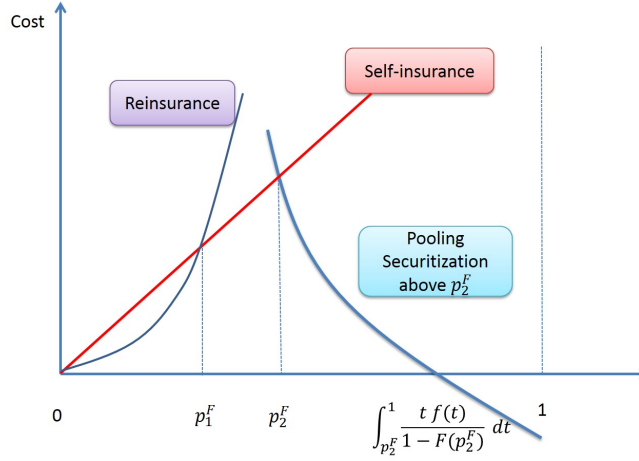


Figure 7 shows the cost function for each risk transfer choice faced by insurers. For all types, the chosen form of risk transfer is the one that has the lowest expected cost. As illustrated in the figure, the expected cost of retention, pC , is increasing and linear in an insurer's type, the expected cost of reinsurance, $\frac{\tilde{B}p\delta}{1-p(1+\delta)}$, is increasing and convex in an insurer's type, and the expected cost of securitization based on the belief that the insurers with types above p_2^F choose pooling securitization, $\frac{\tilde{B} \int_{p_2^F}^1 \frac{p'}{1-F(p_2^F)} dF(p') - p}{1 - \int_{p_2^F}^1 \frac{p'}{1-F(p_2^F)} dF(p')}$, decreases with an insurer's type. Consequently, in general, the equilibrium takes a partition form with three subintervals of insurer types. Insurers with sufficiently low risk in the interval $[0, p_1^F]$ choose full reinsurance, intermediate-risk insurers with types in interval $[p_1^F, p_2^F]$ choose full retention, and high-risk insurers with types in the interval $[p_2^F, 1]$ choose full securitization. The thresholds that determine the various subintervals are the “indifference” points. Depending on the relative magnitudes of the financial distress cost C , the reinsurance markup δ and the loss payment B , we could have $p_1^F = 0$ or $p_1^F = p_2^F$ so that either the interval of risk types that choose reinsurance or

the interval of types that choose retention is empty. (We provide the precise conditions in the proof of Proposition 3.) In these scenarios, the equilibrium is characterized by a single trigger where insurers with risks below the trigger choose either reinsurance or retention, whereas insurers with risks above the trigger choose securitization.

From (7) and the implicit function theorem, we get

$$\frac{dp_2^F}{dB} = \frac{R(p_2^F)}{C - \tilde{B}R'(p_2^F)} \quad (9)$$

The numerator of (9) is positive. Because p_2^F is the unique solution of (7), we can show that the denominator of the R.H.S. of (9) is positive. Thus $dp_2^F/dB > 0$. In other words, p_2^F is an increasing function of B . When the financial distress cost $C \leq \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$, it follows from the above proposition that p_2^F is the threshold risk level above which insurers choose securitization. If $C > \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$, it follows from condition, $\frac{p_3^F\delta}{1-p_3^F(1+\delta)} = R(p_3^F)$, that the trigger level above which insurers choose securitization does not depend on the loss payment B . Taken together, the above discussion shows that an increase in the magnitude of the insurer's aggregate losses weakly *increases* the threshold risk level above which insurers choose securitization so that the securitization subinterval *shrinks*.

Corollary 1 (Effects of Loss Size). *An increase in the size of the net loss payments B reduces the sizes of the subintervals of insurer risk types that choose securitization and reinsurance, respectively.*

An increase in the loss size increases the marginal cost borne by an insurer of subsidizing higher risk types through securitization. An increase in the loss size also increases the marginal cost of the reinsurance markup. Consequently, as the loss size increases, the marginal insurer who is indifferent between retention and securitization has *higher* risk, while the marginal insurer who is indifferent between reinsurance and retention has *lower* risk. Figure 8 illustrates the effects of an increase in the amount of net loss payment.

Catastrophe risks are characterized by low probabilities and large magnitudes of potential losses. The result that an increase in the magnitude of potential losses increases the trigger risk (or loss probability) level above which securitization is chosen suggests that catastrophe risks are less likely to be securitized. Figure 9 supports the prediction that an increase in the loss size increases the trigger level above which risks are securitized. The figure shows that the average issue volume of CAT bond tranches (which directly captures the sizes of losses that are protected by the bonds) with ratings of A

Figure 8: Effect of An Increase in Net Loss Payment

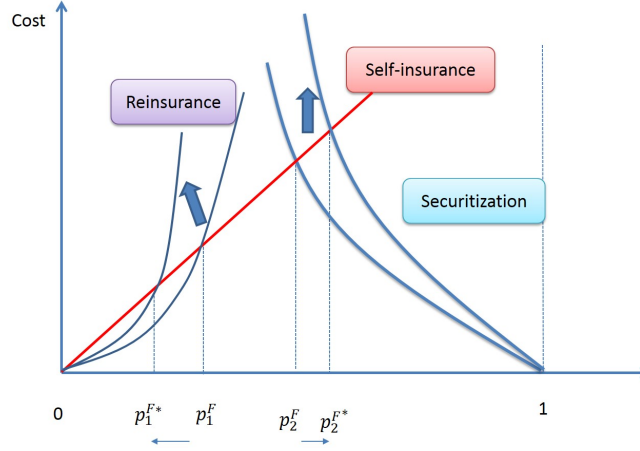
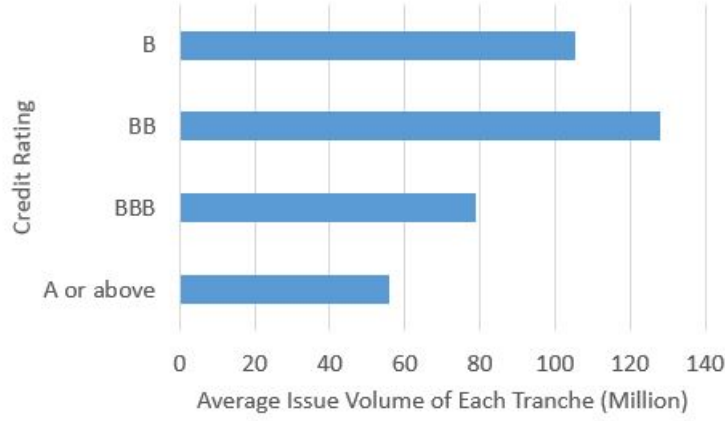


Figure 9: Average Volume of CAT Bond Tranches with Different Ratings



Source: The graph is generated from the data that are collected from Artemis website: http://www.artemis.bm/deal_directory/ and Aon Benfield Research Report (2017) about Insurance-Linked Securitization.

or above is less than the issue volume of tranches with lower ratings. Further, because only high risks are securitized, the corresponding premia are high relative to the ex ante expected loss determined by the average probability $\int_0^1 p dF(p)$. This could also explain why catastrophe-linked securities are usually expensive. Further, credit ratings of many catastrophe bonds, which reflect the loss probabilities, are below investment grade (see Figure 2).

The prediction that an increase in the size of potential losses lowers the likelihood of securitization is consistent with the evidence in Hagendorff et al. (2014) of a negative relation between CAT bond issuance and the size of underwriting losses. They also show a negative relation between CAT bond issuance and the volatility of underwriting losses. As emphasized earlier, the “risk” of an insurer in our

model is its probability of incurring a loss so an increase in the loss size, *ceteris paribus*, also increases the loss volatility. More generally, the volatility of losses, especially in the context of catastrophe risk, is likely to be driven by large losses. The finding of a negative relation between CAT bond issuance and loss volatility, therefore, also supports our theory.

The prediction that only risks above a threshold are securitized also comports with the observed spike in securitization transactions following major catastrophes such as Hurricane Katrina following which actuaries' assessments of future catastrophic events were revised upward (Ahrens et al. (2009)). In support of this implication, Hagendorff et al. (2014) also find that CAT bond issuance is positively related to the likelihood of potential losses.

3.4 Extensions

In online Appendix B, we show that our main implications are robust to an extended model that incorporates index-linked securities whose payouts are tied to the loss of an index, rather than the loss of a particular insurer.

Our model and results can also be directly extended to the scenario in which an insurer is exposed to multiple types of risk with differing distress-triggering probabilities as is the case when the insurer sells multiple lines of insurance. Specifically, in the preceding analysis, we interpret p as the probability that an insurer incurs a distress-triggering loss with different insurers having different distress-triggering probabilities. We can, however, reinterpret the model to focus on a representative insurer that is exposed to multiple classes of risk or insurance lines (e.g. life insurance, property and casualty insurance, etc.) each associated with a probability of inducing a distress-triggering loss for the insurer. In other words, after taking into account potential correlations among different risks in the insurer's overall portfolio, its portfolio can be viewed as divided into different classes or "tranches" of risk with each tranche associated with a different distress-triggering probability. The distress-triggering probabilities of the different tranches take values in the interval $[0, 1]$ and there is asymmetric information about the probabilities. Our analysis and results directly extend to this setting, and imply that it is optimal for the insurer to reinsure the lowest risks, retain the intermediate risks, and securitize the highest risks. These results comport with the observation that insurers often choose both reinsurance and securitization to transfer their *portfolios* of risks of differing distress-triggering probabilities.

4 Variable Financial Distress Costs

In the model that we have analyzed thus far, an insurer incurs a fixed financial distress cost regardless of the extent of the shortfall in its existing capital in meeting its liabilities. To examine the robustness of our main implications, we now modify the model to allow for variable financial distress costs that increase with the size of the insurer's shortfall. For simplicity, we assume that an insurer incurs financial distress costs that are proportional to the insurer's shortfall in the bad state. More precisely, if the insurer chooses to transfer some or all of its risk through reinsurance or securitization, and receives a payment \bar{B} , in the bad state (net of the reinsurance or securitization premium), then the *additional* deadweight financial distress cost is $c \cdot (\tilde{B} - \bar{B})$, where c is a constant. The *maximum* financial distress cost, which occurs when the insurer retains all its risk, is $c \cdot \tilde{B}$. We set $c\tilde{B} = C$ to compare our results in this section with those in the previous one. In this modified model, we also allow for insurers' risk transfer choices to be observable to capital market investors. All other assumptions in the previous section remain the same.

In the presence of variable financial distress costs, separating partial securitization contracts may be the optimal choice for some insurer types in the equilibrium since they benefit from sharing risk with investors in capital markets at the cost of retaining some risk to signal their type. The analysis of the scenario with variable financial distress costs is significantly more complex than the scenario with fixed financial distress costs.

4.1 Reinsurance

We first consider the case where insurers only have access to reinsurance. Because of the presence of the reinsurance markup, it is either optimal for an insurer to choose full reinsurance or no reinsurance at all. Further, because the expected cost of reinsurance, and the expected cost of partial retention, are both linear functions of the reinsurance indemnity net of reinsurance premium, partial reinsurance is suboptimal. Consequently, the insurer's optimal choice between retention and reinsurance, and the optimal reinsurance contract if it chooses reinsurance, are given by Proposition 1. The risk transfer choice and the reinsurance contract are, therefore, the same as in the model with fixed financial distress costs.

4.2 Securitization

Suppose now that insurers only have access to capital markets. The proportional financial distress cost provides low risk insurers the room to bear some risk by choosing partial securitization. The insurer's choice of risk retention level serves as a signal of its type and, thereby, reduces the adverse selection cost due to information asymmetry. An insurer's optimal choice of securitization coverage reflects the tradeoff between the adverse selection/cross-subsidization cost and the expected retention cost.

We conjecture that a candidate PBE is characterized by a threshold risk type p such that insurers with risk types below the threshold partially transfer their risk through separating contracts, while insurers with risk types above the threshold fully transfer their risk through pooling contracts. Insurers who partially transfer their risk through separating securitization contracts reveal their types and, therefore, incur no adverse selection costs, but nonzero expected retention costs arising from partial retention. In contrast, the high risk insurers who fully transfer their risks through the pooling securitization contract incurs zero expected retention costs, but nonzero cross-subsidization costs. The equilibrium threshold p^* is determined by three conditions.

First, for insurers with risk types below the threshold, each type chooses an incentive compatible risk retention level. The incentive compatibility condition implies that the loss amount transferred through separating securitization satisfies the following ordinary differential equation that arises from the local incentive constraint of each risk type (please see the Appendix for the proof)

$$\frac{dB_s^{sep}(p)}{dp} = \frac{B_s^{sep}(p)(1+cp)}{cp(1-p)} \quad (10)$$

The general solution to the above ODE is

$$B_s^{sep}(p) = \exp(\lambda) \exp\left(\int \frac{1+cp}{c(1-p)p} dp\right) \quad (11)$$

where the constant λ is determined endogenously along with the equilibrium threshold p^* .

Second, an insurer with the threshold risk, p^* , is indifferent between the pooling and separating securitization contracts. It incurs nonzero expected retention costs associated with the retention level if it chooses to signal its type, while it bears subsidization costs associated with the full risk transfer if it pools with higher risk insurers. The expected retention cost if the risk type p^* signals its type

by choosing a separating securitization contract is $c \left(\tilde{B} - B_s^{sep}(p^*) + A_s^{sep}(p^*) \right) p^*$. As we discussed in Section 3.2, the subsidization cost incurred by the risk type p^* is $\tilde{B}R(p^*)$ where $R(\cdot)$ is the subsidization ratio function defined in (6).

The equilibrium threshold, p^* , should therefore satisfy the following condition:

$$\underbrace{c \left(\tilde{B} - B_s^{sep}(p^*) + A_s^{sep}(p^*) \right) p^*}_{\text{expected retention costs from separating contracts}} = \underbrace{\tilde{B}R(p^*)}_{\text{subsidization costs from pooling contracts}}. \quad (12)$$

Rearranging the above equation and using (11), we obtain

$$\exp(\lambda) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \quad (13)$$

Third, for p^* to be the equilibrium threshold, it should be sub-optimal for the insurers in the two subintervals to deviate from their securitization choices. For insurers with risk types below p^* , the marginal subsidization costs must exceed the marginal financial distress costs, thereby motivating the insurers to signal their types by retaining some risk. On the other hand, for insurers with risk types above p^* , the expected retention costs must exceed the cross-subsidization costs. As we show in the Appendix, the equilibrium trigger, p^* , satisfies the following condition:

$$c - \left(c + \frac{1}{1-p^*} \right) \left(1 - \frac{R(p^*)}{cp^*} \right) + \frac{1}{1 - \int_{p^*}^1 td\mu_{p^*}(t)} \geq 0 \quad (14)$$

In general, there is a continuum of threshold levels, p^* , satisfying the above inequality. For each such p^* , the marginal expected cost of financial distress is no less than the marginal cost of full securitization. Because the difference between the expected retention cost due to signaling and the expected cross-subsidization cost due to full pooling securitization is a concave function of the insurer's risk type (see proof of Proposition 4) any off-equilibrium deviation is dominated by the equilibrium contract. Therefore, there exists a corresponding λ satisfying (13) so that each p^* determines a PBE of the securitization game. More formally, we define the set \mathcal{P} satisfying (14), that is,

$$\mathcal{P} = \{p^* : c - \left(c + \frac{1}{1-p^*} \right) \left(1 - \frac{R(p^*)}{cp^*} \right) + \frac{1}{1 - \int_{p^*}^1 td\mu_{p^*}(t)} \geq 0\}. \quad (15)$$

The set \mathcal{P} is the set of possible equilibrium threshold risk levels. The following proposition characterizes

the multiple PBEs of the securitization game.

Proposition 4 (Securitization Contracts). *Define the set \mathcal{P} as in (15). For any $p^* \in \mathcal{P}$, the optimal securitization contract, $(A_s^*(p), B_s^*(p))$, is characterized as follows.*

- For an insurer of type $p < p^*$

$$\begin{aligned} A_s^*(p) &= \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}+1}, \\ B_s^*(p) &= \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}} + A_s^*(p). \end{aligned}$$

- For an insurer of type $p > p^*$

$$A_s^*(p) = A_s^* = \frac{\tilde{B} \int_{p^*}^1 t d\mu_{p^*}(t)}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)}, B_s^*(p) = B_s^* = \tilde{B} + A_s^*(p),$$

where

$$\mu_{p^*}(t) = \frac{F(t) - F(p^*)}{1 - F(p^*)}$$

Cross-subsidization costs are basically transfers among insurers. Consequently, among the set of PBEs described in the proposition, the most efficient one minimizes the expected cost of retention including the deadweight financial distress costs incurred by insurers. The efficient PBE is, therefore, the one defined by the threshold \mathbf{p} where

$$\mathbf{p} = \arg \min_{p \in \mathcal{P}} \int_0^p c(\tilde{B} - B_s^{sep}(t, p)) t f(t) dt$$

4.3 Risk Transfer Equilibria

We now consider the scenario where insurers have access to both reinsurance and securitization. In this general scenario, there exist a variety of candidates for PBEs. The reinsurance markup plays a key role in determining the properties of the PBEs. Intuitively, when the reinsurance markup is below a low threshold, reinsurance dominates (partial or full) securitization for low and intermediate risk insurers because the costs due to the reinsurance markup for such insurers are low relative to the expected financial distress costs from partial securitization or the cross-subsidization from full pooling securitization. High risk insurers choose full pooling securitization. If the reinsurance markup is in

an intermediate region, partial securitization becomes attractive to intermediate risk insurers, while low risk insurers choose reinsurance and high risk insurers choose full pooling securitization. If the reinsurance markup exceeds a high threshold, partial securitization dominates reinsurance even for low risk insurers.

To formalize the above intuition, we begin by noting that the expected cost of an insurer with risk type p if it chooses full reinsurance is $\frac{\tilde{B}\delta p}{1-p(1+\delta)}$. The expected cost from choosing a separating partial securitization contract with retention level $\tilde{B} - B_s^{sep}(p) + A_s^{sep}(p)$ is the expected retention cost that is given by $pc \left(\tilde{B} - B_s^{sep}(p) + A_s^{sep}(p) \right)$. By the arguments used to derive (11), incentive compatibility of the securitization contracts implies that

$$B_s^{sep}(p) = \exp(\lambda) \exp \left(\int \frac{1+cp}{c(1-p)p} dp \right). \quad (16)$$

Let p_1 be the risk type that is indifferent between full reinsurance and partial securitization, and p_2 be the risk type that indifferent between partial securitization and full securitization. Any equilibrium of the risk transfer game is characterized by the pair (p_1, p_2) . The set of possible values of (p_1, p_2) is determined by the aforementioned indifference conditions as well as the *equilibrium* conditions that ensure that deviations from the hypothesized equilibrium strategies are suboptimal.

Let us first examine the indifference conditions. Because it represents the point of indifference between full reinsurance and partial securitization, the trigger, p_1 , must satisfy

$$\begin{aligned} \overbrace{\frac{\tilde{B}\delta p_1}{1-p_1(1+\delta)}}^{\text{expected cost of reinsurance}} &= \overbrace{c(\tilde{B} - B_s^{sep}(p_1) + A_s^{sep}(p_1))p_1}^{\text{expected retention cost from choosing partial securitization}} \\ &= c \left(\tilde{B} - \exp(\lambda) \left(\frac{p_1}{1-p_1} \right)^{\frac{1}{c}} \right) p_1, \end{aligned}$$

where the second equality above follows from (16). Rearranging the above equation, we have

$$\exp(\lambda) = \tilde{B} \left(1 - \frac{\delta}{(1-p_1(1+\delta))c} \right) \left(\frac{1-p_1}{p_1} \right)^{\frac{1}{c}} \quad (17)$$

For any p_1 satisfying $p_1 < \frac{c-\delta}{c(1+\delta)}$, a corresponding λ exists satisfying the above equation so that any such p_1 is a candidate indifference point between reinsurance and partial securitization. Accordingly,

we define the set \mathcal{U} as

$$\mathcal{U} = \{p_1 : p_1 < \frac{c - \delta}{c(1 + \delta)}\} \quad (18)$$

In other words, the set \mathcal{U} is the set of *candidate* equilibrium indifference thresholds between reinsurance and partial securitization. Reinsurance is dominated by the full retention for insurers with types greater than $\frac{c - \delta}{c(1 + \delta)}$. The set \mathcal{U} guarantees that partial securitization might dominate reinsurance for certain types of insurers in the equilibrium. We use the term “candidate” because we have not yet imposed the equilibrium conditions. As we see shortly, the imposition of the equilibrium conditions reduces this set.

Given any $p_1 \in \mathcal{U}$, the threshold, p_2 , which represents the point of indifference between partial and full securitization, must satisfy

$$c(\tilde{B} - B_s^{sep}(p_2) + A_s^{sep}(p_2))p_2 = \tilde{B}R(p_2). \quad (19)$$

By (17),

$$B_s^{sep}(p_2) = \tilde{B} \left(1 - \frac{\delta}{(1 - p_1(1 + \delta))c} \right) \left(\frac{1 - p_1}{p_1} \right)^{\frac{1}{c}} \int \frac{1 + cp}{c(1 - p)p} dp.$$

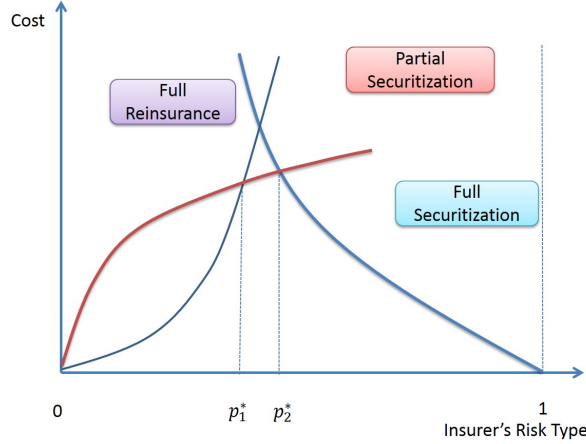
Accordingly, we define the set \mathcal{L} —the set of *candidate* equilibrium indifference thresholds, p_2 —as follows.

$$\mathcal{L} = \left\{ p_2 : 1 - \left(1 - \frac{\delta}{(1 - p_1(1 + \delta))c} \right) \left(\frac{1 - p_1}{p_1} \right)^{\frac{1}{c}} \left(\frac{p_2}{1 - p_2} \right)^{\frac{1}{c}} = \frac{R(p_2)}{cp_2}; \forall p_1 \in \mathcal{U} \right\} \quad (20)$$

We now examine the conditions under which a candidate pair (p_1, p_2) , indeed, characterizes an equilibrium. Let us consider the subset of feasible pairs, (p_1, p_2) where $p_1 < p_2$ so that there is an interval of insurer types who choose partial securitization. For p_2 to be an *equilibrium* threshold, it must be sub-optimal for insurers choosing partial or full securitization to deviate from their respective choices. As we show in the Appendix, this condition ensures that p_2 must satisfy the following inequality for any given $p_1 \in \mathcal{U}$

$$c - \left(c + \frac{1}{1 - p_2} \right) \left(1 - \frac{\delta}{c(1 - p_1(1 + \delta))} \right) \left(\frac{(1 - p_1)p_2}{p_1(1 - p_2)} \right)^{\frac{1}{c}} + \frac{1}{1 - \int_{p_2}^1 t d\mu_{p_2}(t)} \geq 0. \quad (21)$$

Figure 10: Risk Transfer Equilibria with Variable Financial Distress Cost



For each such p_2 , the above inequality implies that the marginal increase in the expected cost of partial securitization is no less than the marginal increase in the expected cost of full securitization due to an increase in the insurer's risk type. It guarantees that any off-equilibrium full securitization contract is dominated by a partial securitization contract for insurers with types below p_2 .

Accordingly, we define the set \mathcal{G} as

$$\mathcal{G} = \left\{ (p_1, p_2) : p_1 < p_2, \right. \\ \left. c - \left(c + \frac{1}{1-p_2} \right) \left(1 - \frac{\delta}{c(1-p_1(1+\delta))} \right) \left(\frac{(1-p_1)p_2}{p_1(1-p_2)} \right)^{\frac{1}{c}} \right. \\ \left. + \frac{1}{1 - \int_{p_2}^1 t d\mu_{p_2}(t)} \geq 0, \forall p_1 \in \mathcal{U}, p_2 \in \mathcal{L} \right\}.$$

In other words, the set \mathcal{G} is the set of equilibria, determined by the pair of thresholds, (p_1, p_2) , that feature reinsurance, partial securitization and full securitization (Figure 10).

We now have the requisite definitions in place to characterize the risk transfer equilibria.

Proposition 5 (Partition Equilibrium). *There exist a set of PBE which are characterized by the pairs of p_1^*, p_2^* such that $\{p_1^*, p_2^*\} \in \mathcal{G}$, insurers with types in the interval $[0, p_1^*]$ choose full reinsurance, insurers with types in the interval $[p_1^*, p_2^*]$ choose separating partial securitization, and insurers with types in the interval $[p_2^*, 1]$ choose pooling full securitization.*

The above proposition shows that the PBE, in general, continue to take the partition form with three intervals when allowing for variable financial distress cost. The general form subsumes several

degenerate cases, which depend on the level of the variable financial distress cost, c , and the reinsurance markup, δ . Specifically, full reinsurance dominates partial risk sharing for low risk insurers if the reinsurance markup is lower than the proportional financial distress cost c . Intermediate risk insurers choose partial securitization provided the proportional financial distress cost is below a threshold. If the proportional financial distress cost exceeds the threshold, however, partial securitization is sub-optimal for all insurers, that is, high risk insurers chooses full securitization, while low risk insurers choose full reinsurance. When the reinsurance markup exceeds the proportional financial distress cost, however, insurers choose partial or full securitization.

Our results are consistent with evidence that CAT bonds mainly provide protection for top layers of the loss distribution. As catastrophe risks are characterized by lower probabilities and high severities, they are more likely to be reinsured or partially securitized, that is, catastrophe-linked securities such as CAT bonds are more likely to be employed (if at all) to protect losses above a threshold.

5 Conclusions

We reconcile the “catastrophe risk” puzzle using a signaling model. Insurers’ risk transfer choices reflect the tradeoff between the lower adverse selection costs associated with reinsurance against reinsurance markups. PBE of the signaling game have a partition form where the lowest risks are reinsured, intermediate risks are partially securitized, and the highest risks are fully securitized. An increase in the loss size increases the threshold risk level above which risks are transferred via securitization. Consequently, catastrophe risk, which is characterized by “low probability-high severity” losses, is less likely to be securitized. Further, catastrophe-linked securities such as CAT bonds are more likely to be employed (if at all) for losses above a threshold. Because only the highest risk insurers choose securitization, they pay high premia in securities markets, which could explain why catastrophe-linked securities are usually expensive, and why catastrophe securities often receive ratings below investment grade.

Our results suggest that, in the scenario in which an insurer is exposed to multiple types of risks, the lowest risks are reinsured, the intermediate risks are partially securitized, and the highest risks are fully securitized. With the entry of more sophisticated investors such as dedicated hedge funds and the advent of blockchain technology, both of which lower the adverse selection costs of securitization relative to reinsurance, the market for insurance-linked securities will grow signifi-

cantly in the future, which supports the optimistic view expressed by key industry professionals (e.g., see <http://www.artemis.bm/blog/2017/10/18/alternative-capital-seen-as-28-of-reinsurance-224bn-by-2021-ey/>).

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Online Appendix A: Proofs

Proof of Proposition 1

Proof. Suppose $\delta < \frac{C}{\tilde{B}}$.

Given the presence of fixed financial distress costs, it is easy to see that it is sub-optimal for an insurer to choose partial reinsurance, that is, if an insurer chooses reinsurance, it chooses full reinsurance. As it is optimal for the insurer to buy just enough reinsurance to cover its net loss in the bad state, the total reinsurance payment $B_r^*(p)$ for it in the bad state is $B_r^*(p) = B - W + A_r^* = \tilde{B}$. The insurer's maximization problem is equivalent to minimizing $A_r(p) - B_r p$ which implies that the constraint (3) is also binding. Hence, the premium is given by $A_r^*(p) = \frac{\tilde{B}p(1+\delta)}{1-p(1+\delta)}$.

The expected payoff of reinsurance for the insurer with type p is $EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p(1+\delta)}$. The expected payoff of full self-insurance for the insurer with type p is $EU_{self}(p) = W + (A(1-p) - Bp) - Cp$. Thus, $EU_r(p) > EU_{self}(p)$ for all $p < \frac{C-\tilde{B}\delta}{C(1+\delta)} = p_1^F$, where p_1^F is defined in (4). Accordingly, reinsurance is sub-optimal for insurers with types $p > p_1^F$, but optimal for insurers with types $p < p_1^F$. ■

Proof of Proposition 2

Proof. Consider first a candidate fully separating equilibrium $(A_s^*(p), B_s^*(p))$, where $(A_s^*(p), B_s^*(p))$ is the securitization contract offered by the insurer with type p . The capital market investors break even, thereby leading the investors' participation condition to be binding. Hence, the premium is $A_s^*(p) = pB_s^*(p)$. However, $(A_s^*(p), B_s^*(p))$ is not incentive compatible because the higher risk insurers are strictly better off by deviating and offering the lower risk insurers' contract. Consequently, we cannot have a fully separating equilibrium. Hence, any equilibrium must necessarily involve some pooling.

Next, we observe that there cannot be an equilibrium in which there exists a quadruple, $\{p_1, p_2, p_3, p_4\}$ with $p_1 \leq p_2 < p_3 \leq p_4$ such that insurers with types in $[p_1, p_2]$ pool together and choose a single full securitization contract, and insurers with types in $[p_3, p_4]$ pool together and choose a single full securitization contract, but the two intervals of insurers choose different contracts. This assertion follows easily from the observation that insurers with types in $[p_3, p_4]$ would prefer the contract offered by the insurers with types in $[p_1, p_2]$.

It follows from the above arguments that it suffices to consider candidate equilibria in which insurers with types below a threshold choose self-insurance, while insurers with types above the threshold choose full pooling securitization. Accordingly, consider a candidate equilibrium defined by a trigger level p . We now examine the conditions for p to be an *equilibrium* threshold. An insurer with type $k \geq p$ chooses full pooling securitization, $B_s^*(k) = \tilde{B} + A_s^*$. The break-even condition of investors requires that the premium be given by

$$A_s^*(k) = A^* = \tilde{B} \frac{\int_p^1 t d\mu_p(t)}{1 - \int_p^1 t d\mu_p(t)},$$

where $\mu_p(t)$ is the investors' posterior beliefs about insurer's types. Therefore, the insurer's expected payoff from securitization is

$$EU_s^{pooling}(k) = W + (A(1 - k) - Bk) - \tilde{B} \frac{\int_p^1 t d\mu_p(t) - k}{1 - \int_p^1 t d\mu_p(t)}. \quad (A1)$$

The insurer, whose type k is less than or equal to p , chooses full retention. Its expected payoff is, therefore, $EU_{self}(k) = W + (A(1 - k) - Bk) - Ck$. It is easy to see that, if $p = p_2^F$ satisfying (7), then $EU_s^{pooling}(p) = EU_{self}(p)$. Hence, the insurer with risk type p_2^F is indifferent between pooling with higher types and self-insurance.

Next, we check that p_2^F is, indeed, the equilibrium threshold. First, we establish incentive compatibility of the set of contracts defined by p_2^F . If an insurer with type $k < p_2^F$ deviates to choose the pooling contract (A^*, B^*) , its expected payoff is

$$EU_s^{deviate}(k) = W + (A(1 - k) - Bk) - \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - k}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)}.$$

It is easy to show that, if $k < p_2^F$, then

$$\tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - k}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} > \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p_2^F}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} = \tilde{B}R(p_2^F) = Cp_2^F > Ck.$$

Thus, $EU_{self}(k) > EU_s^{deviate}(k)$. As a result, the insurer with type $k < p_2^F$ will not choose the pooling contract (A^*, B^*) . If an insurer with type $k > p_2^F$ deviates to choose full self-insurance, the expected payoff is

$$EU_{self}^{deviate}(k) = W + (A(1 - k) - Bk) - Ck.$$

It is easy to see that, if $k > p_2^F$, then

$$\tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - k}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} < \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p_2^F}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} = \tilde{B}R(p_2^F) = Cp_2^F < Ck.$$

Thus, $EU_s^{pooling}(k) > EU_{self}^{deviate}(k)$. Consequently, the insurer whose type is greater than p_2^F will not choose retention.

Now suppose that an insurer with type $k > p_2^F$ finds it profitable to deviate to some other securitization contract (A'_s, B'_s) . Suppose first that the contract involves a full transfer of risk. The deviation is profitable for the insurer iff $A'_s < A^*$. In this case, however, the deviation is also profitable for insurers with higher risk types. Consequently, reasonable off-equilibrium beliefs of investors must necessarily pool insurers with types greater than or equal to k , which makes the hypothesized deviation unprofitable for insurer k . Alternately, applying the D1 refinement, the sets of investor beliefs under which a deviation to the full risk transfer contract (A'_s, B'_s) is profitable increases with the insurer risk

type. Iteratively applying the D1 refinement, therefore, implies that, on observing such a deviation, investors' beliefs assign probability one that the insurer has the highest risk type, which makes it unprofitable for all lower risk insurers to deviate.

Suppose that the deviating contract (A'_s, B'_s) does not involve a full transfer of risk so that $B'_s < B^*$ and the insurer bears the additional financial distress cost C in the bad state. Because the insurer's expected cost under the pooling contract given by (A1) is decreasing and linear in its type k , in this case too, the sets of investor beliefs under which the deviation is profitable are increasing in the insurer type. Iteratively applying the D1 refinement, investors' beliefs assign probability one that the insurer has the highest risk type on observing such a deviation, thereby making it unprofitable for lower risk types.

Similarly, suppose that an insurer with type $k < p_2^F$ finds it profitable to deviate to a securitization contract (A''_s, B''_s) . If the contract involves a full transfer of risk, it must also be profitable for insurers with types in $[k, p_2^F]$. Consequently, reasonable off-equilibrium beliefs must pool together such insurers, which makes the hypothesized deviation unprofitable. Alternately, iteratively applying the D1 refinement, off-equilibrium beliefs following such a deviation assign probability one that the insurer is of type p_2^F , thereby making the deviation unprofitable for all lower risk insurers. If the contract does not involve a full transfer of risk, then the insurer necessarily bears the financial distress cost C in the bad state. In this case too, if such a deviation is profitable for the insurer, it must also be profitable for insurers with types in $[k, p_2^F]$. We can again argue as above that reasonable off-equilibrium beliefs following such a deviation make it unprofitable for the insurer.

Hence, the threshold p_2^F satisfying (7) defines an equilibrium. Moreover, if (7) has a unique solution, then it determines the unique PBE of the risk transfer game. ■

Proof of Proposition 3.

Proof. 1. If $C < \tilde{B}\delta$, where it is sub-optimal for an insurer to choose reinsurance. We are, thus, in the scenario described in Proposition 2.

2. Suppose $\tilde{B}\delta < C < \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$.

It follows from Proposition 1 that insurers with types in the interval $[0, p_1^F]$ prefer full reinsurance to full self-insurance. By Proposition 2, insurers with types in the interval $[p_2^F, 1]$ prefer full pooling securitization to full self-insurance. By condition (8), there is a unique p_2^F satisfying (7).

Since $C < \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$, $\frac{\tilde{B}\delta p_1^F}{1-p_2^F(1+\delta)} > C p_1^F = \frac{\tilde{B}\delta p_1^F}{1-p_1^F(1+\delta)}$. Thus, $p_1^F < p_2^F$.

It follows from the results of Propositions 1 and 2 that p_1^F and p_2^F are two indifference points. Now check whether they are, indeed, the equilibrium thresholds. If an insurer with type in the interval $[0, p_1^F]$, deviates to choose the pooling securitization contract given by Proposition 2, the expected payoff is

$$EU_s^{deviate}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)}.$$

It is easy to see that, since

$$\tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} \geq \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p_2^F}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} = \tilde{B}R(p_2^F) = Cp_2^F > Cp_1^F > Cp,$$

it will not deviate to choose full pooling securitization. Consequently, the insurers with types in the interval $[0, p_1^F]$ will not deviate to choose full pooling securitization. Under restrictions on reasonable off-equilibrium beliefs along the lines of the D1 refinement as in the proof of Proposition 2, an insurer with type in the interval $[0, p_1^F]$ will also not deviate to choose any other securitization contract.

For an insurer with type in the interval $[p_1^F, p_2^F]$, Proposition 1 implies that it will not choose full reinsurance. Proposition 2 implies that it will not choose full pooling securitization or any other securitization contract. As a result, it is optimal for it to choose full self-insurance.

For an insurer with type in the interval $[p_2^F, 1]$, Proposition 2 shows that it will not choose full self-insurance. If it deviates to choose full reinsurance, it pays the additional rents due to reinsurance markup arising from a variety of sources. Thus, the expected payoff is $EU_r^{deviate}(p) = W + (A(1 - p) - Bp) - \frac{\tilde{B}\delta p}{1 - p(1 + \delta)}$. It is easy to show that

$$\frac{\tilde{B}\delta p}{1 - p(1 + \delta)} > Cp$$

since the function $C - \frac{\tilde{B}\delta}{1 - p(1 + \delta)}$ decreases with p and equals zero at p_1^F . Also,

$$Cp > Cp_2^F = \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p_2^F}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)} > \tilde{B} \frac{\int_{p_2^F}^1 t d\mu_{p_2^F}(t) - p}{1 - \int_{p_2^F}^1 t d\mu_{p_2^F}(t)}.$$

As a result, $EU_r^{deviate}(p) < EU_s^{pooling}(p)$ if $p > p_2^F$. By arguments similar to those used in the proof of Proposition 2, which plays restrictions on reasonable off-equilibrium beliefs, it is also sub-optimal for an insurer with type in the interval $[p_2^F, 1]$ to deviate to any other securitization contract. Consequently, it is optimal for insurers with types greater than p_2^F to choose pooling securitization. Further, the conjectured PBE is the unique equilibrium since the values of p_1^F and p_2^F are unique under condition (8) and $\tilde{B}\delta < C < \frac{\tilde{B}\delta}{1 - p_2^F(1 + \delta)}$.

3. Suppose $C > \frac{\tilde{B}\delta}{1 - p_2^F(1 + \delta)}$, that is $p_2^F < p_1^F$. It follows that it is sub-optimal for an insurer to choose self-insurance, thereby leading the equilibria to have a partition form with two subintervals.

First solve for the point of indifference between choosing full reinsurance and pooling with higher risk insurers through securitization. The optimal reinsurance contracts are given by Proposition 1, and the corresponding expected payoff is $EU_r(p) = W + (A(1 - p) - Bp) - \frac{\tilde{B}p\delta}{1 - p(1 + \delta)}$. The optimal pooling securitization coverage is $B_s^* = \tilde{B}$. The indifference point, p_3 , between securitization and reinsurance must solve

$$\frac{\tilde{B}p_3\delta}{1 - p_3(1 + \delta)} = \tilde{B}R(p_3) \tag{A2}$$

Condition $R'(p) < \frac{\delta}{(1-p(1+\delta))^2}$ ensures that there is a unique solution p_3^F to (A2)

Next, we check whether the unique solution p_3^F is the equilibrium threshold. For insurers with types in the interval $[0, p_3^F]$, the expected payoff of full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p(1+\delta)},$$

while the expected payoff of full pooling securitization is

$$EU_s^{deviate}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_3^F}^1 td\mu_{p_3^F}(t) - p)}{1 - \int_{p_3^F}^1 td\mu_{p_3^F}(t)}.$$

For any $p \in [0, p_3^F]$,

$$\frac{\tilde{B}p\delta}{1-p(1+\delta)} < \frac{\tilde{B}p_3^F\delta}{1-p_3^F(1+\delta)} = \frac{\tilde{B}(\int_{p_3^F}^1 td\mu_{p_3^F}(t) - p_3^F)}{1 - \int_{p_3^F}^1 td\mu_{p_3^F}(t)} < \frac{\tilde{B}(\int_{p_3^F}^1 td\mu_{p_3^F}(t) - p)}{1 - \int_{p_3^F}^1 td\mu_{p_3^F}(t)}$$

Then, $EU_r(p) > EU_s^{deviate}(p)$. The insurer types in the interval $[0, p_3^F]$, therefore, will not deviate to choose full securitization. By arguments similar to those used in the earlier proofs, an insurer with type in the interval $[0, p_3^F]$ will also not deviate to choose any other securitization contract.

Similarly, the expected payoff of insurers with types in the interval $[p_3^F, 1]$ from choosing securitization is

$$EU_s(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_3^F}^1 td\mu_{p_3^F}(t) - p)}{1 - \int_{p_3^F}^1 td\mu_{p_3^F}(t)},$$

while the expected payoff of choosing full reinsurance is

$$EU_r^{deviate}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p(1+\delta)}.$$

For any $p \in [p_3^F, 1]$, we have

$$\frac{\tilde{B}(\int_{p_3^F}^1 td\mu_{p_3^F}(t) - p)}{1 - \int_{p_3^F}^1 td\mu_{p_3^F}(t)} < \frac{\tilde{B}(\int_{p_3^F}^1 td\mu_{p_3^F}(t) - p_3^F)}{1 - \int_{p_3^F}^1 td\mu_{p_3^F}(t)} = \frac{\tilde{B}p_3^F\delta}{1-p_3^F(1+\delta)} < \frac{\tilde{B}p\delta}{1-p(1+\delta)}$$

Thus, $EU_s(p) > EU_r^{deviate}(p)$. Therefore, insurers with types in the interval $[p_3^F, 1]$ will not deviate to choose reinsurance. By arguments similar to those used in earlier proofs, they will also not deviate to choose any other securitization contract.

Consequently, the conjectured equilibrium is, indeed, the unique PBE of the signaling game if condition (A2) and $C > \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$ hold.

■

Proof of Corollary 1

Proof. We first show that an increase in the size of the net loss payments B increases the threshold level above which insurers choose full securitization. p_2^F is the point of indifference between the cross-subsidization costs from pooling with higher types and expected costs from retaining risk. Given the “single crossing” condition (8), there exists a unique solution to (7), p_2^F ; that is

$$g(p_2^F) = Cp_2^F - \tilde{B}R(p_2^F) = 0$$

Applying the implicit function theorem to the above equation, we get $\frac{\partial p_2^F}{\partial B} = \frac{R(p_2^F)}{C - \tilde{B}R'(p_2^F)}$. Under the “single crossing” condition (8), we can show that $\frac{\partial p_2^F}{\partial B} > 0$. An increase in the size of the net loss payments, B , increases the threshold level above which insurers choose full securitization.

We then show that an increase in the size of the net loss payments B increases the threshold level below which insurers choose full Reinsurance. It is easy to show $\frac{\partial P_1^F}{\partial B} = -\frac{\delta}{C(1+\delta)} < 0$. It implies an increase in the size of the net loss payments, B , decreases the threshold level above which insurers choose reinsurance. Consequently, the subinterval of insurer risk types that choose securitization or reinsurance decreases.

In sum, an increase in the size of net loss payment, B , reduces the subinterval of insurer risk types that choose securitization and increases the total subinterval of insurer risk types that choose reinsurance and securitization. Given that insurers choose securitization if only if their probability of potential losses is very high, catastrophe risks that are characterized by “low probability-high severity” are, therefore, more likely to be either retained by insurers or reinsured. ■

Proof of Proposition 4

Proof. We first show that PBEs cannot be fully pooling or fully separating.

Consider first a candidate pooling equilibrium where all insurers offer the same contract $(A_s^*(p), B_s^*(p))$ given by Proposition 2. Because financial distress costs now depend on an insurer’s shortfall in meeting its liabilities, the lower risk insurers have incentives to retain some risk to signal their types, thereby reducing the subsidization costs from pooling securitization contracts.

Now consider a candidate fully separating equilibrium where each insurer type chooses corresponding securitization contracts at fair price since its risk type is perfectly revealed in the capital markets. Thus, the optimal risk retention level $\tilde{B} - B_s^{sep}(p) + A_s^{sep}(p)$ (or the optimal total risk coverage $B_s^{sep}(p)$) solves

$$\max_{\tilde{p}} W + (A(1-p) - Bp) - (A_s(\tilde{p}) - B_s^{sep}(\tilde{p})p) - c \left(\tilde{B} - B_s^{sep}(\tilde{p}) + A_s^{sep}(\tilde{p}) \right) p$$

such that

$$A_s(\tilde{p}) - B_s^{sep}(\tilde{p})\tilde{p} \geq 0 \tag{A3}$$

The break-even condition (A3) for capital markets is binding. The above is, therefore, equivalent to

$$\min_{\tilde{p}} B_s^{sep}(\tilde{p} - p) + c \left(\tilde{B} - B_s^{sep}(\tilde{p}) + A_s^{sep}(\tilde{p}) \right) p$$

The first order condition is

$$B_s^{sep'}(\tilde{p})(\tilde{p} - p) + B_s^{sep}(\tilde{p}) + c p \tilde{p} B_s^{sep'}(\tilde{p}) + c p B_s^{sep}(\tilde{p}) - c B_s^{sep'}(\tilde{p}) p = 0.$$

Setting $\tilde{p} = p$, we obtain

$$(1 + cp) B_s^{sep}(p) = \frac{dB_s^{sep}(p)}{dp} cp(1 - p) \quad (\text{A4})$$

The general solution of the above ordinary differential equation is given by (10), that is

$$B_s^{sep}(p) = \exp(\lambda) \exp\left(\int \frac{1 + cp}{cp(1 - p)} dp\right),$$

where λ is the constant of integration. It is easy to show that, for any λ , there is a \tilde{p} where $0 < \tilde{p} < 1$, such that $B_s^{sep}(\tilde{p}) = \tilde{B} + A_s^{sep}(\tilde{p})$. It follows that the pure separating equilibrium is also violated since not all insurers are able to signal their types.

Using arguments similar to those used in the proof of Proposition 2, we can show that it suffices to consider candidate *semi pooling* equilibria characterized by a threshold risk type p^* such that insurers with types below it partially transfer their risks through separating contracts, while insurers with risk types above it fully transfer their risks through pooling contracts. Insurers who choose separating contracts reveal their types and, therefore, incur no adverse selection costs, but nonzero expected costs from the partial retention. The insurer of type p^* should be indifferent between a separating and pooling contract.

The expected cost to an insurer of type p from choosing a separating contract that reveals its type is

$$C_s^{sep}(p)p = c \left(\tilde{B} - \exp(\lambda) \left(\frac{p}{1 - p} \right)^{\frac{1}{c}} \right) p$$

The expected cost to the insurer with type p from choosing a pooling contract is $\tilde{B}R(p)$, where $R(p)$ is defined by equation (6).

Thus, an indifference threshold p^* is determined by

$$c \left(\tilde{B} - \exp(\lambda) \left(\frac{p^*}{1 - p^*} \right)^{\frac{1}{c}} \right) p^* = \tilde{B}R(p^*).$$

Any p^* satisfying the above equation is a candidate for the threshold that supports the conjectured

semi pooling PBE. The indifference point p^* also determines the incentive compatible pooling and separating contracts in terms of the value of λ . Rearranging (12) and using (11), we obtain (13), that is

$$\exp(\lambda) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}}.$$

Clearly, $\forall p^* \in [0, 1]$, there exists a corresponding λ such that p^* is the point of indifference between pooling and separating contracts.

For the given indifference point p^* , plugging (13) into (11), we obtain the corresponding separating contracts for the insurer with type $p < p^*$.

$$B_s^{sep*}(p) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}} + A_s^{sep*}(p) \quad (\text{A5})$$

$$A_s^{sep*}(p) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*} \right) \left(\frac{1-p^*}{p^*} \right)^{\frac{1}{c}} \left(\frac{p}{1-p} \right)^{\frac{1}{c}+1} \quad (\text{A6})$$

The break-even condition for capital markets requires the pooling contract premium to be

$$A_s^* = \frac{\tilde{B} \int_{p^*}^1 t d\mu_{p^*}(t)}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)}$$

We now show that $p^* \in [0, 1]$ is an *equilibrium* indifference threshold if it satisfies condition (14).

For an insurer with type $p \in [0, p^*]$, the expected payoff of choosing partial securitization is

$$EU_s^{sep}(p) = W + (A(1-p) - Bp) - c \left(\tilde{B} - B_s^{sep}(p) + A_s^{sep}(p) \right) p$$

If it deviates to the pooling contract, the expected payoff is

$$EU_s^{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p^*}^1 t d\mu_{p^*}(t) - p}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)}$$

We now show that $EU_s^{deviatepool}(p) \leq EU_s^{sep}(p)$ if condition (14) holds. Define

$$G_1(p) = cp \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) - \tilde{B} \frac{\int_{p^*}^1 t d\mu_{p^*}(t) - p}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)}$$

where $\tilde{B}_s^{sep}(p)$ is given by $B_s^{sep}(p) - A_s^{sep}(p) = \tilde{B} \left(1 - \frac{R(p^*)}{cp^*}\right) \left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}} \left(\frac{p}{1-p}\right)^{\frac{1}{c}}$. We have

$$\begin{aligned} G_1'(p) &= c \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) - cp \tilde{B}_s^{sep'}(p) + \frac{\tilde{B}}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)} \\ &= c \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) - \frac{\tilde{B}_s^{sep}(p)}{1-p} + \frac{\tilde{B}}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)} \\ G_1''(p) &= -c \tilde{B}_s^{sep'}(p) - \frac{\tilde{B}_s^{sep'}(p)(1-p) + \tilde{B}_s^{sep}(p)}{(1-p)^2} \leq 0. \end{aligned}$$

Thus, $G_1(p)$ is a concave function of p . So we have

$$\frac{\partial G_1(p)}{\partial p} \Big|_{p < p^*} \geq \frac{\partial G_1(p)}{\partial p} \Big|_{p = p^*}$$

Next, note that

$$\begin{aligned} \frac{\partial G_1(p)}{\partial p} \Big|_{p=p^*} &= c \left(\tilde{B} - \tilde{B} \left(1 - \frac{R(p^*)}{cp^*}\right) \right) - \frac{\tilde{B} \left(1 - \frac{R(p^*)}{cp^*}\right)}{1-p^*} + \frac{\tilde{B}}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)} \\ &= \tilde{B} \left(c - \left(c + \frac{1}{1-p^*} \right) \left(1 - \frac{R(p^*)}{cp^*}\right) + \frac{1}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)} \right). \end{aligned}$$

Under condition (14), $\frac{\partial G_1(p)}{\partial p} \Big|_{p < p^*} \geq 0$, that is $G_1(p)$ is an increasing function of p for $p < p^*$ so that $G_1(p) < G_1(p^*) = 0$. Consequently,

$$cp \left(\tilde{B} - B_s^{sep}(p) \right) < \tilde{B} \frac{\int_{p^*}^1 t d\mu_{p^*}(t) - p}{1 - \int_{p^*}^1 t d\mu_{p^*}(t)},$$

and $EU_s^{sep}(p) > EU_s^{deviatepool}$. Hence, the insurers with risk types below p^* will not deviate to pooling securitization by (14). Because the Spence-Mirrlees single-crossing condition holds (due to the linear objective function of insurers), the “local” incentive compatibility condition (A4) ensures that an insurer with risk type $p \leq p^*$ will also not deviate to choose the partial securitization contract of some other type $p' \leq p^*$. Finally, as in the proof of Proposition 2, we can show that, under reasonable off-equilibrium beliefs, it is sub-optimal for an insurer with risk type $p \leq p^*$ to deviate to some other arbitrary securitization contract (A_s, B_s) that is not chosen by another risk type $p' \leq p^*$. If such a deviation were profitable for the insurer of type $p < p^*$, it would also be profitable for types $p' \in (p, p^*]$. Consequently, on observing such an off-equilibrium deviation, the beliefs of capital market investors would pool the insurer of type p with the insurers of types $p' \in (p, p^*]$, thereby making the deviation unprofitable. Alternatively, iteratively applying the D1 refinement, investors believe that the deviating insurer is of the risk type p^* with probability one, which makes the deviation unprofitable for all lower

risk types.

For insurers with types $p \in [p^*, 1]$, the expected payoff of choosing full pooling securitization is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p^*}^1 td\mu_{p^*}(t) - p}{1 - \int_{p^*}^1 td\mu_{p^*}(t)}$$

The expected payoff of mimicking an arbitrary lower-risk insurer of type $\hat{p} < p^*$ is

$$EU_s^{deviatesep}(p) = W + (A(1-p) - Bp) - \underbrace{c(\tilde{B} - \tilde{B}_s^{sep}(\hat{p}))p}_{\text{expected financial distress costs of mimicking}} - \underbrace{\frac{\tilde{B}_s^{sep}(\hat{p})(\hat{p} - p)}{1 - \hat{p}}}_{\text{benefits from mimicking}}$$

Define

$$G_2(p) = c(\tilde{B} - \tilde{B}_s^{sep}(\hat{p}))p + \frac{\tilde{B}_s^{sep}(\hat{p})(\hat{p} - p)}{1 - \hat{p}} - \tilde{B} \frac{\int_{p^*}^1 td\mu_{p^*}(t) - p}{1 - \int_{p^*}^1 td\mu_{p^*}(t)}$$

The first derivative is

$$\begin{aligned} G_2'(p) &= c(\tilde{B} - \tilde{B}_s^{sep}(\hat{p})) - \frac{\tilde{B}_s^{sep}(\hat{p})}{1 - \hat{p}} + \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu_{p^*}(t)} \\ &= c(\tilde{B} - \tilde{B}_s^{sep}(\hat{p})) + \left(\frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu_{p^*}(t)} - \frac{\tilde{B}_s^{sep}(\hat{p})}{1 - \hat{p}} \right) > 0 \end{aligned}$$

It is obvious that $G_2(p)$ is an increasing function of $p \in [p^*, 1]$. Thus, $G_2(p) \geq G_2(p^*) = 0 \quad \forall p \geq p^*$. That is, $c(\tilde{B} - \tilde{B}_s^{sep}(\hat{p}))p + \frac{\tilde{B}_s^{sep}(\hat{p})(\hat{p} - p)}{1 - \hat{p}} > \tilde{B} \frac{\int_{p^*}^1 td\mu_{p^*}(t) - p}{1 - \int_{p^*}^1 td\mu_{p^*}(t)}$. Hence, it is easy to show that $EU_s^{pool}(p) > EU_s^{deviate}(p) \quad \forall p > p^*$. As a result, insurers with risk types greater than p^* will not deviate to choose separating contracts. As earlier, we can iteratively apply the D1 refinement to show that an insurer with risk type $p > p^*$ will also not deviate to choose some other arbitrary securitization contract.

By the above arguments, each candidate threshold $p^* \in \mathcal{P}$ defined in (15) defines a semi-pooling PBE. ■

Proof of Proposition 5

Proof. 1. Suppose first that

$$\delta < c \tag{A7}$$

It then follows from Proposition 1 that the insurer with risk type below a threshold chooses full reinsurance. By Proposition 4, higher risk insurers prefer pooling securitization, while lower risk insurers prefer separating securitization. Therefore, we conjecture that there are two types of PBEs

under condition (A7). The differences between the two types of PBEs lie in the intermediate risk insurers' choice between full reinsurance and partial securitization.

First considers the candidates for a pair of triggers (p_1, p_2) , where p_1 is the point of indifference between full reinsurance and partial securitization, and p_2 is the point of indifference between partial securitization and full securitization. The intermediate interval is nonempty iff $p_1 < p_2$. By our earlier arguments, p_1 must satisfy

$$\frac{\tilde{B}\delta p_1}{1 - p_1(1 + \delta)} = c(\tilde{B} - \exp(\lambda) \left(\frac{p_1}{1 - p_1} \right)^{\frac{1}{c}}) p_1 \quad (\text{A8})$$

where the constant of integration, λ , is determined by p_1 if it is the equilibrium threshold.

Rearranging the above equation, we obtain (17), where

$$\exp(\lambda) = \tilde{B} \left(1 - \frac{\delta}{(1 - p_1(1 + \delta))c} \right) \left(\frac{1 - p_1}{p_1} \right)^{\frac{1}{c}} \quad (\text{A9})$$

The trigger, p_2 , must satisfy equation (19), that is, $c(\tilde{B} - B_s^{sep}(p_2))p_2 = \tilde{B}R(p_2)$, where it follows from (17) that

$$\tilde{B}_s^{sep}(p_2) = \tilde{B} \left(1 - \frac{\delta}{(1 - p_1(1 + \delta))c} \right) \left(\frac{1 - p_1}{p_1} \right)^{\frac{1}{c}} \left(\frac{p_2}{1 - p_2} \right)^{\frac{1}{c}}$$

The above two equations lead to the following relationship between p_1 and p_2 :

$$1 - \left(1 - \frac{\delta}{(1 - p_1(1 + \delta))c} \right) \left(\frac{1 - p_1}{p_1} \right)^{\frac{1}{c}} \left(\frac{p_2}{1 - p_2} \right)^{\frac{1}{c}} = \frac{R(p_2)}{cp_2}$$

We define the set \mathcal{U} by (18), which comprises of all possible indifference points p_1^* . We define the set \mathcal{L} by (20), which contains all possible indifference points p_2^* .

Suppose that $p_1^* < p_2^*$. Conjecture a partition equilibrium where insurers with types in the range $[0, p_1^*]$ choose full reinsurance, insurers with types in the range $[p_1^*, p_2^*]$ choose separating partial securitization, and insurers with types in the range $[p_2^*, 1]$ choose pooling full securitization. We now show that the pair of indifference points are, indeed, equilibrium thresholds.

For insurers with types in the range $[0, p_1^*]$, their expected payoff of full reinsurance is

$$EU_r(p) = W + (A(1 - p) - Bp) - \frac{\tilde{B}\delta p}{1 - p(1 + \delta)}.$$

If they deviate to choose partial securitization by choosing the corresponding coverage, where

$$\tilde{B}_s^{sep}(p) = B_s^{sep}(p) - A_s^{sep}(p) = \tilde{B} \left(1 - \frac{\delta}{(1 - p_1^*(1 + \delta))c} \right) \left(\frac{1 - p_1^*}{p_1^*} \right)^{\frac{1}{c}} \left(\frac{p}{1 - p} \right)^{\frac{1}{c}} \quad (\text{A10})$$

their expected payoff is

$$EU_{deviatesep}(p) = W + (A(1 - p) - Bp) - c(\tilde{B} - \tilde{B}_s^{sep}(p))p.$$

If they deviate to choose full pooling securitization, where

$$B_s^* = \tilde{B} + A_s^*; \quad A_s^* = \frac{\tilde{B} \int_{p_2^*}^1 t d\mu(t)}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}^*(t)}, \quad (\text{A11})$$

the expected payoff is

$$EU_{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}^*(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}^*(t)}.$$

Define

$$\Phi(p) = \frac{\tilde{B}\delta p}{1 - p(1 + \delta)} - cp \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right).$$

It is easy to show that $\Phi(p)$ is a convex function. Since $\Phi(0) = \Phi(p_1^*) = 0$, then, $\Phi(p) \leq 0 \forall p \in [0, p_1^*]$. That is

$$\frac{\tilde{B}\delta p}{1 - p(1 + \delta)} < c \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) p$$

Define

$$\Psi(p) = cp \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) - \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}^*(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}^*(t)}.$$

It is easy to see that function $\Psi(p)$ is a concave function of p . Then $\frac{\partial \Psi(p)}{p} \big|_{p < p_2^*} > \frac{\partial \Psi(p)}{p} \big|_{p = p_2^*}$ and

$$\begin{aligned} \frac{\partial \Psi(p)}{p} \big|_{p = p_2^*} &= \tilde{B} \left(c - \left(c + \frac{1}{1 - p_2^*} \right) \left(1 - \frac{\delta}{c(1 - p_1^*(1 + \delta))} \right) \left(\frac{(1 - p_1^*)p_2^*}{p_1^*(1 - p_2^*)} \right)^{\frac{1}{c}} \right. \\ &\quad \left. + \frac{1}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}^*(t)} \right). \end{aligned}$$

For any $p_1^* \in \mathcal{L}$, it follows that $\frac{\partial \Psi(p)}{p} > 0$ for all $p < p_2^*$ if condition (21) holds. Therefore, $\Psi(p)$ is an increasing function of p . So $\Psi(p) \leq \Psi(p_2^*) = 0$ for all $p < P_2^*$; that is

$$cp \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) < \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}^*(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}^*(t)},$$

.

Since

$$\frac{\tilde{B}\delta p}{1 - p(1 + \delta)} < c \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) p < \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}^*(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}^*(t)},$$

$$EU_r(p) > EU_{deviatesep}(p), \quad EU_r(p) > EU_{deviatepool}(p)$$

Therefore, insurers with types in the range $[0, p_1^*]$ will not deviate to choose either separating partial securitization or pooling full securitization. Iteratively applying the D1 refinement, they will also not deviate to choose some other arbitrary securitization contract.

Now consider insurers with types in the range $[p_1^*, p_2^*]$. If they choose partial securitization contracts given by (A10), the expected payoff is $EU_s^{sep}(p) = W + (A(1-p) - Bp) - c(\tilde{B} - B_s^{sep}(p))p$.

If they deviate to choose full reinsurance contracts given by Proposition 1, the expected payoff is

$$EU_{deviatere}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p(1+\delta)}.$$

If they deviate to choose pooling securitization given by (A11), the expected payoff is

$$EU_{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}(t)}.$$

Since $\Phi(p)$ is a convex function with $\Phi(0) = \Phi(p_1^*) = 0$, $\Phi(p) > 0$ for $p > p_1^*$, that is, $\frac{\tilde{B}\delta p}{1-p(1+\delta)} > cp(\tilde{B} - \tilde{B}_s^{sep}(p))$. Thus $EU_{deviatere}(p) < EU_s^{sep}(p)$.

Also, when $p < p_2^*$, it follows that $c(\tilde{B} - B_s^{sep}(p))p < \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}(t)}$. Hence, $EU_s^{sep}(p) > EU_{deviatepool}(p)$. Therefore, insurers with types in the range $[p_1^*, p_2^*]$ will choose neither full reinsurance nor full pooling securitization. Because the Spence-Mirrlees single-crossing condition holds, the “local” incentive compatibility condition (A4) for the partial securitization contracts ensures that an insurer with risk type $p \in [p_1^*, p_2^*]$ will also not deviate to choose some other type’s partial securitization contract. Finally, iteratively applying the D1 refinement, they will also not deviate to choose some other arbitrary securitization contract that is not chosen by another type.

We now consider the insurers with types in the range $[p_2^*, 1]$.

If they choose pooling securitization given by (A11), the expected payoff is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}(t)}.$$

If they deviate to choose full reinsurance, the expected payoff is

$$EU_{deviatere} = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p(1+\delta)}.$$

By the property of the function $\Phi(p)$, it is easy to show that, when $p > p_2^* > p_1^*$, $\Phi(p) > 0$. Thus,

$$\frac{\tilde{B}\delta p}{1-p(1+\delta)} > cp(\tilde{B} - \tilde{B}_s^{sep}(p)).$$

By the property of function $\Psi(p)$, it is easy to show that, when $p > p_2^* > p_1^*$,

$$cp \left(\tilde{B} - \tilde{B}_s^{sep}(p) \right) > \tilde{B} \frac{\int_{p_2^*}^1 t d\mu_{p_2^*}(t) - p}{1 - \int_{p_2^*}^1 t d\mu_{p_2^*}(t)}.$$

As a result, $EU_s^{pooling}(p) > EU_{deviatere}(p)$. The insurers, therefore, will not deviate to choose full reinsurance. It follows from the results of Proposition 4 that the insurers on this interval would not choose partial securitization. There are multiple possible PBEs, where the thresholds $\{p_1^*, p_2^*\} \in \mathcal{G}$ and $p_1^* < p_2^*$.

Now consider the case where $p_1^* > p_2^*$ so that partial securitization is sub-optimal for insurers. In this case, we conjecture a PBE with two partitions, where insurers with type in the range $[0, p_3^*]$ choose full reinsurance, while insurers with types in the range $[p_3^*, 1]$ choose pooling full securitization. We are, therefore, in the scenario as characterized by Part 3 of Proposition 3.

2. Suppose $\delta > c$. It follows that full reinsurance is the sub-optimal choice for all insurers. Consequently, we are in the scenario as characterized by Proposition 4. ■

Online Appendix B: Index-Linked Securitization

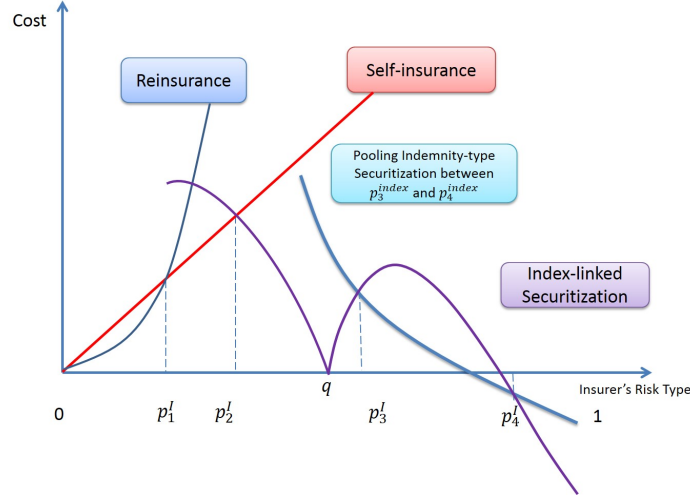
We now extend our model to incorporate index-linked securities whose payouts are tied to the loss of an index, rather than the loss of a particular insurer. Specifically, there is a publicly observable index I that incurs a loss with probability q , where q is *public information*. The index-linked securitization contract, (A_{index}, B_{index}) , specifies the premium, A_{index} , received by investors and the payment, B_{index} , made by investors if the index incurs a loss. Because the index loss probability, q , is public information, and capital markets are competitive, we must have $A_{index} = qB_{index}$. Further, the presence of the fixed financial distress cost, C , implies that it is optimal for an insurer to choose full insurance, that is, the insurer sets, $B_{index} = \tilde{B} + A_{index}$. Hence,

$$A_{index} = \frac{q}{1-q} \tilde{B} \tag{A12}$$

The index contract can mitigate the adverse selection problem suffered by the securitization contract that is tied to an insurer's loss—that is, an indemnity-based contract—since the index loss probability, q , is public information and known by investors. However, an index-linked contract introduces basis risk because the loss distribution of the index need not be perfectly correlated with the loss distribution of an insurer. We model the imperfect correlation between insurer loss distributions and the index loss distribution as follows.

For an insurer with loss probability $p < q$, the set of states of the world under which the insurer incurs a loss is a *subset* of the set of states under which the index incurs a loss. Consequently, the probability that the index incurs a loss, but the insurer does not is $q - p$, and the *joint probability* that both the insurer and the index incur losses is simply p . Analogously, for an insurer with loss

Figure 11: Partition Equilibrium with Index Securitization



probability $p > q$, the set of states under which the insurer incurs a loss is a *superset* of the set of states under which the index incurs a loss. Hence, the probability that the insurer incurs a loss, but the index does not is $p - q$, and the *joint probability* that both incur losses is q .

The cost to an insurer with loss probability $p < q$ from choosing the index contract to transfer its risk arises from the fact that it must pay a higher premium that corresponds to the loss probability of the index, q , rather than its true loss probability, p . The cost to an insurer with loss probability $p > q$ stems from basis risk, that is, the insurer will not receive any payout if it incurs a loss, but the index doesn't in which case it is insolvent and incurs the financial distress cost, C . On the other hand, an insurer with loss probability $p > q$ benefits from the fact that it pays a premium that corresponds to the index loss probability, q , rather than its own true loss probability, $p > q$.

It follows from the above and (A12) that the expected cost of the index contract for an insurer with risk type, p , is

$$\text{Expected Cost of Index Contract} = \underbrace{\left(\frac{q}{1-q} - \frac{p}{1-p} \right) \tilde{B}}_{\text{expected benefit/cost of index contract premium}} + \underbrace{(p-q)_+ C}_{\text{expected cost of basis risk}}, \quad (\text{A13})$$

where $(p-q)_+ = \max(p-q, 0)$. By (A13), the expected cost of the index-linked securitization contract is a non-monotonic function of the insurer's risk type p as the "purple curve" shown in Figure 11. Specifically, the expected cost decreases with insurer's risk type when it is less than the index loss probability, q ; increases with insurer's risk type when $q < p < 1 - \sqrt{\frac{\tilde{B}}{C}}$; and decreases with insurer's risk type when $p > 1 - \sqrt{\frac{\tilde{B}}{C}}$. The expected cost equals zero in two cases: (i) the insurer's risk type, p , is equal to the index loss probability, q , so that the insurer faces no basis risk and pays a premium that corresponds to its true loss probability; and (ii) the insurer's risk type, p , is equal to, \bar{p} , so that the insurer's benefit from paying a premium associated to the index loss probability exactly offsets the cost of basis risk, where \bar{p} satisfies $(\frac{\bar{p}}{1-\bar{p}} - \frac{q}{1-q})\tilde{B} = (\bar{p} - q)C$.

The following proposition shows that the PBE of the risk transfer game continues to have a partition form.

Proposition 6 (Partition Equilibrium with Index-linked Securitization). *There exist four triggers, $p_1^I, p_2^I, p_3^I, p_4^I$ with $p_1^I \leq p_2^I \leq p_3^I \leq p_4^I$ such that the unique PBE (under the D1 refinement) is characterized as follows. Insurers with types in the interval $[0, p_1^I]$ choose full reinsurance, insurers with types in the interval $[p_1^I, p_2^I]$ choose full retention, insurers with types in the intervals $[p_2^I, p_3^I]$ and $[p_4^I, 1]$ choose index-linked securitization and insurers with types in the interval $[p_3^I, p_4^I]$ choose full pooling “indemnity-based” securitization.*

The above proposition shows that the PBE is, in general, characterized by five intervals. Insurers with risk types in the lowest interval choose reinsurance, followed by retention, index-linked securitization, indemnity-based securitization, and index-linked securitization again respectively.⁹ As shown by Figure 11, the interplay between the costs associated with basis risk due to index-linked securitization, and the costs associated with pooling with higher risk types in indemnity-based securitization, causes the relative benefit of index-based vis-a-vis indemnity-based securitization to vary *non-monotonically* with the insurer’s risk type. Consequently, the interval over which securitization—index-based or indemnity-based—is chosen consists of three subintervals in general. The lower and higher subintervals of risk types choose index-based securitization, whereas the intermediate subinterval of risk types choose indemnity-based securitization. Importantly, however, the key implication of Proposition 3 is robust to the incorporation of index-linked securitization. There exists a trigger risk level such that insurers with risks above the trigger choose securitization; index-linked or indemnity-based.

Proof of Proposition 6

Proof. The proof follows using logic that is quite similar to the proof of Proposition 3. First, consider the scenario where $\tilde{B}\delta < C \leq \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$ and $q_1 < q < q_2$, where

$$q_1 = \frac{p_1^F \tilde{B} + p_1^F(1-p_1^F)C}{\tilde{B} + p_1^F(1-p_1^F)C} \quad (\text{A14})$$

$$q_2 = \frac{p_2^F \tilde{B} + p_2^F(1-p_2^F)C}{\tilde{B} + p_2^F(1-p_2^F)C} \quad (\text{A15})$$

It follows from the proof of Proposition 1 and the properties of the expected cost function of index-linked securitization that insurers with low risk type prefer full reinsurance to both full self-insurance and index-linked securitization. Insurers with risk type higher than \bar{p} prefer index-linked securitization because the benefit from paying the index-linked premium rather than the indemnity-based premium is greater than the cost of basis risk, where $(\frac{\bar{p}}{1-\bar{p}} - \frac{q}{1-q})\tilde{B} = (\bar{p} - q)C$. In addition, the net benefit from index-linked securitization approaches infinity as insurer’s risk type approaches one. Consequently,

⁹Depending on the relative magnitudes of the financial distress cost and insurer loss size, either the interval over which retention is chosen or the interval over which reinsurance is chosen could be empty. Further, depending on the index loss probability, q , one or more of the subintervals over which index-linked securitization is chosen could be empty. Please see the proof of Proposition 6 for the detailed conditions.

there exists a threshold level of risk above which index-linked securitization dominates indemnity-based securitization.

By the above arguments, it suffices to consider candidate equilibria with five partitions such that insurers with risk types in the lowest subinterval choose reinsurance, followed by retention, index-linked securitization, indemnity-based securitization, and index-linked securitization again respectively. Accordingly, we examine the partition equilibrium defined by four thresholds p_1^I , p_2^I , p_3^I and p_4^I . Specifically, $p_1^I = p_1^F$ is the indifference point between full reinsurance and full self-insurance as defined by Proposition 1. p_2^I is the indifference point between full self-insurance and index-linked securitization such that

$$\left(\frac{q}{1-q} - \frac{p_2^I}{1-p_2^I}\right)\tilde{B} = p_2^I C \quad (\text{A16})$$

p_3^I and p_4^I are the two indifference points between index-linked securitization and pooling of indemnity-based securitization, which determines the subinterval where insurers choose pooling of indemnity-based securitization and are jointly determined by the following:

$$\left(\frac{q}{1-q} - \frac{p_3^I}{1-p_3^I}\right)\tilde{B} + (p_3^I - q)C = \tilde{B} \frac{\int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t) - p_3^I}{p_4^I - \int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t)} \quad (\text{A17})$$

$$\left(\frac{q}{1-q} - \frac{p_4^I}{1-p_4^I}\right)\tilde{B} + (p_4^I - q)C = \tilde{B} \frac{\int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t) - p_4^I}{p_4^I - \int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t)} \quad (\text{A18})$$

where $d\mu_{p_3^I}(t) = \frac{dF(t)}{F(p_4^I) - F(p_3^I)}$.

(i) Under restrictions on reasonable off-equilibrium beliefs along the lines of the D1 refinement as in the proof of Proposition 1, insurers with types in the interval $[0, p_1^F]$ prefer full reinsurance to full self-insurance and any other indemnity-based securitization. Further, insurers with types in this interval will not deviate from full reinsurance to index-linked securitization because, for any $0 < p < p_1^F$,

$$\frac{\tilde{B}\delta p}{1 - (1 + \delta)p} < \frac{\tilde{B}\delta p_1^F}{1 - (1 + \delta)p_1^F} < p_2^I C = \left(\frac{q}{1-q} - \frac{p_2^I}{1-p_2^I}\right)\tilde{B} < \left(\frac{q}{1-q} - \frac{p}{1-p}\right)\tilde{B}$$

(ii) We now show it is optimal for insurers with types in the interval $[p_1^F, p_2^I]$ to choose full self-insurance. Since $q_1 < q < q_2$, we have $p_1^F < p_2^I < p_2^F$. By the proof of Proposition 1, insurers with types in this interval prefer self-insurance to reinsurance as well as other indemnity-base securitization. Also, insurers in this interval will not deviate to choose index-linked securitization because for any $p \in [p_1^F, p_2^I]$

$$pC < p_2^I C = \left(\frac{q}{1-q} - \frac{p_2^I}{1-p_2^I}\right)\tilde{B} < \left(\frac{q}{1-q} - \frac{p}{1-p}\right)\tilde{B}$$

(iii) We now show that it is optimal for insurers with types in interval $[p_2^I, p_3^I]$ to choose index-linked securitization. By the proof of Proposition 1, it is sub-optimal for insurers to choose either

reinsurance and self-insurance. It is easy to show that $p_2^I < q < p_3^I$ since $q_1 < q < q_2$. It is sub-optimal for insurers to choose pooling indemnity-type securitization with all higher risk insurers. Also because pooling indemnity-type securitization within two separate intervals are not incentive compatible, it is optimal for insurers with types in interval $[p_2^I, p_3^I]$ to choose index-linked securitization because for any $p \in [p_2^I, p_3^I]$

$$\left(\frac{q}{1-q} - \frac{p}{1-p}\right)\tilde{B} + (p-q)_+C < \tilde{B} \frac{\int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t) - p}{p_4^I - \int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t)}$$

(iv) Similarly, we can show that insurers with types in the interval $[p_3^I, p_4^I]$ prefer pooling indemnity-based securitization. Under the restrictions on reasonable off-equilibrium beliefs along the lines of the D1 refinements, insurers will not deviate to choose any other indemnity-based securitization. Also because for any $p \in [p_3^I, p_4^I]$

$$\tilde{B} \frac{\int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t) - p}{p_4^I - \int_{p_3^I}^{p_4^I} t d\mu_{p_3^I}(t)} < \left(\frac{q}{1-q} - \frac{p}{1-p}\right)\tilde{B} + (p-q)_+C \quad (\text{A19})$$

it is optimal for insurers to choose pooling indemnity-type securitization.

(v) We now show it is optimal for insurers with types in the interval $[p_4^I, 1]$ to choose index-linked securitization. The benefits from paying index-linked securitization dominates the cost of basis risk for insurers with sufficiently high risk. Thus the net benefits from index-linked securitization will exceed the subsidization benefit from pooling indemnity-type securitization with lower risk insurers. Consequently, insurers prefer index-linked securitization to all other alternatives.

For completeness, we next analyze other cases where one or more of the intervals that characterize the partition equilibrium described above may be empty.

1(a). Suppose $C < \tilde{B}\delta$, it is sub-optimal for an insurer to choose reinsurance. By Proposition 3, insurers with types below p_2^F choose full retention while insurers with types above p_2^F choose pooling indemnity type securitization. If $q < q_2$, there exist unique p_2^I , p_3^I and p_4^I such that $p_2^I < p_2^F < p_3^I$. According to the properties of cost function of index-linked securitization contract, intermediate risk insurers with types in the interval $[p_2^I, p_3^I]$ and $[p_4^I, 1]$ prefer index-linked securitization contract to either retention or pooling securitization with indemnity trigger. Consequently, there exists a unique PBE (under the D1 refinement) with four partitions determined by the thresholds p_2^I , p_3^I and p_4^I . Insurers with types in the interval $[0, p_2^I]$ choose full self-insurance, insurers with types both in the interval $[p_2^I, p_3^I]$ and $[p_4^I, 1]$ choose index-linked securitization, and insurers with types in the interval $[p_3^I, p_4^I]$ choose full pooling “indemnity-based” securitization.

1(b). Suppose $C < \tilde{B}\delta$ and $q > q_2$, we have $p_2^I > p_2^F$. It is too costly to choose index-linked securitization rather than indemnity-type securitization. There exists two thresholds p_2^{I*} and p_4^{I*} , where p_2^{I*} is the indifference point between full retention and pooling of indemnity-type securitization and p_4^{I*} is the indifference point between pooling of indemnity-type securitization and index-linked

securitization. p_2^{I*} and p_4^{I*} are jointly determined by

$$Cp_2^{I*} = \tilde{B} \frac{\int_{p_4^{I*}}^{p_2^{I*}} td\mu_{p_2^{I*}}(t) - p_2^{I*}}{p_4^{I*} - \int_{p_2^{I*}}^{p_4^{I*}} td\mu_{p_2^{I*}}(t)} \quad (\text{A20})$$

$$\left(\frac{q}{1-q} - \frac{p_4^{I*}}{1-p_4^{I*}}\right)\tilde{B} + (p_4^{I*} - q)C = \tilde{B} \frac{\int_{p_4^{I*}}^{p_2^{I*}} td\mu_{p_2^{I*}}(t) - p_4^{I*}}{p_4^{I*} - \int_{p_2^{I*}}^{p_4^{I*}} td\mu_{p_2^{I*}}(t)} \quad (\text{A21})$$

where $d\mu_{p_2^{I*}}(t) = \frac{dF(t)}{F(p_4^{I*}) - F(p_2^{I*})}$.

Consequently, there exists a unique PBE (under the D1 refinement) with three partitions determined by the threshold p_2^{I*} and p_4^{I*} . Insurers with types in the interval $[0, p_2^{I*}]$ choose full self-insurance, insurers with types in the interval $[p_2^{I*}, p_4^{I*}]$ choose full pooling “indemnity-based” securitization, and insurers with types in the interval and $[p_4^{I*}, 1]$ choose index-linked securitization.

2(a). Suppose $\tilde{B}\delta < C < \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$ and $q < q_1$ where q_1 is defined by (A14), there exist unique p_5^I defined by

$$\left(\frac{q}{1-q} - \frac{p_5^I}{1-p_5^I}\right)\tilde{B} = \frac{\tilde{B}p_5^I\delta}{1-p_5^I(1+\delta)} \quad (\text{A22})$$

such that $p_5^I < p_1^F$. It follows from Proposition 1 that insurers with types in the interval $[0, p_5^I]$ prefer full reinsurance to full self-insurance. In addition, insurers with types in the interval $[0, p_5^I]$ also prefer full reinsurance to index-linked securitization. As stated earlier, insurers with types in the interval $[p_5^I, p_3^I]$ prefer index-linked securitization contract to either retention or pooling securitization with indemnity trigger, and insurers with types in the interval $[p_3^I, 1]$ prefer pooling securitization with indemnity trigger to other alternatives. Consequently, the unique equilibrium (under the D1 refinement) is characterized by four intervals. Specifically, insurers with types in the interval $[0, p_5^I]$ choose full reinsurance, insurers with types both in the interval $[p_5^I, p_3^I]$ and $[p_4^I, 1]$ choose full index-linked securitization, and insurers with types in the interval $[p_3^I, p_4^I]$ choose full pooling “indemnity-based” securitization.

2(b). Suppose $\tilde{B}\delta < C < \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$ and $q > q_2$. Reinsurance dominates retention and index-linked securitization for insurers with risk type lower than p_1^F . Hence, the situation for insurers with risk types greater than p_1^F is the same as the scenario stated in proof of 1(b). Consequently, there exists a unique PBE (under the D1 refinement) with four partitions determined by three thresholds, p_1^F , p_2^{I*} and p_4^{I*} . Specifically, insurers with types in the interval $[0, p_1^F]$ choose full reinsurance, insurers with types in the interval $[p_1^F, p_2^{I*}]$ choose full self-insurance, insurers with types in the interval $[p_2^{I*}, p_4^{I*}]$ choose full pooling “indemnity-based” securitization, and insurers with types in the interval and $[p_4^{I*}, 1]$ choose index-linked securitization.

3. Suppose $C > \frac{\tilde{B}\delta}{1-p_2^F(1+\delta)}$, that is $p_2^F < p_1^F$. It is suboptimal to choose self-insurance. Define

$$q_3 = \frac{p_3^F \tilde{B} + p_3^F(1 - p_3^F)C}{\tilde{B} + p_3^F(1 - p_3^F)C}$$

Suppose $q < q_3$, there exists one indifference point between reinsurance and index-linked securitization p_5^I , where $p_5^I < q$. Thus it is clear that insurers with risk below p_5^I prefer reinsurance to other alternatives. For insurers with risk higher than p_5^I , we are, thus, in the scenario described in the first case for insurer with risk above p_1^F . Consequently, there exists three thresholds, p_5^I , p_3^I and p_4^I such that insurers with types in the interval $[0, p_5^I]$ choose full reinsurance, insurers with types in the interval $[p_5^I, p_3^I]$ and $[p_4^I, 1]$ choose index-linked securitization, and insurers with types in the interval $[p_3^I, p_4^I]$ choose full pooling “indemnity-based” securitization. ■