# Capital, Systemic Risk, Insurance Prices and Regulation


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**Abstract**

We develop a unified equilibrium model of competitive insurance markets that incorporates the demand and supply of insurance as well as insurers' asset and liability risks. Insurers' assets may be exposed to both idiosyncratic and systemic shocks. We obtain new insights into the relationship between insurance premia and insurers' internal capital that potentially reconcile the conflicting predictions of previous theories that investigate the relation using partial equilibrium frameworks. Equilibrium effects lead to a non-monotonic U-shaped relation between insurance price and internal capital. We study the effects of systemic risk on the optimal asset and liquidity management by insurers as well as risk-sharing between insurers and insurees. In the first-best benchmark scenario, we show that, when systemic risk is low, both insurees and insurers hold no liquidity reserves, insurees are fully insured, and insurers bear all the systemic risk. When systemic risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share systemic risk with insurers. When systemic risk is high, however, both insurees and insurers hold some liquidity reserves, and insurees partially share systemic risk with insurers. The first best asset and liquidity management policies as well as the systemic risk allocation can be implemented through a regulatory intervention policy that combines a minimum liquidity requirement, *ex post* taxation contingent on the aggregate state, comprehensive insurance policies, and reinsurance. We also provide implications for the solvency regulation of insurers facing systemic risk.

## 1 Introduction

Financial insitutions such as insurers and banks are usually required to hold sufficient equity capital on the liability side of their balance sheets and liquid reserves on the asset side as a buffer against the risk of insolvency, especially when their loss portfolios are imperfectly diversified and/or returns on their assets shrink dramatically. The financial crisis of 2007-2008 was precipitated by the presence of insufficient liquidity buffers and excessive debt levels in the financial system that made financial institutions vulnerable to large aggregate negative shocks. In the context of insurers, the imperfect incorporation of the externality created by systemic risk on their investment decisions may lead them to hold insufficient liquidity buffers. The resulting increase in insurer insolvency risk has an impact on the amount of insurance they can supply to insurees and, therefore, the degree of risk-sharing in the insurance market. Indeed, empirical evidence shows that, in response to Risk Based Capital (RBC) requirements, under-capitalized insurers not only increase their capital holdings to meet minimum capital requirements, but also take more risks to reach higher returns (Cummins and Sommer, 1996; Shim, 2010; Sager, 2002). Insurers' propensity to "reach for yield" contributes to their overall insolvency risk.[1] Systemic risk may, therefore, lead to a misallocation of capital and suboptimal risk sharing among insurees and insurers. To the best of our knowledge, however, the above arguments have yet to be theoretically formalized in an equilibrium framework that endogenizes the demand and supply of insurance as well as insurers' asset and liability risks. Such a framework could potentially shed light on the optimal regulation of insurance firms taking into account both the asset and liability sides of insurers' balance sheets.

We contribute to the literature by developing an equilibrium model of competitive insurance markets where insurers' assets may be exposed to idiosyncratic and aggregate/systemic shocks. In the unregulated economy, we show that the equilibrium insurance price varies non-monotonically in a U-shaped manner with the level of internal capital held by insurers. In other words, the insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the internal capital is above the threshold. We thereby reconcile conflicting predictions in previous literature on the relation between insurance premia and internal capital that are obtained in partial equilibrium frameworks that focus on either demand-side or

[1] Cox(1967) describes bank's tendency to invest in high risk loans with higher returns. Becker and Ivashina (2013)) support insurers' reaching for yield behavior by examining insurers' bond investment decisions

supply-side forces. We also obtain the additional testable implications that an increase in insurers' asset risk, which raises the default probability, raises insurance premia and reduces coverage. We then proceed to derive insights into the solvency regulation of insurers by studying the benchmark "first best" economy where systemic risk is fully internalized. We analyze the effects of systemic risk on the allocation of insurer capital to liquidity reserves and risky assets as well as risk sharing among insurees and insurers. We show that, when systemic risk is below a threshold, it is optimal for insurers and insurees to hold zero liquidity reserves, insurees are fully insured, and insurers bear all systemic risk. When systemic risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share systemic risk with insurers. When the systemic risk is high, however, both insurees and insurers hold nonzero liquidity reserves, and insurees partially share systemic risk with insurers. We demonstrate that the first best allocations can be implemented through regulatory intervention that comprises of comprehensive insurance policies that combine insurance and investment, reinsurance, a minimum liquidity requirement, and ex post budget-neutral taxation and subsidies contingent on the realized aggregate state.

Our model features two types of agents: a continuum of ex ante identical, risk averse insurees each facing a risk of incurring a loss in their endowment of capital, and a continuum of ex ante identical risk neutral insurers each endowed with a certain amount of internal "equity" capital. There is a storage technology/safe asset that provides a constant risk free return and a continuum of risky assets that generate higher expected returns than the risk free asset. Although both insurees and insurers can directly invest in the safe asset, only insurers have access to the risky assets. In addition to their risk-sharing function, insurance firms, therefore, also serve as intermediaries to channel individual capital into productive risky assets. Insuree losses are independently and identically distributed, but insurers' assets are exposed to aggregate/systemic risk. Specifically, a certain proportion of insurers is exposed to a common asset shock, while the remaining insurers' asset risks are idiosyncratic. A priori, it is unkown whether a particular insurer is exposed to the common or idiosyncratic shock. The proportion of insurers who are exposed to the common shock is, therefore, the natural measure of the systemic risk in the economy. Insurees invest a portion of their capital in the risk-free asset and use the remaining capital to purchase insurance. Insurers invest their internal capital and the external capital raised from selling insurance claims in the risk-free and risky assets.

We first derive the market equilibrium of the unregulated economy. In the unregulated economy, insurees make their insurance purchase decisions rationally anticipating insurers' investment strategy and default risk given their observations of insurers' internal capital, the size of the insurance pool, and the insurance price (the premium per unit of insurance). *Ceteris paribus*, an increase in insurers' internal capital or a decrease in asset risk increases the demand for insurance due to the lower likelihood of insurer insolvency. An increase in the risk of insuree losses leads to a decrease in insurance demand because it increases the proportion of insurees who suffer losses and, therefore, decreases the amount that each insuree recovers if he incurs a loss, but the insurance company is insolvent. Insurers, in turn, choose how much insurance to sell taking as given the insurance price and the loss profile of their insuree pools. Competition among insurers ensures that, in equilibrium, each insurer earns zero expected economic profits that incorporate the opportunity costs of internal capital that is used to make loss payments when insurers are insolvent. An increase in the insurance price, therefore, lowers the amount of insurance that each insurer sells in equilibrium leading to a downward sloping "competitive" insurer supply curve. An increase in the internal capital or an increase in asset risk, *ceteris paribus*, increases the opportunity costs of providing insurance, thereby increasing the amount of insurance that provides zero economic profits to insurers in competitive markets. An increase in the loss proportion increases the cost of claims, thereby pushing up the competitive supply level.

In competitive equilibrium, the insurance price is determined by market clearing—the demand for insurance must equal the supply—and zero economic profits for insurers. The insurance demand curve and the competitive supply curve are both downward sloping with the demand curve being steeper due to the risk aversion of insurees. Consequently, any factor that increases the insurance demand curve, *ceteris paribus*, decreases the equilibrium price, while a factor that increases the competitive supply curve has a positive effect. We analytically characterize the competitive equilibrium of the economy and explore its comparative statics.

We demonstrate that there is a U-shaped relation between the insurance price and insurers' internal capital. Specifically, the insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the internal capital is above the threshold. The intuition for the non-monotonic U-shaped relation hinges on the influence of both demand-side and supply-side factors. An increase in insurers' internal capital increases the competitive supply of insurance coverage because of the increased opportunity costs of internal capital. Because insurers are risk-neutral, however, the change in the competitive supply of insurance coverage is linear in the internal capital. On the demand side, an increase in insurers' internal capital increases insurers' insolvency buffer, thereby increasing the demand for insurance coverage. An increase in internal capital also increases the funds available for investment that further has a positive impact on the demand for insurance. The demand, however, is concave in the internal capital due to insurees' risk aversion. Because the insurance supply varies linearly with capital, while the insurance demand is concave, there exists a threshold level of capital at which the demand effect equals the supply effect. Consequently, the demand effect dominates the supply effect so that the equilibrium premium rate goes down when the internal capital level is lower than the threshold. When the capital is above the threshold, the supply effect dominates so that the premium rate increases.

As suggested by the above discussion, equilibrium effects that integrate both demand side and supply side forces play a central role in driving the U-shaped relation between the insurance price and insurer capital. Our results, therefore, reconcile and further refine the opposing predictions for the relation in the literature that stem from a focus on only demand or only supply effects in partial equilibrium frameworks. Specifically, the "capacity constraints" theory, which focuses on the supply of insurance, predicts a negative relationship between insurance price and capital by assuming that insurers are free of insolvency risk (Gron, 1994; Winter, 1994). In contrast, the "risky debt" theory incorporates the default risk of insurers, but predicts a positive relationship between insurance price and capital (Doherty and Garven, 1986; Cummins, 1988, Cummins and Danzon, 1997). Empirical evidence on the relationship is also mixed. We make the simple, but fundamental point that the insurance price reflects the effects of capital on both the demand for insurance and the supply of insurance in equilibrium. We show that the relative dominance of demand-side and supply-side forces depends on the level of internal capital, thereby generating a U-shaped relation between price and internal capital.

Next, we show that an increase in insurers' asset risk, which increases their insolvency probability, increases the insurance price and reduces the insurance coverage in equilibrium. The intuition for the results again hinges on a subtle interplay between the effects of an increase in asset risk on insurance supply and demand. A positive shock to insurers' asset risk, *ceteris paribus*, has the *direct* effect of increasing the competitive supply of insurance coverage, that is, the level of insurance supply at which insurers earn zero economic profits. Consequently, the amount of funds available to pay loss claims in distress increases, thereby having the *indirect* effect of increasing the demand for insurance. On the other hand, an increase in the asset risk increases the insurers' insolvency probability that has a negative effect on the demand for insurance. We show that, under reasonable conditions, the direct effect outweighs the indirect effect. Consequently, an increase in asset risk reduces insurance demand, but increases the zero-economic-profit supply level, thereby increasing the insurance price and decreasing the coverage level in equilibrium. Our results imply that the response to the increased asset risk of insurance firms is the shift of insuree's capital accumulation from indirect investment in risky assets to direct storage in safe assets.

We then proceed to analyze the implications of our framework for the solvency regulation of insurers by analyzing the benchmark "first best" economy in which systemic risk is fully internalized. We derive the optimal allocation of insurer capital between the safe asset (liquidity reserves) and risky assets as well as the sharing of risk between insurers and insurees. When the systemic risk is low, there is sufficient aggregate capital in the economy to provide full insurance to insurees so that insurers bear all the aggregate risk. Further, because the expected return from risky assets exceeds the risk-free return, it is optimal to allocate all capital to risky assets so that neither insurers nor insurees have holdings in the risk-free asset. When systemic risk takes intermediate values, insurees cannot be provided with full insurance because of the limited liability of insurers in the bad aggregate state. Consequently, insurers and insurees share aggregate risk, but it is still optimal to exploit the higher expected surplus generated by the risky assets so that all the capital in the economy is invested in the risky assets. When systemic risk is very high, however, risk-averse insurees would bear excessively high losses in the bad aggregate state if all capital were invested in risky assets. Consequently, both insurees and insurers hold positive liquidity reserves, and share aggregate risk.

We demonstrate that a regulator/social planner can implement the first-best allocation policies through a combination of comprehensive insurance policies sold by insurers that combine insurance with investment, reinsurance, a minimum liquidity requirement, and ex post budget-neutral taxation that is contingent on the aggregate state. The comprehensive insurance policies provide direct access to the risky assets for insurees. Reinsurance achieves risk-sharing among insurers, while ex post taxation transfers funds from solvent to insolvent insurers. The minimum liquidity requrement, which is only imposed when systemic risk exceeds a threshold, forces insurers to maintain the first best level of liquidity reserves.

The plan for the paper is as follows. We further discuss related literature in Section 2. We present the model in Section 3 and derive the equilibrium of the unregulated economy in Section 3.1. We analyze the impact of systemic risk and regulation in Section 4. Section 5 and we relegate all proofs to the Appendix.

## 2 Related Literature

Our paper is related to two lines of literature that investigate the relation between capital and price. The first branch proposes the "capacity constraint" theory, which assumes that insurers are free from insolvency risk. The prediction of an inverse relation between insurance price and capitalization crucially hinges on the assumption that insurers are limited by regulations or by infinitely risk averse policyholders so that they can only sell an amount of insurance that is consistent with zero insolvency risk (e.g.,Gron, 1994; Winter, 1994). Winter(1994) explains the variation in insurance premia over the "insurance cycle" using a dynamic model. Empirical tests using industry-level data prior to 1980 support the predicted inverse relation between insurance capital and price, but data from the 1980s do not support the prediction. Gron (1994) finds support for the result using data on short-tail lines of business. Cagle and Harrington (1995) predict that the insurance price increases by less than the amount needed to shift the cost of the shock to capital given inelastic industry demand with respect to price and capital.

Another significant stream of literature—the "risky corporate debt" theory—incorporates the possibility of insurer insolvency and predicts a positive relation between insurance price and capitalization (e.g., Doherty and Garven, 1986; Cummins, 1988). The studies in this strand of the literature emphasize that, because insurers are not free of insolvency risk in reality, the pricing of insurance should incorporate the possibility of insurers' financial distress. Higher capitalization levels reduce the chance of insurer default, thereby leading to a higher price of insurance associated with a higher amount of capital. Cummins and Danzon (1997) show evidence that the insurance price declines in response to the loss shocks in the mid-1980s that depleted insurer's capital using data from 1976 to 1987. While the "capacity constraint" theory concentrates on the supply of insurance, “the pricing of risky debt" theory focuses on capital’s influence on the quality of insurance firms and, therefore, the demand for insurance. The empirical studies support the mixed results for different periods and business lines.

We complement the above streams of the literature by integrating demand-side and supply-side forces in an equilibrium framework. We show that there is a U-shaped relation between price and internal capital. In contrast with the literature on “risk debt pricing", which assumes an exogenous process for asset value, we endogenize the asset value which depends on the total invested capital including both internal capital and capital raised through the selling of insurance policies. Insurers’ assets and total liabilities are, therefore, simultaneously determined in equilibrium in our analysis.

Our paper is also related to the studies that examine the relation between capital holdings and risk taking of insurance companies. Cummins and Sommer (1996) empirically show that insurers hold more capital and choose higher portfolio risks to achieve their desired overall insolvency risk using data from 1979 to 1990. It is argued that insurers response to the adoption of RBC requirements in both property-liability and life insurance industry by increasing capital holdings to avoid regulation costs, and by investing in riskier assets to obtain high yields (e.g.,Baranoff and Sager, 2002; Shim, 2010). Insurers are hypothesized to choose risk levels and capitalization to achieve target solvency levels in response to buyers’ demands for safety. Our paper fits into the literature by studying the response of the market price to exogenous shocks to internal capital and assets in an equilibrium framework. Our results shed some light on insurance investment regulation. Relaxing liquidity constraints on insurers’ assets induces insurees to choose relatively greater self-insurance, thereby shrinking the insurance markets and, therefore, the channeling of insurance capital to value-creating assets.

Our paper also contributes to the literature on capital allocation and insurance pricing. Zanjani (2002) argues that price differences across markets are driven by different capital requirements to maintain solvency assuming that capital is costly to hold. Our paper endogenizes the cost of capital in terms of the opportunity cost of holding internal capital, which is used to pay for loss claims when insurers default. We highight insurees’ and insurers’ responses to internal capital shocks. Consequently, insurance prices reflect insurees’ demand for safety and insurers’ abilities to provide insurance with imperfect protection.

## 3 The Model

We consider a single-period economy with two dates 0 and 1. There is a single consumption/capital good. There are two types of agents: a continuum of measure 1 of risk-averse *insurees* or *policy holders* and a continuum of measure 1 of risk-neutral *insurance firms*. Each insuree is endowed with 1 unit of capital at date 0 and has a logarithmic utility function. Each insurance firm is endowed with $K$ units of "internal" capital. There is a storage technology/safe asset that is in sufficiently large supply that it provides a constant return of $R_f$ per unit of capital invested.

At date 1, an insuree $i$ can incur a loss $l \leq 1$ so that a portion of each insuree's endowment is at risk. Losses are independently and identically distributed across insurees. Each insuree's loss probability is $p$. At date 0, each insuree invests a portion of her capital in the safe asset and the remainder in buying an insurance policy at a premium $\kappa$ per unit of loss, where $\kappa$ is endogenously determined in equilibrium. Insurance markets are competitive so that insurees and insurers act as price takers by taking the insurance premium rate as given in making their decisions.

Insurers have internal capital $K$ and raise external capital by selling insurance policies. Each insurance firm $j$ has access to a risky technology that generates a return of $R_H$ per unit of invested capital with probability $1-q$ when it "succeeds" but $R_L < R_H$ with probability $q$ when it "fails." A proportion $1-\tau$of insurance firms are exposed to idiosyncratic technology shocks, that is, the technology shocks are independently and identically distributed for this group of insurance firms. The remaining proportion $\tau$ of insurers are, however, exposed to a *common* shock, that is, the technology shock described above is the *same* for these insurers. Although insurers know that a proportion $\tau$ of them is exposed to a common shock, an individual insurer does not know whether it is exposed to an idiosyncratic or common shock a priori. $\tau$ is a measure of the *systemic risk* in the economy.

We assume that

$$(1-q)R_H + qR_L \geq R_f. \tag{1}$$

The above condition ensures that the expected return on the risky project is at least as great as the risk-free rate. While policy holders can directly invest in the safe asset, only insurance firms have access to the production technology. Consequently, in addition to the provision of insurance to policy holders, insurance firms also play important roles as financial intermediaries who channel the capital supplied by policy holders to productive assets.

If the insurance premium per unit of loss is $\kappa$, the total external capital raised by an insurer $j$ is $\kappa C_j$, where $C_j$ is the face value of insurance claims sold by the insurer. Each insurer can invest its total capital, $K + \kappa C_j$ in a portfolio comprising of the risk-free storage technology and the risky project. In an autarkic economy with no regulation, it follows from condition (1) that it is optimal for risk-neutral insurance firms to invest their entire capital in the risky technology.

By our earlier discussion, the total liability of an insurer is $pC_j$ because a proportion $p$ of its pool of insurees incur losses. Insurers default if their total liability cannot be covered by the total investment returns when the risky technology fails, that is when

$$pC_j > (K + \kappa C_j)R_L. \tag{2}$$

In the event of default, the total available capital of an insurer is split up among insurees in proportion to their respective indemnities. The internal capital plays the role of a buffer that increases an insurer's capacity to meet its liabilities and, thereby, the amount of insurance it can sell. The cost of holding internal capital in our model is an opportunity cost, which refers to the returns from the invested internal capital that are depleted to pay out liabilities when insurers default.

Insurees observe the total capital, $K + \kappa C_j$, held by each insurer $j$. In making the decision on the level of insurance coverage to purchase, insurees rationally anticipate the possibility of default, and the amount they will be paid for a loss when insurers' asset returns are insufficient to pay out the aggregate loss claims as shown by (2).

### 3.1 The Equilibrium

We now derive the equilibrium of the insurance markets by analyzing the demand and supply of insurance by insurees and insurers, respectively. In equilibrium, the demand for insurance equals the supply.

### 3.1.1 Insurance Demand

Each insuree chooses its portfolio, which comprises of his investment in the safe asset (self-insurance) and his choice of insurance coverage, to maximize his expected utility. Without insurance, each insuree $i$'s expected utility is given by the autarkic utility level,

$$\text{Autarkic Utility } = p\ln(R_f - l) + (1-p)\ln(R_f). \tag{3}$$

Insurees are competitive and take the insurance price as given in making their purchase decisions. They observe the total capital held by insurers and, therefore, rationally anticipate the possibility that they may not be fully indemnified in the scenario where they incur losses, but the insurer is insolvent. Insurees also rationally incorporate insurers' investment portfolio choices in making their insurance demand decisions. As previously stated, insurers invest all their capital in the risky technology, thereby causing insurers to be likely to default in the "bad" state where the technology fails. The likelihood that insurees' loss claims may not be fully indemnified is then affected by the risk in the investment portfolio of insurance firms and the total liabilities insured by them. In general, the loss payment obtained by each insuree is determined by three factors: the proportion of insurees in the insurer's pool who incur losses, the total amount of capital held by the insurer, and the return of the insurer's investment project.

Given that insurees and insurers are ex ante identical, we focus on symmetric equilibria where insurees make identical portfolio choices and insurers have ex ante identical pools of insurees. Without loss of generality, therefore, we focus on a representative insurer. Suppose that the amount of coverage purchased by a representative insuree is $C_d$. If $\kappa$ is the insurance premium per unit of coverage and $C_s$ is the total face value of the insurance policies sold by the insurer, its total capital is $K + \kappa C_s$. The insurer's available capital if its project fails is, therefore, $(K + \kappa C_s)R_L$. Consequently, the payment received by each insuree who incurs a loss when the insurer's project fails is $\min(C_d, \frac{(K+\kappa C_s)R_L}{p})$. It is clear from our subsequent results that it is suboptimal for the insurer to sell so much coverage that it is unable to meet losses in the "good" state where its project succeeds. In the following, therefore, we assume this result to avoid unnecessarily complicating the exposition.

The insuree's optimal demand for insurance coverage maximizes its expected utility subject to its budget constraint, that is, it solves

$$\max_{C_d} \overbrace{p(1-q)\ln\left[(1-\kappa C_d)R_f - l + C_d\right]}^{\text{insuree incurs loss in insurer's "good" state}} +$$
$$\overbrace{pq\ln\left[(1-\kappa C_d)R_f - l + \min(C_d, \frac{(K+\kappa C_s)R_L}{p})\right]}^{\text{insuree incurs loss in insurer's "bad" state}}$$
$$+\overbrace{(1-p)\ln\left[(1-\kappa C_d)R_f\right]}^{\text{insuree does not incur loss}} \tag{4}$$

such that

$$\kappa C_d \leq 1 \tag{5}$$

As is clear from the above, an atomistic insuree makes his insurance purchase decision based on his probability of a loss and the probability that the insurer's project fails. Because he observes the insurer's total capital when he makes his decision, the insuree's decision rationally incorporates the expected proportion of the population of insurees that will incur losses.

The properties of the logarithmic utility function guarantee that it is suboptimal for insurees to invest all their capital in risky insurance so that the budget constraint, (5) is not binding. The necessary and sufficient first order condition for the insuree's optimal choice of insurance coverage, $C_d^*$, is

$$\left\{\frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f}\right\} \cdot \mathbf{1}_{\{pC \geq (K+\kappa C_s)R_L\}}$$
$$+\left\{\frac{p(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f}\right\} \cdot \mathbf{1}_{\{pC_s < (K+\kappa C_s)R_L\}} = 0 \tag{6}$$

The solution to the above equation can be expressed as a function, $C_d^*(K, C_s, \kappa)$, where we suppress the dependence of the optimal demand on the liability and asset risk parameters, $p$ and $q$, and the safe asset return, $R_f$, to simplify the notation.

The following lemma characterize the insuree's optimal demand for insurance coverage for a given insurance premium rate, $\kappa$.

**Lemma 1** • *Suppose insurees assume that insurers will defaut in the "bad" state where the technology fails. For a given insurance premium rate, $\kappa$, the optimal insurance demand $C_d^*$ is* *given by*

$$C_d^* = C_d^*(K, C_s, \kappa), \tag{7}$$

*where* $C_d^*(K, C_s, \kappa)$ *satisfies equation*

$$\frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \tag{8}$$

- *Suppose insurees assume that insurers do not default in the "bad" state where the technology fails. For a given insurance premium rate,* $\kappa$*, the optimal insurance demand* $C_d^*$*is given by*

$$C_d^* = C_d^*(\kappa), \tag{9}$$

*where* $C_d^*(\kappa)$ *satisfies*

$$\frac{p(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \tag{10}$$

By (8) and (10), we note that the insurer's internal capital, $K$, total supply, $C_s$, and asset risk parameter, $q$, influence the optimal demand for insurance coverage only when insurees foresee insurer insolvency in the "bad" state, where its assets fail. For generality, we allow for the case that the market insurance premium rate might lead to over insurance, i.e., $C_d > l$.

The following lemma shows how the optimal demand for insurance coverage varies with the fundamental parameters of the model that will be useful when we derive the equilibrium of the insurance market.

**Lemma 2 (Variation of Insurance Demand)** *The optimal demand for insurance,* $C_d^*$*, (i) decreases with the premium rate,* $\kappa$*; (ii) decreases with the return,* $R_f$*, on the safe asset; (iii) increases with insurers' internal capital,* $K$*; (iv) increases with the total face value of policies sold by the insurer,* $C_s$*; increases with the insurer's asset return in the low state,* $R_L$*; and (v) decreases with the insurer's expected probability of failure;* $q$*.*

The optimal demand for insurance claims reflects the tradeoff between self-insurance through investments in the safe asset and the purchase of insurance coverage with potential default risk for the insurer and, therefore, imperfect insurance for the insuree. Capital allocated in safe assets plays an alternative role in buffering the losses that cannot be indemnified by insurers when their assets fail. The insurance demand decreases with the insurance premium rate, that is, the demand curve is downward-sloping, since the utility function of insurees satisfies the properties highlighted by Hoy and Robson (1981) for insurance to be a normal good. An increase in the risk-free return raises the autarkic utility level, thereby diminishing the demand for insurance coverage.

In addition to functioning as a risk warehouse, which absorbs and diversifies each insuree's idiosyncratic loss, insurance firms also serve as financial intermediaries who channel external capital supplied by policyholders to productive assets. In our model, the overall insolvency risk faced by insurance firms are simultaneously determined by the asset and liability sides of insurer's balance sheets. An increase in the aggregate loss proportion of the insuree pool; a decrease in the internal capital held by insurers; a decrease in the amount of external capital raised by the insurer from selling insurance; and a decrease in the asset return in the low state all lower the insurance coverage of an insuree when the insurer is insolvent so that the optimal insurance demand declines.

### 3.1.2 Competitive Insurance Supply

Each insurer chooses its optimal supply of insurance coverage to maximize its total net expected payoffs from providing insurance for insurees and investing the capital it raises. As discussed earlier, in the absence of regulatory intervention, it is optimal for each insurer to invest its entire capital in the risky project due to its risk neutrality and the asset return condition (1). Insurers are competitive and take the insurance premium rate as given in making their insurance supply decisions. An insurer chooses to supply insurance if and only if its expected net profits are at least as great as its autarkic expected payoff, that is, its expected payoff from not selling insurance and investing its internal capital. An insurer's autarkic expected payoff is

$$\text{Autarkic Expected Payoff} = K\left(\left(1-q\right)R_H + qR_L\right). \tag{11}$$

Each insurer makes its supply decision knowing the expected proportion of insurees, $p$, who will incur losses. In the bad state where its technology fails, if its available capital is lower than the total loss payments to insurees, then the capital is divided equally among the insurees. The optimal supply of insurance coverage level, therefore, solves

$$\max_{C_s} \{(1-q)\left((K+\kappa C_s) R_H - pC_s\right)\} + \{q\left((K+\kappa C_s) R_L - pC_s\right)\} \cdot \mathbf{1}_{\{pC_s \leq (K+\kappa C_s)R_L\}} \tag{12}$$

such that

$$\begin{aligned}&\{(1-q)\left((K+\kappa C_s) R_H - pC_s\right)\} + \{q\left((K+\kappa C_s) R_L - pC_s\right)\} \cdot \mathbf{1}_{\{pC_s \leq (K+\kappa C_s)R_L\}} \\ &\geq K\left((1-q) R_H + qR_L\right) \qquad (P.C)\end{aligned} \tag{13}$$

The participation constraint, (13), ensures that the insurer chooses to sell a nonzero amount of coverage if and only if its expected net profit exceeds its expected payoff in autarky. From (12) and (13), it is clear that it is optimal for the insurer to supply no coverage if the premium rate, $\kappa < \frac{p}{R_H}$ and infinite coverage if $\kappa > \frac{p}{R_L}$. In equilibrium, therefore, we must have $\kappa \in [\frac{p}{R_H}, \frac{p}{R_L}]$. It also follows from the linearity of the objective function and the participation constraint that the participation constraint, (13), must bind in equilibrium, that is, insurers make zero expected *economic profits*. Consequently, the *competitive supply* of insurance coverage for any premium rate $\kappa \in [\frac{p}{R_H}, \frac{p}{R_L}]$ is

$$C_s^*(K, \kappa) = \frac{qKR_L}{(1-q)(\kappa R_H - p)} \tag{14}$$

**Lemma 3 (Competitive Insurance Supply)** *For $\kappa \in (\frac{p}{R_H}, \frac{p}{R_L})$, the competitive insurance supply level, $C_s^*(K, \kappa)$, (i) decreases with the insurance premium rate, $\kappa$; (ii) increases with insurers' internal capital, $K$; (iii) increases with insurers' expected default probability, $q$; (iv) increases with the asset return, $R_L$, in the bad state; and (v) increases with the loss probability of insurees, $p$.*

An increase in the premium rate increases the expected return from supplying insurance and, therefore, decreases the coverage level at which each insurer's participation constraint, (13), is binding. For given $\kappa \in (\frac{p}{R_H}, \frac{p}{R_L})$, an increase in the insurer's internal capital, asset risk, or the aggregate risk of the pool of insurees lowers the expected returns from providing insurance and, therefore, increases the competititive insurance supply level.

### 3.1.3 Competitive Equilibria of Unregulated Insurance Market

We now derive the insurance market equilibrium that is characterized by the insurance price (per unit of coverage) $\kappa^*$. The equilibrium satisfies the following conditions.

1. Given the equilibrium price $\kappa^*$, the face value of coverage supplied by each insurer is $C_s^*(K,\kappa^*)$ given by (14) and insurers make zero expected economic profits.

2. Given the equilibrium price $\kappa^*$ and the competitive supply level $C_s^*(K,\kappa^*)$, the coverage purchased by each insuree is $C_d^*(K, C_s^*(\kappa^*), \kappa^*)$ given by (7) and (9).

3. The equilibrium price $\kappa^*$ clears the market, that is, $C_d^*(K, C_s^*(\kappa^*), \kappa^*) = C_s^*(K,\kappa^*) = C^*$.

The following proposition characterizes the equilibria of the insurance market. We begin with some necessary definitions. Define the expected return from the insurer's risky technology,

$$ER = (1-q)R_H + qR_L. \tag{15}$$

Define the excess demand function

$$F(K,\kappa) = C_d^*(K, C_s^*(\kappa), \kappa) - C_s^*(K,\kappa), \tag{16}$$

where $C_d^*(K, C_s^*(\kappa), \kappa)$ is the demand function described by Lemma (1) and $C_s^*(K,\kappa)$ is given by ((14)).

**Proposition 1 (Competitive Equilibria)** • *Suppose $K \leq \overline{K}_1$, where $\overline{K}_1$is given by*

$$F(K = \overline{K}_1, \kappa = \frac{p}{ER}) = 0 \tag{17}$$

*In equilibrium, insurers default in the "bad" state when their assets fail. The equilibrium price, $\kappa^*$, satisfies:*

$$F(K, \kappa^*) = 0. \tag{18}$$

- *Suppose $K > \overline{K}_1$. In equilibrium, insurers do not default in the "bad" state where their assets fail. The equilibrium insurance price is $\kappa^* = \frac{p}{ER}$ and the equilibrium coverage level, $C^*$, is* *given by*

$$C^* = \frac{p}{\kappa^*} - \frac{(1-p)(R_f - l)}{1 - \kappa^* R_f} > l.$$

The above proposition shows that there are two possible equilibria that are determined by the internal capital of insurers. When the internal capital is lower than the threshold level $\overline{K}_1$, the representative insurer defaults in the "bad" state that is rationally foreseen by all agents. When the internal capital is higher than the threshold $\overline{K}_1$, the insurer faces no insolvency risk and this is rationally anticipated by all agents. In the equilibrium, therefore, the supply of insurance is completely elastic, and the price is determined by the aggregate loss proportion of insurees adjusted by the expected return from the risky technology, $\frac{p}{ER}$, at which the insurer's participation constraint (13) is binding.

We focus on the more interesting first scenario in which insurers with insufficient internal capital default after their technologies fail. Figure 1 shows the equilibrium. The equilibrium price $\kappa^*$ and coverage level $C^*$, satisfy the system of equations (33),(34) and(35).

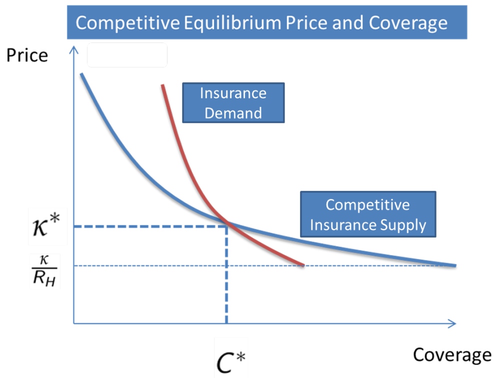
In addition, the condition (36) ensures that the insurer, indeed, defaults in the bad state. The equilibrium premium rate must be greater than $\frac{p}{R_H}$. To ensure that the insurer defaults in its "bad" state, the equilibrium insurance rate must be less than $\frac{p}{ER}$. The demand curve for insurance coverage and the competitive insurance supply curve are both downward sloping, but the demand curve is steeper than the supply curve. Thus the existence condition for a solution $\kappa^*$ to the system of equations is

$$F(\kappa)|_{\kappa \to \frac{p}{ER}} = \lim_{\kappa \to \frac{p}{ER}} C_d^*(\kappa, C_s^*(\kappa)) - \lim_{\kappa \to \frac{p}{ER}} C_s^*(\kappa) > 0. \tag{19}$$

Any solution, $\kappa^*$, to the system of equations must also satisfy the following constraint:

$$\frac{qKR}{(1-q)(\kappa^* R_H - p)} < \frac{1}{\kappa^*}$$

so that $\kappa^*$ is indeed the equilibrium premium rate. As there may be multiple possible equilibria corresponding to premium rates that satisfy the above conditions, we choose the smallest equilibrium premium that maximizes the expected utility of insurees and, therefore, the social welfare, that is the smallest $\kappa^*$, at which $\frac{\partial F(\kappa)}{\partial \kappa}|_{\kappa^*} > 0$.




We next identify the effects of shocks in the economy on the equilibrium insurance price, coverage level and social welfare.

## 3.2 The Effects of Capital and Risk

### 3.2.1 Internal Capital

Internal capital influences the equilibrium insurance price through the demand for and supplyof insurance. By (14), an increase in internal capital increases the competitive insurance supply level. There are both direct and indirect effects of an increase in internal capital on the demand for insurance. An increase in internal capital has the direct effect of increasing the demand for insurance because of the higher available capital to meet insurance claims. The demand for insurance coverage is further enlarged by the insurees' anticipation of the increase in the competitive supply of insurance with the increase in internal capital. Consequently, the overall effect of internal capital on the demand for insurance is also positive. The net effects of an increase in internal capital on the equilibrium premium rate depend on the relative dominance of demand-side and supply-side effects.

The smallest equilibrium price $\kappa^*$ satisfies $\frac{\partial F(K,\kappa)}{\partial \kappa}|_{\kappa=\kappa^*} > 0$. The marginal effects of internal capital on the insurance price can be understood through its effects on the excess demand function.

$$\frac{\partial F(K,\kappa^*)}{\partial K} = \underbrace{\frac{\partial C_d^*(K,C_s^*,\kappa^*)}{\partial K}}_{\text{direct effect on demand}} + \underbrace{\frac{\partial C_d(K,C_s^*,\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(K,\kappa^*)}{\partial K}}_{\text{indirect effect on demand}} - \underbrace{\frac{\partial C_s^*(K,\kappa^*)}{\partial K}}_{\text{direct effect on competitive supply}}.$$

The following proposition describes the effects of internal capital on the equilibrium insurance price.

**Proposition 2 (The Effects of Internal Capital)**

- *Suppose* $\frac{\partial F(K,\kappa^*)}{\partial K}|_{K\to 0} > 0$. *There exist a threshold* $\widetilde{K}$ *such that the equilibrium premium rate* $\kappa^*$ *decreases with internal capital when* $K < \widetilde{K}$, *and increases when* $K > \widetilde{K}$.

- *Suppose* $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} < 0$, *the equilibrium premium rate increases with the amount of internal capital.*

*Let* $\widetilde{\kappa}$ *be the equilibrium premium rate corresponding to the internal capital level* $\widetilde{K}$ . $\widetilde{K}$ *and are jointly determined by the following two equations:*

$$\frac{\partial F(K,\kappa^*)}{\partial K}|_{K=\widetilde{K},\kappa^*=\widetilde{\kappa};} = 0 \tag{20}$$

$$C_d^*\left(\widetilde{K}, C_s^*(\widetilde{\kappa},\widetilde{K}), \widetilde{\kappa},\right) = C_s^*(\widetilde{K},\widetilde{\kappa}). \tag{21}$$

The above proposition shows that the insurance premium decreases with insurers' internal capital when the internal capital level is relatively low, while it increases with insurers' internal capital when its level is relatively high. This result reconciles the conflicting predictions on the relation between insurance price and capital in previous literature. The “capacity constraint" theory relies on the assumption that insurance firms are free of insolvency. Winter(1990) argues that insurance firms can only write the volume of business consistent with zero insolvency due to regulation. The total capital amount determines the capacity of the insurance market. A significant negative shock to insurer capitalshrinks the supply of insurance in imperfect capital markets. It follows that the insurance price increases and insurance coverage declines while the demand for insurance is not affected in the absence of insurer insolvency. The “pricing of risky debt" theory incorporates the insolvency risk of insurance firms. Cummins and Sommer (1996) theoretically show both a postive and negative relation between price and a retroactive loss shock based on an optimal endogenous capitalization structure of insurance firms.

As mentioned earlier, an increase in internal capital increases the insurance demand and supply so the net impact depends on which of the two effects is dominant. By (14), the competitive supply of insurance is linear in the internal capital level. Because insurees are risk-averse, the demand for insurance is concave in the insurer's internal capital. Consequently, the excess demand function, $F(K,\kappa^*)$, is concave in the internal capital, that is, If $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} > 0$, then there exists, in general, a threshold level of internal capital, $\widetilde{K}$, at which the marginal effect of internal capital on the excess demand is zero. It follows from the concavity of the excess demand that the marginal effect of internal capital on the excess demand is positive for $K < \widetilde{K}$ and negative for $K > \widetilde{K}$. In other words, the risk aversion of insurees causes the "demand effect" of an increase in internal capital on the insurance price to dominate the "supply effect" for $K < \widetilde{K}$ and vice versa for $K > \widetilde{K}$. Hence, the equilibrium insurance premium varies in a U-shaped manner with the level of internal capital. If $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} \leq 0$, then the marginal effect of internal capital on the excess demand is always non-positive so that the equilibrium insurance premium increases with internal capital.

#### 3.2.2 The Effects of Asset Risk

We now address the impacts of the representative insurer's asset risk on the equilibrium premium rate and insurance coverage. The presence of asset induced insolvency complicates the decisions on both the demand and supply sides. The impact of asset risk on insurance supply indirectly influences insurance demand by affecting the total capital available to the insurer to meet liabilities in insolvency. Specifically, it follows from (14) that an increase in asset risk increases the competitive insurance supply level. Because insurees rationally foresee the likelihood that their losses will not be fully indemnified by insurers, the direct effect of an increase in asset risk on insurance demandis negative. The increase in the competitive supply level with asset risk, however, increases the amount each insuree is able to recover if it incurs a loss, but the insurer is insolvent. The indirect impact of an increase in asset risk on insurancey demand is, therefore, positive.Thenet impact of asset risk on the equilibrium premium rateis determined via its effect on the excess demand function,

$$\frac{\partial F(\kappa^*, q)}{\partial q} = \underbrace{\frac{\partial C_d(\kappa^*, q)}{\partial q}}_{\text{direct effect on demand}<0} + \underbrace{\frac{\partial C_d(\kappa^*, q)}{\partial C_s^*}\frac{\partial C_s^*}{\partial q}}_{\text{indirect effect on supply}>0} - \underbrace{\frac{\partial C_s^*(\kappa^*, q)}{\partial q}}_{\text{direct effect on zero-economic-profit supply}>0}, \tag{22}$$

where we explicitly indicate the dependence of the demand and supply functions on the asset risk parameter, $q$. The following proposition characterizes the effect of asset risk on the equilibrium insurance price and coverage.

**Proposition 3 (The Effects of Asset Risk)** *Suppose* $\frac{R_L}{p} < R_f$*. The equilibrium insurance price increases with the asset risk,* $q$*, while the coverage level declines. If* $\frac{R_L}{p} \geq R_f$*, then the effect of asset risk on the insurance price is ambiguous.*

The intuition for the condition $\frac{R_L}{p} < R_f$ is as follows. $\frac{R_L}{p}$ captures the marginal contribution of an increase in the supply of insurance claims to the marginal utility of each insuree in the default state is, while $R_f$ measures the marginal contribution of an increase in insurance demand to marginal utility of each insuree in the default state. Consequently, the condition $\frac{R_L}{p} < R_f$ implies that the marginal contribution of insurance supply to the marginal utility is less than that of insurance demand. In other words, one unit increase in insurance supply will induce less than one unit increase in insurance demand. It follows that the indirect effect of an increase in asset risk on insurance demand through the increase in the competitive insurance supply level is less than the direct effect on the competitive supply level. Consequently, the excess demand decreases with asset risk so that the equilibrium price increases.

# 4 Regulation

There are three sources of inefficiencies in the unregulated economy as analyzed in the previous section. First, each insurer makes its insurance supply decisions and investment decisions incorporating its *individual* asset return distribution without fully internalizing the potential correlation of asset returns across insurers arising from the fact that a proportion $\tau$ of insurers is exposed to a common shock. Without considering systemic risk, insurers may hold insufficient liquidity reserves and over-invest their capital in risky assets. Second, insurees' idiosyncratic losses may not be fully insured by insurers when each insurer's internal capital is relatively low. Insurees bear insurers' default risk driven by the asset side of their balance sheets when there is no effective risk sharing mechanisms among insurers to insure their idiosyncratic asset risk. Third, each insuree has no access to the risky assets, and insurance firms serve as the only intermediaries that channels the insuree's capital into more productive risky assets. Insurers, however, cannot effectively share the investment risk with insurees through the insurance policies that only protect insuree's losses without combining investment returns to insurees. The equilibrium price and insurance coverage level in the unregulated economy, therefore, do not internalize the externalities created by systemic risk of insurers' assets and the lack of instruments that achieve full risk-sharing. Consequently, we potentially have a misallocation of insuree capital to the purchase of insurance and misallocation of insurer capital to safe and risky assets.

Regulatory intervention could improve allocative efficiency by internalizing the externalities created by systemic risk, imposing necessary liquidity reserves requirement, and also providing *risk sharing mechanisms* through *ex post* taxation transfers among insurers. We now consider a regulated economy in which the regulator possesses the same information as the agents in the economy including the proportion of insurees who incur losses as well as the proportion of insurance firms whose assets fail. We first study a hypothetical benchmark scenario in which both insurees and insurers have access to risky technology, insurers fully insure insuree's idiosyncratic risk of incurring losses and fully/partially share the asset returns risk depending on the proportion $\tau$ of insurers exposed to a common shock. We then derive how regulatory tools can be combined to achieve optimal asset allocation and risk allocation in the hypothetical benchmark scenario.

## 4.1 Benchmark First Best Scenario

We begin by studying a hypothetical benchmark scenario that full internalizes the inefficiencies in the unregulated economy due to systemic risk and ineffective risk sharing mechanisms among insurees and insurers. In this economy, we assume that there are effective risk sharing mechanisms among insurers so that insuree's idiosyncratic losses can be fully insured by insurers. Without loss of generality, we can assume that there is a single representative risk averse insuree with 1 unit of capital good and a single representative risk neutral insurer with $K$ units of capital goods. We assume that both the insuree and the insurer have access to risky assets that may be subject to aggregate shocks. With probability $q$, the economy operates in the "bad" aggregate state where a proportion $\tau$ of risky investments earn a low return $R_L$. Thus the return per unit capital invested is $M^L$, where $M^L = (1-q)(1-\tau)R_H + q(1-\tau)R_L + \tau R_L$. With probability $1-q$ , the economy operates in the "good" aggregate state where a proportion $\tau$ of the risky investments earn a high rate of return $R_H$. It follows that the return per unit capital invested is $M^H$, where $M^H = (1-q)(1-\tau)R_H + q(1-\tau)R_L + \tau R_H$. The Insurer provides insurance to cover the insuree;s idosyncratic losse, but also share the aggregate risk associated with investments in the risky assets.

Let $C^H$and $C^L$ be the insurer's combined returns from investing capital in risky technology and selling insurance in the good and bad aggregate states, respectively. Insurers decide the proportion $\alpha$ of safe assets as liquidity reserves buffer. In the equilibrium, insurer's payoff from investing in risky assets and selling insurance is as great as its autarkic payoff, that is

$$\alpha K R_f + [(1-q)C_H + qC_L] = K[(1-q)R_H + qR_L]$$

The insuree invests a proportion $\beta$ of its capital in safe assets and the rest in purchasing risky insurance. Let $D^H$and $D^L$ be the bundled insurance claims received by insurees including both individual losses and returns from risky assets and/or insurance. The optimal returns from investment in insurance policies and the optimal investment in safe assets are solved as follows:

$$\max_{\beta,\alpha,D^L,D^H} \underbrace{q\ln(\beta R_f + D^L)}_{\text{bad aggregate shock}} + \underbrace{(1-q)\ln(\beta R_f + D^H)}_{\text{good aggregate shock}} \tag{23}$$

subject to

$$\alpha K R_f + [(1-q)C^H + qC^L] = K[(1-q)R_H + qR_L] \tag{24}$$

$$D^L + C^L = [(1-\beta) + (1-\alpha)K]M^L - pl \tag{25}$$

$$D^H + C^H = [(1-\beta) + (1-\alpha)K]M^H - pl \tag{26}$$

$$\alpha K R_f + C^L \geq 0 \tag{27}$$

$$\alpha K R_f + C^H \geq 0$$

The following proposition shows the optimal asset allocation between risky and safe assets, and the optimal risk allocation among the representative insuree and insurer.

**Proposition 4 (Benchmark Asset allocation and Risk Sharing among Insurees and Insurers)** *Suppose (i)* $q < 0.5$, *and* $K < \frac{(ER-pl)(ER-R_f)}{ER \cdot R_f}\frac{1-q}{1-2q}$, *or when* $q > 0.5$, *and for any* $K$*;*

*(ii)*$(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$ *, then*

- *when* $\tau \leq \tau_1$, *where* $\tau_1 = \frac{K \cdot ER}{(1+K)(ER-R_L)}$ *, both insuree and insurer hold zero liquidity reserves buffer, such that* $\beta^* = 0, \quad \alpha^* = 0$ *. The optimal return per unit capital invested in insurance* *policies in good and bad aggregate states are the same, and insurees are fully insured, that is*

$$D^H = D^L = D^* = ER - pl$$

- *when* $\tau_1 < \tau < \tau_2$, *where*

$$\begin{aligned}\tau_2 \quad &= \quad \frac{(1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)}{2(1-q)(1+K)(R_H - ER)(ER - R_L)} \\ &+ \quad \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H - ER)(ER - R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big) \cdot ER + pl(1-q)\Big)(ER - R_f)\end{array}}}{2(1-q)(1+K)(R_H - ER)(ER - R_L)}\end{aligned}$$

*both insuree and insurer still holds zero liquidity reserves buffer such that* $\beta^* = 0$, *and* $\alpha^* = 0$. *Insurees are imperfectly insured, and the optimal returns per unit capital invested in insurance products in good and bad aggregate states are respectively:*

$$D^H = (1+K)M^H - pl - \frac{ER}{1-q}K, \qquad\qquad D^L = (1+K)M^L - pl$$

*Insurer's limited liability constraint 27 binds, and its optimal returns per unit capital of investing in risky assets and providing insurance in good and bad aggregate states are respectively*

$$C^H = \frac{ER}{1-q}K \qquad\qquad C^L = 0$$

*In this case, insurers and insurees share the systemic risk.*

- *when* $\tau > \tau_2$, *both insuree and insurer in total have to hold some proportion of safe assets such that*

$$\beta^* + \alpha^* K = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)}$$

*Insurees are imperfectly insured, and the opitmal return per unit capital invested in insurance* *products in good and bad aggregate states are respectively:*

$$D^H = \big(1 + K - (\beta^* + \alpha^* K)\big)M^H - pl - \frac{ER}{1-q}K + \alpha^* K R_f$$

$$D^L = \big(1 + K - (\beta^* + \alpha^* K)\big)M^L - pl + \alpha^* K R_f$$

*Insurer's limited liability constraint 27 binds, and its optimal returns per unit capital of investing in risky assets and providing insurance in good and bad aggregate states are respectively*

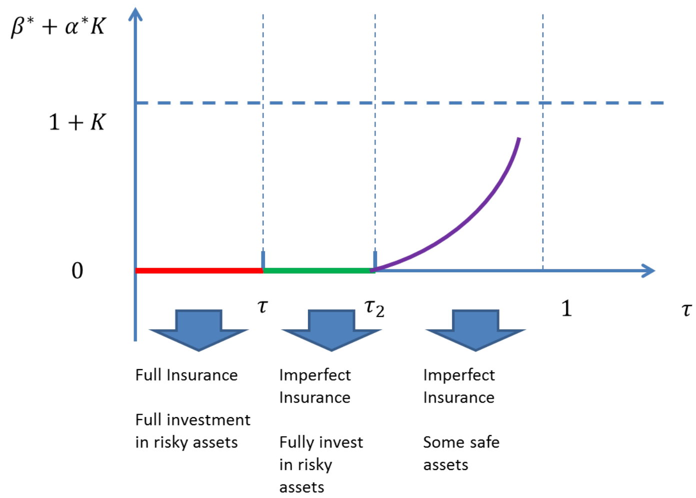
$$C^H = \frac{ER}{1-q}K - \alpha^* K \cdot R_f \qquad\qquad C^L = -\alpha^* K \cdot R_f$$

*In this case, insurers and insurees share the systemic risk.*

The above proposition shows the effects of systemic risk on the optimal asset allocation and risk sharing arrangement when the systemic risk are hypothetically internalized. When the systemic risk measure $\tau$ is relatively low, insuree's idosyncratic losses and returns from investment in risky assets can be fully insured by insurers. Thus it is optimal to invest all social capital in risky assets to produce the highest level of allocative social capital. When the systemic risk measure $\tau$ is in the intermediate level, insurers may not have enough capital to payoff its insurance claims in the bad aggregate so that insurer defaults and its limited liability constraint in that state is binding. It may still opitmal for both insurers and insurees to invest all in risky technology to achieve the highest level of total allocative capital.[2]When the systemic risk measure $\tau$ is above a threshold, however, the marginal increase in the total expceted allocative capital returns from risky assets is insufficient to compensate from the disutility to the representative insuree arising from the imperfect insurance payoffs due to aggregate shocks. It is, therefore, to hold certain amount of safe assets. Figure 4.1 summarizes the relationship between systemic risk measure $\tau$ and the optimal investment in safe assets. It reflects the tradeoffs between total allocative returns from investments and the risk sharing among insurees and insurers. When the systemic risk is low, the total allocative capital reaches the maximum level, and insurees are fully insured, and insurers take all systemic risk. When

[2]When asset default probability is sufficiently high and insurer's internal capital is relatively low, the marginal increase in total expected allocative capital returns from risky assets may be insufficient to compenate the disutility arising from the imperfect insurance payoffs due to aggregate shocks. It, thus may be opitmal to hold some safe assets as in the third case.

Figure 1: Systemic Risk and Liquidity Reserves Buffer




the systemic risk is in the intermediate level, the total allocative capital also reaches the highest level, and insurees are imperfectly insured, and insurees and insurers share the systemic asset risk. When the systemic risk is high, the marginal decrease in the total allocative capital due to some investment in safe assets trades off the wedge between insurance claims received by insurees in good and bad aggregate states.

We next analyze how the benchmark level of investment portfolios and risk sharing can be implemented through regulatory intervention.

## 4.2 Regulatory intervention

The inefficient investment allocation and risk sharing gap between the autarkic economy and the hypothetical benchmark arises from three factors: the lack of effective idiosyncratic asset risk sharing among insurees, lack of the effective systemic asset risk sharing among both insurees and insurers, and lack of the effective mechanism internalizing effect of systemic risk on insurer's investment portfolios. The above three factors provide regulators the room to reduce the market inefficiency using comprehensive tools.

**Liquidity Requirement and Systemic Risk**

Proposition 4and Figure 4.1 imply the opitmal investment in safe assets. Without internalizing systemic risk, insurers always invest all available capital into risky assets. This provides regulators the room to imposing the minimum amount of liquidity reserves when systemic risk is high enough.

Optimal regulatory liquidity requirement can enforce insurers to reach optimal investment decisions by internalizing its contribution to systemic risk.

**Taxation and Idiosyncratic Risk**

In the unregulated economy, there is no effective idosyncratic risk sharing mechanism among insurers. It follows that insurees bear the insurer default risk driven by the idosyncratic asset risk when insurer's internal capital is sufficiently low. In the regulated economy, regulators can serve as an "reinsurer" by taxing the insurers whose risky assets succeed and refunding the insurers whose risky assets fail. This *ex post* taxation contingent on the aggregate state is very similar to "insurance guarantee funds" run by state regulators. This mechnism can fully insure insurer's idiosyncratic risk but the systemic risk. Thus the taxation and refunds depends on the aggregate state of the economy. Let $T_S^H$ and $T_F^H$ be the taxation transfers from sucessful and failed insurers respectively in the good aggregate state. Also, let $T_S^H$ and $T_F^H$ be the taxation transfers from sucessful and failed insurers respectively in the bad aggregate state. The positive transfer means receiving tax refunds, while the negative transfer means paying taxes. Thus the tax balance condition in both good and bad aggregate state are:

$$\big((1-q)(1-\tau)+\tau\big)T_S^H + q(1-\tau)T_F^H = 0$$

$$(1-q)(1-\tau)T_S^L + \big(q(1-\tau)+\tau\big)T_F^L = 0$$

**Comprehensive Insurance and Optimal Risk Sharing**

In the unregulated economy, insurers only provide insurees insurance of their individual losses, even though they also serve as financial intermediation that channels insuree's capital to more productive technology by collecting insurance premium. Therefore, insurance policy that only provides loss coverages cannot be the opitmal risk sharing form. In the regulated economy, insurers sell comprehensive insurance by bundling both loss proection and investment returns, through which insurers can fully take the idiosyncratic risk of incurring losses and idiosyncratic asset risk given the regulatory tax schemes, insurees and insurers can also effectively share the systemic asset risk when systemic is relatively high. Let $d^H$ be the returns per unit of capital invested in comprehensive insurance policies in the good aggregate state, and $d^L$ be the returns per unit of capital invested in comprehensive insurance policies in the bad aggregate state.

The following proposition describes how the above comprehensive tools can be used to achieve the first best benchmark level of investment allocation and systemic risk sharing among insurees and insurers.

**Proposition 5 (Regulatory Intervention)** *Suppose (i) $q < 0.5$, and $K < \frac{(ER-pl)(ER-R_f)}{ER \cdot R_f} \frac{1-q}{1-2q}$, or when $q > 0.5$, and for any $K$;*

*$(ii)(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$ , then*

- *when $\tau \leq \tau_1$, regulators impose no liquidity requirement, both insuree and insurer invest everything in risky technology, such that*

$$\beta^* = 0, \qquad \alpha^* = 0$$

*The optimal returns per unit capital invested the comprehensive insurance product in good and bad aggregate states are the same, that is*

$$d^H = d^L = d^* = ER$$

*Insurees are fully insured, idiosyncratic losses, idiosyncratic asset and systemic risk fully taken by insurers through the following taxation scheme*

$$\begin{cases} T_S^L &= (1+K)(M^L - R_H) \\ T_F^L &= (1+K)(M^L - R_L) \\ T_S^H &= (1+K)(M^H - R_H) \\ T_F^H &= (1+K)(M^H - R_L) \end{cases} \tag{28}$$

- *when $\tau_1 < \tau \leq \tau_2$, regulators impose no liquidity requirement, both insuree and insurer invest everything in risky technology, such that*

$$\beta^* = 0, \qquad \alpha^* = 0$$

*The optimal returns per unit capital invested the comprehensive insurance product in good and bad aggregate states are the same, that is*

$$d^H = (1+K)M^H - \frac{ER}{1-q}K \qquad\qquad d^L = (1+K)M^L \tag{29}$$

*Insuree' idiosyncratic losses and idiosyncratic asset risk are fully taken by insurers through the taxation scheme as 28, however, the systemic risk are share among insurees and insurers through the comprehensive insurance policy 29.*

- *when $\tau > \tau_2$, regulators impose no liquidity requirement, both insuree and insurer invest everything in risky technology, such that*

$$\alpha^* \in \left( \max\left\{ \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K},\ 0 \right\},\ 1 \right)$$

*and* $\beta^* = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha^* K$

- *The optimal returns per unit capital invested the comprehensive insurance product in good and bad aggregate states are the same, that is*

$$\begin{aligned} d^H &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f}{1-\beta^*} \\ d^L &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f}{1-\beta^*} \end{aligned} \tag{30}$$

*Insuree' idiosyncratic losses and idiosyncratic asset risk are fully taken by insurers through the taxation scheme as follows:*

$$\begin{cases} T_S^L &= (1 + K - (\beta^* + \alpha^* K))(M^L - R_H) \\ T_F^L &= (1 + K - (\beta^* + \alpha^* K))(M^L - R_L) \\ T_S^H &= \big(1 + K - (\beta^* + \alpha^* K)\big)(M^H - R_H) \\ T_F^H &= \big(1 + K - (\beta^* + \alpha^* K)\big)(M^H - R_L) \end{cases} \tag{31}$$

*however, the systemic risk are share among insurees and insurers through the comprehensive insur-*

*ance policy as 30.*

The above proposition implies that the comprehensive tools can be used by regulators to reduce the inefficiencies of unregulated economy. *Ex post* taxation contingent on the aggregate state, plays the role of "insurance guarantee funds", which improves the efficiency of insurers to fully absorb insuree's idiosyncratic loss risk when their internal capital is relatively low. *Comprehensive insurance policies* combining insurance with investment, together with *ex post* taxation, improve the efficiency of insurers to fully absorb the insuree's idiosyncratic asset risk, and also improve the degree of the systemic risk sharing between insurees and insurers. *Liquidity requirement* adjusts ineffciencies arising from insurer's misallocation of their assets and the optimal systemic risk sharing among insurees and insurers. Thus, when systemic risk is high enough, the optimal investment allocation reflects the tradeoff between the growth of total assets and insuree's aversion to systemic risk.

# 5 Conclusion

We develop an equilibrium model of competitive insurance markets where insurers' assets may expose to both idiosyncratic shocks and systemic shocks. In the unregulated economy, without considering the systemic risk of insurers' assets, insurers may over-invest their capital into risky assets, which increases their default risk borne by insurees. We reconcile the conflicting predictions in previous literature and provide new insights into the relationship between insurance premia and internal capital that stem from the influence of both demand and supply side forces. The insurance price varies non-monotonically in a U-shaped manner with the level of internal capital held by insurers. In other words, the insurance price decreases with a positive shock to internal capital when the internal capital is below a lower threshold, increases when the internal capital is above the threshold, and keep constant at the fair premium level when the internal capital is above a higher threshold. We also obtain additional testable implications for the effects of insurers' idiosyncratic asset risks on premia and the level of insurance coverage. An increase in asset risk, that raises the default probability, raises insurance premia and reduces coverage.

We then study a regulated economy where the systemic risk can be internalized, and explore the effects of systemic risk on optimal asset and liquidity management by insurers, and risk sharing among insurees and insurers. Specifically, when the systemic risk is low, both insurees and insuers should hold zero liquidity reserves, insurees are fully insured, and insurers take all systemic risk. When the systemic risk is in the intermediate level, both insurees and insurers still hold zero liquidity reserves, however, insurees should partially share systemic risk with insuers. When the systemic risk is high, both insurees and insurers should hold some liquidity reserves, and insurees should partially share the systemic risk with insuers. The benchmark level of liquidity reserves and optimal systemic risk sharing can be implemented through regulatory intervetion including imposing minimum amount of liquidity reserves and *ex post* taxation contingent on the aggregate state, as well as comprehensive insurance policies. We use our results to shed some light on the solvency regulation of insurers facing systemic asset risk.

# Appendix

### Proof of Lemma 2

**Proof.**

Suppose insurees assume that insurers will defaut in the "bad" state where the technology fails, the optimal demand for insurance coverage is given by equation 8. Define $\mathcal{G}(C_d^*, \kappa, p, q, R_f, R_L, K, C_s)$ as

$$\mathcal{G}(C_d^*, \kappa, p, q, R_f, R_L, K, C_s) = \frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} \tag{32}$$

Let

$$\begin{aligned} W_1 &= (1-\kappa C_d^*)R_f - l + C_d^* \\ W_2 &= (1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s^*)R_L}{p} \\ W_3 &= (1-\kappa C_d^*)R_f \end{aligned}$$

Since

$$\frac{\partial \mathcal{G}(C_d)}{\partial C_d} = -\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} - \frac{pq\kappa^2 R_f^2}{W_2^2} - \frac{(1-p)\kappa^2 R_f^2}{W_3^2} < 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial \kappa} = -\frac{p(1-q)R_f(R_f - l)}{W_1^2} - \frac{pqR_f(R_f - l + \frac{KR_L}{p})}{W_2^2} - \frac{(1-p)R_f^2}{W_3^2} < 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial p} = \frac{(1-q)(1-\kappa R_f)}{W_1} - \frac{q\kappa R_f}{W_2} - \frac{q\kappa R_f(K+\kappa C_s^*)R_L}{W_2^2 p} + \frac{\kappa R_f}{W_3} =? \quad \Rightarrow \text{umbigious}$$

$$
\begin{aligned}
\frac{\partial \mathcal{G}(C_d)}{\partial R_f} &= -\frac{p(1-q)\kappa}{W_1} - \frac{p(1-q)(1-\kappa R_f)(1-\kappa C_d)}{W_1^2} - \frac{pq\kappa}{W_2} + \frac{pq\kappa R_f(1-\kappa C_d)}{W_2^2} \\
&\quad -\frac{(1-p)\kappa}{W_3} + \frac{(1-p)\kappa R_f(1-\kappa R_f)}{W_3^2} \\
&= -\frac{p(1-q)\kappa}{W_1} - \frac{p(1-q)(1-\kappa R_f)(1-\kappa C_d)}{W_1^2} - \frac{pq\kappa(l - \frac{(K+\kappa C_s)R_L}{p})}{W_2^2} \\
&\quad -\frac{(1-p)\kappa R_f(R_f - C_d)}{W_3^2} < 0
\end{aligned}
$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial K} = \frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2} > 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial C_s} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{W_2^2} > 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial R_L} = \frac{q\kappa R_f(K+\kappa C_s)}{W_2^2} > 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial q} = -\frac{p(1-\kappa R_f)}{W_1} - \frac{p\kappa R_f}{W_2} < 0$$

■

**Proof of Lemma3**

**Proof.**

The competitive insurance supply $C_s^*$ is given by equation (14).

It is obvious that

$$\frac{\partial C_s^*}{\partial \kappa} = -\frac{qKR_LR_H}{(1-q)(\kappa R_H - p)^2} < 0$$

$$\frac{\partial C_s^*}{\partial K} = \frac{qR_L}{(1-q)(\kappa R_H - p)} > 0$$

$$\frac{\partial C_s^*}{\partial q} = \frac{KR_L}{(1-q)^2(\kappa R_H - p)} > 0$$

$$\frac{\partial C_s^*}{\partial R_L} = \frac{qK}{(1-q)(\kappa R_H - p)} > 0$$

$$\frac{\partial C_s^*}{\partial p} = \frac{qKR_L}{(1-q)(\kappa R_H - p)^2} > 0$$

■

**Proof of Proposition 1**

**Proof.**

1. We first consider the case when $K \leq \overline{K}_1$, where $\overline{K}_1$satisfies (17). We conjecture that in the equilibrium insurees form rational expectations that insurers default in the "bad" state where their technology fails. If there exists a solution $\kappa^*$to the following system of equations

$$\frac{p(1-q)(1-\kappa^* R_f)}{(1-\kappa^* C_d^*)R_f - l + C_d^*} - \frac{pq\kappa^* R_f}{(1-\kappa^* C_d^*)R_f - l + \frac{(K+\kappa^* C_s^*)R_L}{p}} - \frac{(1-p)\kappa^* R_f}{(1-\kappa^* C_d^*)R_f} = 0 \tag{33}$$

$$C_s^* = \frac{qKR}{(1-q)(\kappa^* R_H - p)} \tag{34}$$

$$C_s^* = C_d^* = C^* \tag{35}$$

and $\kappa^*$satisfies condition

$$pC^* \geq (K + \kappa^* C^*)R_L \tag{36}$$

and (5), $\kappa^*$is the equilibrium insurance price.

However, it is suboptimal for insurees to spend their entire capital in purchasing risky insurance due to the properties of logarithm utility function. Condition (5), therefore, is automatically satisfied if there exists a solution $\kappa^*$to the system of equations. To solve for the equilibrium insurance price $\kappa^*$, we first derive the existance condition to the system of equations. The solution $\kappa^*$to the system of equations can be simplified to the solution to the excess demand equation defined as (39).

We first determine the boundaries for the candidate $\kappa^*$. The lower bound for $\kappa^*$ is $\frac{p}{R_H}$ since insurers would not like to supply any amount of insurance coverage for the price lower than $\frac{p}{R_H}$. Further, condition (36) holds as long as $\kappa^* \leq \frac{p}{ER}$. Thus upper bound for $\kappa^*$ is $\frac{p}{ER}$. Therefore, we only need to check the existence condition of $\kappa^*$ in the range $(\frac{p}{R_H}, \frac{p}{ER}]$.

We now check of sign of $F(\kappa)$ at the lower boundary of $\kappa$.

$$F(\kappa|K)|_{\kappa\to\frac{p}{R_H}} = \lim_{\kappa\to\frac{p}{R_H}} C_d^*(\kappa, C_s^*(\kappa)|K) - \lim_{\kappa\to\frac{p}{R_H}} C_s^*(\kappa|K)$$

When the premium rate is just above the level of $\frac{p}{R_H}$, insurers have to sell a very large finite amount so that condition (13)is binding,, otherwise they will not supply any insurance when insurance price is below the level of $\frac{p}{R_H}$. Thus $\lim_{\kappa\to\frac{p}{R_H}} C_s^*(\kappa|K) \to +\infty$. Given $\frac{p}{R_H} < \frac{p}{ER}$, no matter how large insurers can sell their coverages and collect premiums, insurers will always default.

Thus when $\kappa \to \frac{p}{R_H}$, the implicit equation for insurance demand is

$$\begin{aligned}
&\frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \\
\Rightarrow &C_d^*|_{\kappa\to\frac{p}{R_H}} = C_d^*(\kappa \to \frac{p}{R_H}|K) < \frac{R_H}{p} < +\infty
\end{aligned}$$

So $\lim_{\kappa\to\frac{p}{R_H}} C_d^*(\kappa, C_s^*(\kappa)|K) < +\infty$. We can show that

$$F(\kappa|K)|_{\kappa\to\frac{p}{R_H}} = \underbrace{\lim_{\kappa\to\frac{p}{R_H}} C_d^*(\kappa, C_s^*(\kappa)|K)}_{<+\infty} - \underbrace{\lim_{\kappa\to\frac{p}{R_H}} C_s^*(\kappa|K)}_{\to+\infty} < 0$$

We then determine the sign of $\frac{\partial F(\kappa^*)}{\partial \kappa^*}$ where

$$\frac{\partial F(\kappa^*|K)}{\partial \kappa^*} = \frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial \kappa^*} + \frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*} - \frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*} \tag{37}$$

From (34), the sign of $\frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*}$ is negative. We need to determine the sign of $\frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial \kappa^*}$ and $\frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial C_s^*}$.

According to the proof of Lemma (2), it is easy to see

$$\frac{\partial F(\kappa)}{\partial \kappa} = \underbrace{\frac{\partial C_d^*(\kappa, C_s^*(\kappa))}{\partial \kappa}}_{<0} + \underbrace{\frac{\partial C_d^*(\kappa, C_s^*(\kappa))}{\partial C_s^*}}_{>0}\underbrace{\frac{\partial C_s^*(\kappa)}{\partial \kappa}}_{<0} - \underbrace{\frac{\partial C_s^*(\kappa)}{\partial \kappa}}_{<0} \tag{38}$$

The sign of $\frac{\partial F(\kappa)}{\partial \kappa}$ is indeterminate. Suppose

$$F(\kappa|K)|_{\kappa=\frac{p}{ER}} = \lim_{\kappa=\frac{p}{ER}} C_d^*(\kappa, C_s^*(\kappa)|K) - \lim_{\kappa=\frac{p}{ER}} C_s^*(\kappa|K) \geq 0 \tag{39}$$

there exists at least one $\kappa^* \in (\frac{p}{R_H}, \frac{p}{ER}]$ such that $F(\kappa^*) = 0$ and also satisfies condition (36).

We now examine that when $K \leq \overline{K}_1$, condition (39) is satisfied. In other words, we have to show that $F(K|\kappa = \frac{p}{ER})$ is a decreasing function and $\overline{K}_1$is the solution to (17).

Since

$$F(K \to 0|\kappa = \frac{p}{ER}) = \underbrace{C_d^*(K \to 0|\kappa = \frac{p}{ER})}_{>0} - \underbrace{C_s^*(K \to 0|\kappa = \frac{p}{ER})}_{\to 0} > 0 \tag{40}$$

When $\kappa = \frac{p}{ER}$,the insurance claims received by each insuree who incurs losses are equal to the insurance claims seld by each insurer. Thus

$$\frac{\partial F(K|\kappa = \frac{p}{ER})}{\partial K} = \frac{\partial C_d^*(K|\kappa = \frac{p}{ER})}{\partial C_s^*} \frac{\partial C_s^*}{\partial K} - \frac{\partial C_s^*}{\partial K} < 0 \ (if\ R_f > \frac{R_L}{p}) \tag{41}$$

Conditions (40) and (41) imply that there exists a solution $\overline{K}_1$to the equation (17). Condition(41)also implies for any $K \leq \overline{K}_1$,$F(K|\kappa = \frac{p}{ER}) \geq 0$. Thus when $K < \overline{K}_1$,there exists at least one equilibrium insurance price. However, since $\frac{\partial EU(\kappa^*)}{\kappa^*} < 0$, we focus on the equilibrium with the smallest price $\kappa$, at which $\frac{\partial F(\kappa)}{\partial \kappa}|_{\kappa^*} > 0$ and social welfare are maximized.

- 2. Now we consider the case $K > \overline{K}_1$.We conjecture thatin equilibrium insurers will not default iin the "bad" state where their technology fails. As we shown in previous case,when $K > \overline{K}_1$, the equilibrium where insurers default in its "bad" state cannot be maintained. It is because given higher than $\overline{K}_1$amount of insuer capital, the possible solution to equation (39) might be greater than $\frac{p}{ER}$, which violates condition (36).According to previous argument, insurees' demand for insurance coverage is not binding. In this case, the insurers face no opportunity cost and earns zero profit at the actuarially fair price $\frac{p}{ER}$, at which insurers would like to provide as much insurance as they can. The equilibrium insurance coverage is determined by insurance demands, that is

$$\frac{p(1-\kappa R_f)}{(1-\kappa C^*)R_f - l + C^*} = \frac{(1-p)\kappa R_f}{(1-\kappa C^*)R_f}$$

where $\kappa = \frac{p}{ER}$. And we can show that the solution to the above implicit function$C^*$is

$$C^* = \left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}$$

We now verfy $C^*$ is the opitmal equilibrlium insurance coverage so that the insurer would not default in its"bad" state as we conjecture, that is,$(C^*\kappa + K)R_L > pC^*$. It implies that the insurer's internal capital has to satisfy the following:

$$K > C^*(\frac{p}{R_L} - \frac{p}{ER})$$

that is

$$K > \underbrace{\left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}(\frac{p}{R_L} - \frac{p}{ER})}_{let=\overline{K}_2}$$

We now show that $\overline{K}_1 = \overline{K}_{2.}$According to condition(17), we have

$$C_d^*(\overline{K}_1|\kappa \to \frac{p}{ER}) - \frac{q\overline{K}_1 R_L}{(1-q)(\frac{p}{ER}R_H - p)} = 0$$

Thus $\overline{K}_1 = C_d^*(\overline{K}_1|\kappa \to \frac{p}{ER})\frac{(1-q)(\frac{p}{ER}R_H - p)}{qR_L}$. Since the equilibrium insurance demand $C_d^*(K = \overline{K}_1)$is equal to $C^*(K > \overline{K}_2)$. In other words, $C_d^*(K = \overline{K}_1) = C^* = \left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}$. Now we have $\overline{K}_1 = \left(p - (1-p)\frac{R_f - l}{\frac{ER}{p} - R_f}\right)\frac{ER}{p}\frac{(1-q)(\frac{p}{ER}R_H - p)}{qR_L}$. It is easy to show that $\frac{(1-q)(\frac{p}{ER}R_H - p)}{qR_L} = \frac{p}{R_L} - \frac{p}{ER}$, therefore, $\overline{K}_1 = \overline{K}_{2.}$ Thus when $K > \overline{K}_1$, the solution $C^*$is the equilibrium opitmal insurance coverage.

■

**Proof of Proposition 2**

**Proof.** We check the sign of $\frac{\partial \kappa^*}{\partial K} = -\frac{\frac{\partial F(\kappa^*, K)}{\partial K}}{\frac{F(\kappa^*, K)}{\partial \kappa^*}}$. It is clear that

$$\frac{\partial F(\kappa^*)}{\partial K} = \underbrace{\frac{\partial C_d(\kappa^*)}{\partial K}}_{\text{direct effect on demands}} + \underbrace{\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*)}{\partial K}}_{\text{indirect effect on demands}} - \underbrace{\frac{\partial C_s(\kappa^*)}{\partial K}}_{\text{direct effect on zero-expected-profit supply}}$$

We know

$$\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*}{\partial K} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}\frac{qR_L}{(1-q)(\kappa^* R_H - p)} > 0$$

$$\frac{\partial C_s(\kappa^*)}{\partial K} = \frac{qR_L}{(1-q)(\kappa^* R_H - p)} > 0$$

Now we determine the sign of $\frac{\partial C_d^*}{\partial K}$. From (32), we have

$$\frac{\partial \mathcal{G}(C_d^*, K)}{\partial K} = \frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2} > 0$$

Thus

$$\frac{\partial C_d^*}{\partial K} = -\frac{\frac{\partial \mathcal{G}(C_d^*)}{\partial K}}{\frac{\partial \mathcal{G}(C_d^*)}{\partial C_d}} = \frac{\frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} > 0$$

Therefore,

$$\frac{\partial F(\kappa^*, K)}{\partial K} = \underbrace{\frac{\partial C_d(\kappa^*, K)}{\partial K}}_{>0} + \underbrace{\frac{\partial C_d(\kappa^*, K)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*, K)}{\partial K}}_{>0} - \underbrace{\frac{\partial C_s(\kappa^*, K)}{\partial K}}_{>0}$$

Further, we know $\frac{\partial F(\kappa^*, K)}{\partial \kappa^*} > 0$. Thus the sign of $\frac{\partial F(\kappa^*, K)}{\partial K}$ is indeterminate, and the effects of internal capital on the equilibrium price is non-monotonic.

Also,

$$
\begin{aligned}
\frac{\partial^2 F(\kappa^*,K)}{\partial K^2} =& \frac{\partial^2 C_d(\kappa^*,K)}{\partial K^2} + \frac{\partial\left(\frac{\partial C_d(\kappa^*,K)}{\partial C_s^*}\frac{\partial C_s^*}{\partial K}\right)}{\partial K} \\
=& \underbrace{-\frac{2W_2 pq\kappa^* R_f \frac{R_L}{p}\left[\left(\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}+\frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}\right)\frac{\partial W_2}{\partial K}\right]}{\left[\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2\right]^2}}_{\text{direct effect part}<0} \\
& \underbrace{-\frac{2W_2 pq\kappa^{*2} R_f \frac{R_L}{p}\left[\left(\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}+\frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}\right)\frac{\partial W_2}{\partial K}\right]\frac{\partial C_s^*}{\partial K}}{\left[\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2\right]^2}}_{\text{indirect effect part}<0} < 0
\end{aligned} \tag{42}
$$

Thus the marginal effect of internal capital $K$ on insurance demand $C_d$ is decreasing, while the effect on insurance supply at which insurers get zero profits is constant. Consequently, the overall effects are decreasing.

Since

$$
\begin{aligned}
\frac{\partial F(\kappa^*,K)}{\partial K} =& \frac{\frac{pq\kappa^* R_f \frac{R_L}{p}}{W_2^2}}{\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2} + \frac{pq\kappa^{*2}R_f^2}{W_2^2} + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}} \\
&+ \frac{pq\kappa^{*2} R_f \frac{R_L}{p}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2}\frac{qR_L}{(1-q)(\kappa^* R_H - p)} \\
&- \frac{qR_L}{(1-q)(\kappa^* R_H - p)}
\end{aligned} \tag{43}
$$

Therefore, suppose $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} > 0$, there exists a threshold level of $\widetilde{K}$ and the corresponding price $\widetilde{\kappa}$ such that the equilibrium price $\kappa^*$ decrease with the amount of internal capital when $K < \widetilde{K}$, while increase with an increase in the amount of internal capital when $K > \widetilde{K}$; suppose $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} < 0$, the equilibrium price increases with the amount of internal capital, where $K$ and

$\bar{\kappa}$ are jointly been determined by the following two equations

$$\frac{\partial F(\kappa^*, K)}{\partial K}|_{\kappa=\widetilde{\kappa};K=\widetilde{K}} = 0$$
$$C_d^*\left(\widetilde{\kappa}, \widetilde{K}, C_s^*(\widetilde{\kappa}, \widetilde{K})\right) = C_s^*(\widetilde{\kappa}, \widetilde{K})$$

■

**Proof of Proposition 3**

**Proof.** We check the sign of $\frac{\partial \kappa^*}{\partial q} = -\frac{\frac{\partial F(\kappa^*)}{\partial q}}{\frac{F(\kappa^*)}{\partial \kappa^*}}$. Since

$$\frac{\partial \mathcal{G}(C_d)}{\partial q} = -\frac{p(1-\kappa R_f)}{W_1} - \frac{p\kappa R_f}{W_2}$$

We have

$$\frac{\partial C_d^*(\kappa^*)}{\partial q} = -\frac{\frac{p(1-\kappa R_f)}{W_1} + \frac{p\kappa R_f}{W_2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}}$$

$$\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*)}{\partial q} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}\frac{JR_L}{(1-q)^2(\kappa R_{\mathcal{H}} - p)}$$

$$\frac{\partial C_s^*(\kappa^*)}{\partial q} = \frac{JR_L}{(1-q)^2(\kappa R_{\mathcal{H}} - p)}$$

Thus

$$
\begin{aligned}
\frac{\partial F(\kappa^*)}{\partial q} =& \frac{\partial C_d(\kappa^*)}{\partial q} + \frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*}{\partial q} - \frac{\partial C_s^*(\kappa^*)}{\partial q} \\
=& -\frac{\frac{p(1-\kappa R_f)}{W_1} + \frac{p\kappa R_f}{W_2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} \\
&+ \frac{pq\kappa^2 R_f \frac{R_L}{p}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}\frac{JR_L}{(1-q)^2(\kappa R_{\mathcal{H}} - p)} - \frac{JR_L}{(1-q)^2(\kappa R_{\mathcal{H}} - p)} \\
=& -\frac{\frac{p(1-\kappa R_f)}{W_1} + \frac{p\kappa R_f}{W_2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} \\
&+ \underbrace{\frac{pq\kappa^2 R_f \frac{R_L}{p} - pq\kappa^2 R_f^2}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}}_{<0 \quad \text{if} \quad \frac{R_L}{p}<R_f}\frac{JR_L}{(1-q)^2(\kappa R_{\mathcal{H}} - p)} \\
&- \frac{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}\frac{JR_L}{(1-q)^2(\kappa R_{\mathcal{H}} - p)}
\end{aligned}
$$

Thus $\frac{\partial F(\kappa^*)}{\partial q} < 0$ when $\frac{R_L}{p} < R_f$. Therefore, $\frac{\partial \kappa^*}{\partial q} > 0$.The equilibrium premium rate increase with an increase in the asset risk. When $\frac{R_L}{p} < R_f$, the indirect effects of asset risk on insurance demand is offset by the direct effects on zero-expected-profit insurance supply. Consequently, the demand effects dominates so that the equilibrium price goes up and the equilibrium coverage shrinks.

■

**Proof of Proposition (4)**

**Proof.**

According to the insuree's optimization problem that maximize (23) subject to (24),(25),(26)(27). Thus we have

$$
\begin{aligned}
\mathcal{L} \quad = \quad & q\ln(\beta R_f + W^L - C^L) + (1-q)\ln(\beta R_f + W^H - C^H) \\
& + \lambda\{\alpha K R_f + [(1-q)C^H + qC^L] - K[(1-q)R_H + qR_L]\} \\
& + \mu\{\alpha K R_f + C^L\}
\end{aligned}
$$

$$\partial C^L : -\frac{q}{\beta R_f + W_L - C_L} + \lambda q + \mu = 0$$
$$\partial C^H : -\frac{1-q}{\beta R_f + W_H - C_H} + \lambda(1-q) = 0$$

CASE ONE: Suppose $\mu = 0$

So $W^H - C^H = W^L - C^L$, thus

$$C^H - C^L = W^H - W^L = [(1-\beta) + (1-\alpha)K]\tau(R_H - R_L)$$

$$C^H = C^L + [(1-\beta) + (1-\alpha)K]\tau(R_H - R_L)$$

plug into constraint, we have

$$C^L = K\big((1-q)R_H + qR_L - \alpha R_f\big) - (1-q)[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L)$$

$$C^H = K\big((1-q)R_H + qR_L - \alpha R_f\big) + q[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L)$$

$$\begin{aligned} D^H = D^L &= [(1-\beta) + (1-\alpha)K]\big((1-q)R_H + qR_L\big) - pl - K\big((1-q)R_H + qR_L) - \alpha R_f\big) \\ &= (1-\beta-\alpha K)\big((1-q)R_H + qR_L\big) + \alpha K R_f - pl \\ &= (1-\beta)\big((1-q)R_H + qR_L\big) - \alpha K\big((1-q)R_H + qR_L - R_f\big) - pl \end{aligned}$$

we have

$$\begin{aligned} C^{L*} &= K\big(ER - \alpha R_f\big) - (1-q)[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) \\ C^{H*} &= K\big(ER - \alpha R_f\big) + q[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) \end{aligned}$$

$$D^* = D^{H*} = D^{L*} = (1-\beta)ER - \alpha K\big(ER - R_f\big) - pl$$

Thus the insuree is fully insured, and its utility is:

$$\begin{aligned} EU_{\text{insuree}} &= \ln\big(\beta R_f + D^*\big) \\ &= \ln\big( - \beta(ER - R_f) - \alpha K(ER - R_f) + ER - pl\big) \end{aligned}$$

The opitmal level of investment in safe assets is

$$\begin{aligned} & \max_{\alpha,\beta} \ln\big( - \beta(ER - R_f) - \alpha K(ER - R_f) + ER - pl\big) \\ \Rightarrow \quad & \max_{\alpha,\beta} \ln\big( - (\beta + \alpha K)(ER - R_f) + ER - pl\big) \end{aligned}$$

subject to

$$\begin{aligned} \alpha K R_f + C^{L*} &> 0 \\ 0 \leq \beta + \alpha K &\leq 1 + K \end{aligned}$$

Since the objective function is a decreasing function of $(\beta + \alpha K)$, thus $\beta^* + \alpha^* K = 0$

The above constraint can be simplified as follows:

$$\alpha K R_f + K\big(ER - \alpha R_f\big) - (1-q)[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) > 0$$

that is

$$\begin{aligned} & K \cdot ER > (1-q)(1+K)\tau(R_H - R_L) \\ \Rightarrow \quad & \tau < \tfrac{K \cdot ER}{(1-q)(1+K)(R_H - R_L)} \\ \Rightarrow \quad & \tau < \tfrac{K \cdot ER}{(1+K)(ER - R_L)} \end{aligned}$$

When the aggregate shock $\tau$ is low enough such that

$$\tau < \frac{K \cdot ER}{(1+K)(ER - R_L)}$$

both insurees and insurers invest all capital in risky assets, that is $\alpha^* = 0$ , and $\beta^* = 0$ , and the returns from investment in insurance policies is $D^H = D^L = D^* = ER - pl$.

CASE TWO: Suppose $\mu > 0$, that is $\alpha K R_f + C^L = 0$. Thus $C^L = -\alpha K R_f$ and $C_H = \frac{ER-(1-q)\alpha R_f}{1-q} K$.

So

$$\begin{aligned} D^L &= W^L - C^L \\ &= [(1-\beta) + (1-\alpha)K]M^L - pl + \alpha K R_f \\ &= [1 + K - (\beta + \alpha K)]M^L - pl + \alpha K R_f \end{aligned}$$

$$\begin{aligned} D^H &= W^H - C^H \\ &= [(1-\beta) + (1-\alpha)K]M^H - pl - \frac{ER - (1-q)\alpha R_f}{1-q} K \\ &= [1 + K - (\beta + \alpha K)]M^H - pl - \frac{ER}{1-q} K + \alpha K R_f \end{aligned}$$

Thus

$$\begin{aligned} N^L &= \beta R_f + D^L = (1+K)M^L + (\beta + \alpha K)(R_f - M^L) - pl \\ N^H &= \beta R_f + D^H = (1+K)M^H + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q} K \end{aligned}$$

The opitmal level of investment in safe assets is

$$\max_{\alpha,\beta} q \ln\big((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\big) + (1-q)\ln((1+K)M^H$$
$$+(\beta + \alpha K)(R_f - M^H) - pl - \tfrac{ER}{1-q} K)$$

subject to

$$0 \leq \beta + \alpha K \quad \leq \quad 1 + K$$

From $\partial(\beta + \alpha K)$ we have F.O.C that is

$$\frac{q(R_f - M^L)}{N^L} - \frac{(1-q)(M^H - R_f)}{N^H} + \lambda_1 - \lambda_2 = 0$$

Suppose $\lambda_1 = \lambda_2 = 0$, then $\frac{q(R_f - M^L)}{N^L} = \frac{(1-q)(M^H - R_f)}{N^H}$

That is

$$\begin{aligned} & q(R_f - M^L)\big((1+K)M^H + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\big) \\ = \quad & (1-q)(M^H - R_f)\big((1+K)M^L + (\beta + \alpha K)(R_f - M^L) - pl\big) \end{aligned}$$

We have

$$\beta + \alpha K = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)}$$

Now we have to check $0 < \beta + \alpha K < 1 + K$.

We first check $\beta + \alpha K > 0$, that is

$$(1-q)(1+K)(ER \cdot R_f - M^H M^L) + pl(1-q)(ER - R_f) - qER \cdot K \cdot (R_f - M^L) > 0$$

$$(1-q)(1+K)ER \cdot R_f + pl(1-q)(ER - R_f) - q \cdot ER \cdot K \cdot R_f$$

$$> (1-q)(1+K)M_H M_L - q \cdot ER \cdot K \cdot M_L$$

$\Rightarrow$

$$\begin{aligned} (1-q)(1+K)(R_H - ER)(ER - R_L)\tau^2 - \Big( & (1-q)(1+K)(R_H + R_L - 2ER)ER \\ +q \cdot ER \cdot K(ER - R_L)\Big)\tau + \Big(\big(qK - (1-q)(1+K)\big) \cdot ER + pl(1-q)\Big)(ER - R_f) > 0 \end{aligned}$$

Let

$$
\begin{aligned}
\tau_2' \;=\;& \frac{(1-q)(1+K)(R_H+R_L-2ER)ER+q\cdot ER\cdot K(ER-R_L)}{2(1-q)(1+K)(R_H-ER)(ER-R_L)} \\
& - \frac{\sqrt{\begin{gathered}\Big((1-q)(1+K)(R_H+R_L-2ER)ER+q\cdot ER\cdot K(ER-R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H-ER)(ER-R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big)\cdot ER+pl(1-q)\Big)(ER-R_f)\end{gathered}}}{2(1-q)(1+K)(R_H-ER)(ER-R_L)}
\end{aligned}
$$

$$
\begin{aligned}
\tau_2 \;=\;& \frac{(1-q)(1+K)(R_H+R_L-2ER)ER+q\cdot ER\cdot K(ER-R_L)}{2(1-q)(1+K)(R_H-ER)(ER-R_L)} \\
& + \frac{\sqrt{\begin{gathered}\Big((1-q)(1+K)(R_H+R_L-2ER)ER+q\cdot ER\cdot K(ER-R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H-ER)(ER-R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big)\cdot ER+pl(1-q)\Big)(ER-R_f)\end{gathered}}}{2(1-q)(1+K)(R_H-ER)(ER-R_L)}
\end{aligned}
$$

We now compare $\tau_1$ and $\tau_2'$

When $\tau=\big(\frac{K}{1+K}\big)\big(\frac{ER}{ER-R_L}\big)$, we check whether

$$\beta+\alpha K\big|_{\tau=\left(\frac{K}{1+K}\right)\left(\frac{ER}{ER-R_L}\right)}<0$$

$$\beta+\alpha K=\frac{(1+K)(ER\cdot R_f-M^H M^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}}{(M^H-R_f)(R_f-M^L)}$$

that is whether

$$(1+K)(ER\cdot R_f-M^H M^L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M^L)}{1-q}<0$$

The above inequality is equivalent to

$$\big(1+K\frac{1-2q}{1-q}\big)\cdot ER\cdot R_f < ER^2 - pl(ER-R_f)$$

Therefore, when $q<0.5$, and $K<\frac{(ER-pl)(ER-R_f)}{ER\cdot R_f}\frac{1-q}{1-2q}$, or when $q>0.5$, and for any $K$, we have $\beta+\alpha K|_{\tau=\left(\frac{K}{1+K}\right)\left(\frac{ER}{ER-R_L}\right)}<0$ . In other words, $\tau_2'<\tau_1$

We next check when $\tau=1$, whether $\beta+\alpha K<1+K$.

When $\tau=1$, we have $M^L=R_L$, and $M^H=R_H$.

$$\begin{aligned}\beta+\alpha K-(1+K) &= \frac{(1+K)(ER\cdot R_f-R_HR_L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-R_L)}{1-q}}{(R_H-R_f)(R_f-R_L)}-(1+K)\\ &= \frac{\begin{array}{c}(1+K)(ER\cdot R_f-R_HR_L)+pl(ER-R_f)\\ -\frac{qER\cdot K\cdot(R_f-R_L)}{1-q}+(1+K)(R_f-R_H)(R_f-R_L)\end{array}}{(R_H-R_f)(R_f-R_L)}\end{aligned}$$

which is equivalent to

$$(1+K)(ER\cdot R_f-R_HR_L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-R_L)}{1-q}+(1+K)(R_f-R_H)(R_f-R_L)<0$$

that is

$$pl(ER-R_f)+\big((ER+R_f-R_H-R_L)R_f\big)<K\big(\frac{q}{1-q}ER(R_f-R_L)-(ER+R_f-R_H-R_L)R_f\big)$$

Suppose

$$(ER+R_f-R_H-R_L)R_f+pl(ER-R_f)<0$$

it is easy to see $\tau_2<1$.

Therefore, when $\tau>\tau_2$,

$$\beta^*+\alpha^*K=\frac{(1+K)(ER\cdot R_f-M_HM_L)+pl(ER-R_f)-\frac{qER\cdot K\cdot(R_f-M_L)}{1-q}}{(M_H-R_f)(R_f-M_L)}$$

When $\tau_1 \leq \tau \leq \tau_2$, $\beta^* + \alpha^* K = 0$

$\blacksquare$

**Proof of Proposition** (5)

**Proof.**

**CASE ONE:** $\tau < \tau_1 = \frac{K \cdot ER}{(1+K)(ER-R_L)}$ , insurees are fully insured so that

$$\beta R_f + d^H(1-\beta) - pl = ER - (\beta + \alpha K)(ER - R_f) - pl$$

$$\beta R_f + d^L(1-\beta) - pl = ER - (\beta + \alpha K)(ER - R_f) - pl$$

Thus

$$d^H = d^L = \frac{(1-\beta)ER - (\beta + \alpha K)(ER - R_f)}{1-\beta}$$

So insuree's utility is

$$\max_{\beta} \ln \big( ER - (\beta + \alpha K)(ER - R_f) - pl \big)$$

subject to

$$0 \leq \beta \leq 1$$

Thus

$$\beta^* = 0$$

Thus regulator's problem is

$$\max_{\alpha} \ln \big( ER - \alpha K (ER - R_f) - pl \big)$$

subject to

$$0 \leq \alpha \leq 1$$

Thus

$$\alpha^* = 0$$

Therefore, the optimal insurance contract is

$$d^{L*} = d^{H*} = ER$$

Now we solve for the opitmal taxation rate and taxation refund.

- in bad aggregate state,
  - successful insurer's payoff: $\big(1+K\big)R_H + T_S^L - d^L$
  - failed insurer's payoff: $\big(1+K\big)R_L + T_F^L - d^L$
  - each insurer don't bear idiosycratic risk

$$\begin{aligned}
& \big(1+K\big)R_H + T_S^L - d^L = \big(1+K\big)R_L + T_F^L - d^L \\
= \quad & C^{L*} = K \cdot ER - (1-q)(M^H - M^L)(1+K) \\
\Rightarrow \quad & \begin{cases} T_S^L & = C^{L*} + d^L - (1+K)\cdot R_H = (1+K)(M^L - R_H) < 0 \\ T_F^L & = C^{L*} + d^L - (1+K)\cdot R_L = (1+K)(M^L - R_L) > 0 \end{cases}
\end{aligned}$$

$$\Rightarrow \begin{cases} T_S^L & = (1+K)(M^L - R_H) \\ T_F^L & = (1+K)(M^L - R_L) \end{cases}$$

- in bad aggregate state, the tax budget balance constraints:
  - the proportion of failed insurers: $q(1-\tau)+\tau$
  - the proportion of successful insurers: $(1-q)(1-\tau)$
  - taxation balance: $\big(q(1-\tau)+\tau\big)\cdot T_F^L + (1-q)(1-\tau)\cdot T_S^L = (1+K)(M^L - M^L) = 0$

Similarly if in the good aggregate state:

- in good aggregate state,
  - successful insurer's payoff: $\big(1+K\big)R_H + T_S^H - d^H$
  - failed insurer's payoff: $\big(1+K\big)R_L + T_F^H - d^H$

– each insurer don't bear idiosycratic risk

$$
\begin{aligned}
& \big((1+K-t\cdot K\big)R_H + T_S^H - d^H = \big(1+K-t\cdot K\big)R_L + T_F^H - d^H \\
= \quad & C^{H*} = K\cdot ER + q(M^H - M^L)(1+K) \\
\Rightarrow \quad & \begin{cases} T_S^H & = C^{H*} + d^H - (1+K)\cdot R_H = (1+K)(M^H - R_H) < 0 \\ T_F^H & = C_H^* + d^H - (1+K)\cdot R_L = (1+K)(M^H - R_L) > 0 \end{cases}
\end{aligned}
$$

$$
\Rightarrow \begin{cases} T_S^H & = (1+K)(M^H - R_H) \\ T_F^H & = (1+K)(M^H - R_L) \end{cases}
$$

- in good aggregate state, the tax budget balance constraints:
  - the proportion of failed insurers: $q(1-\tau)$
  - the proportion of successful insurers: $(1-q)(1-\tau)+\tau$
  - taxation balance:

$$
\big((1-q)(1-\tau)+\tau\big)\cdot T_S^H + q(1-\tau)\cdot T_F^H = (1+K)(M^H - M^H) = 0
$$

$$
\begin{cases} T_S^L & = (1+K)(M^L - R_H) \\ T_F^L & = (1+K)(M^L - R_L) \\ T_S^H & = (1+K)(M^H - R_H) \\ T_F^H & = (1+K)(M^H - R_L) \end{cases}
$$

Insurees invest all their capital in buying risky insurance contracts, insurers invest all their capital in risky assets, and regulators's taxes transfers are given as above.

CASE TWO: $\frac{K\cdot ER}{(1+K)(ER-R_L)} \leq \tau \leq \tau_2$, insurees cannot be perfectly insured, thus the insuree's payoffs in the two aggregate states are:

- in bad aggregate state: $\beta R_f + d^L(1-\beta) - pl = (1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl$
- in good aggregate state: $\beta R_f + d^H(1-\beta) - pl = (1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K$

Thus insuee's problem becomes:

$$\max_{\beta}(1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)+q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)$$

subject to

$$0 \leq \beta \leq 1$$

Thus

$$\begin{aligned}\mathcal{L} &= (1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)\\ &\quad +q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)+\lambda_1\beta+\lambda_2(1-\beta)\end{aligned}$$

FOC:

$$\frac{(1-q)(R_f-M^H)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}+\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}+\lambda_1-\lambda_2=0$$

that is

$$\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}-\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}=\lambda_2-\lambda_1$$

Suppose $\lambda_2=\lambda_1=0$, that is $0<\beta<1$,

However, we can solve the solution to function

$$\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}=\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$$

such that $\beta* < 0$, which violates $0<\beta<1$.

Since $\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}-\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$ is a decreasing function of $\beta$, thus we need $\lambda_2=0$, and $\lambda_1>0$, that is

$$\beta^*=0$$

Similarly, we can solve for the regulator's problem as follows:

$$\max_{\alpha}(1-q)\ln\big((1+K)M_H+\alpha K(R_f-M_H)-pl-\frac{ER}{1-q}K\big)+q\ln\big((1+K)M_L+\alpha K(R_f-M_L)-pl\big)$$

subject to

$$0\leq\alpha\leq 1$$

The similar logic as we analyze opitmal $\beta$, we can solve the optimal $\alpha^*$, that is $\alpha^*=0$

Therefore the optimal insurance contracts are:

- in bad aggregate state: $d^L=(1+K)M^L$
- in good aggregate state: $d^H=(1+K)M^H-\frac{ER}{1-q}K$

Now we derive the **optimal taxation scheme**:

In the bad aggregate state:

- in bad aggregate state,
  - successful insurer's payoff: $\big(1+K\big)R_H+T_S^L-d^L=(1+K)(R_H-M_L)+T_S^L$
  - failed insurer's payoff: $\big(1+K\big)R_L+T_F^L-d^L=(1+K)(R_L-M_L)+T_F^L$
  - each insurer doesn't bear idiosycratic risk

$$\begin{aligned}&(1+K)R_H+T_S^L-d^L=(1+K)R_L+T_F^L-d^L=C^{L*}=0\\ \Rightarrow\ &\begin{cases}T_S^L &= d^L-(1+K)R_H=(1+K)(M^L-R_H)<0\\ T_F^L &= (1+K)M^L-(1+K)R_L=(1+K)(M^L-R_L)>0\end{cases}\end{aligned}$$

- in bad aggregate state, the tax budget balance constraints:
  - the proportion of failed insurers: $q(1-\tau)+\tau$
  - the proportion of successful insurers: $(1-q)(1-\tau)$
  - taxation balance:

$$\big(q(1-\tau)+\tau\big)\cdot T_S^L+(1-q)(1-\tau)\cdot T_F^L=(1+K)(M^L-M^L)=0$$

Similarly if in the good aggregate state:

- in good aggregate state,
  - successful insurer's payoff: $\big((1+K\big)R_H + T_S^H - d_H$
  - failed insurer's payoff: $\big(1+K\big)R_L + T_F^H - d_H$
  - each insurer don't bear idiosycratic risk

$$\big((1+K\big)R_H + T_S^H - d^H = \big(1+K\big)R_L + T_F^H - d^H = C^{H*} = \frac{ER \cdot K}{1-q}$$

$$\Rightarrow \begin{cases} T_S^H & = d^H + \frac{ER \cdot K}{1-q} - (1+K)R_H = (1+K)(M^H - R_H) < 0 \\ T_F^H & = d^H + \frac{ER \cdot K}{1-q} - (1+K)R_L = (1+K)(M^H - R_L) > 0 \end{cases}$$

- in good aggregate state, the tax budget balance constraints:
  - the proportion of failed insurers: $q(1-\tau)$
  - the proportion of successful insurers: $(1-q)(1-\tau)+\tau$
  - taxation balance:

$$\big((1-q)(1-\tau)+\tau\big) \cdot T_S^H + q(1-\tau) \cdot T_F^H = (1+K)(M^H - M^H) = 0$$

**In sum:**

$$\begin{cases} T_S^L & = (1+K)(M^L - R_H) \\ T_F^L & = (1+K)(M^L - R_L) \\ T_S^H & = (1+K)(M^H - R_H) \\ T_F^H & = (1+K)(M^H - R_L) \end{cases}$$

Insurees invest all their capital in buying risky insurance contracts, insurers invest all their capital in risky assets, and regulators's taxes transfers are given as above.

**CASE THREE**: $\tau > \tau_2$, insurees cannot be perfectly insured, thus the insurees' payoff in two states are:

- in bad aggregate state: $\beta R_f + d^L(1-\beta) - pl = (1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl$
- in good aggregate state:

$\beta R_f + d^H(1-\beta) - pl = (1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K$

Thus insuee's problem becomes:

$$\max_{\beta}(1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)+q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)$$

subject to

$$0 \leq \beta \leq 1$$

Thus

$$\begin{aligned}\mathcal{L} &= (1-q)\ln\big((1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\big) \\ &\quad +q\ln\big((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\big) + \lambda_1\beta + \lambda_2(1-\beta)\end{aligned}$$

FOC:

$$\frac{(1-q)(R_f - M^H)}{(1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K} + \frac{q(R_f - M^L)}{(1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl} + \lambda_1 - \lambda_2 = 0$$

that is

$$\frac{q(R_f - M^L)}{(1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl} - \frac{(1-q)(M^H - R_f)}{(1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K} = \lambda_2 - \lambda_1$$

Suppose $\lambda_2 = \lambda_1 = 0$, that is $0 < \beta < 1$

Thus the opitmal $\beta^*$is

$$\beta^* = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha K$$

Thus insuree' utility is given by

$$(1-q)\ln\Big((1+K)M^H - \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(R_f - M^L)} - pl - \frac{ER}{1-q}K\Big)$$
$$+q\ln\Big((1+K)M^L + \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)} - pl\Big)$$

for any $\alpha$, $\beta = \beta^*(\alpha)$ such that insuree's welfare won't change. Thus we need

$$\beta^* = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha K < 1$$

If $\frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} \leq 1$, $\alpha^*$can be any number between 0 and 1. If $1 < \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} < 1 + K$, then $\alpha^*$has to be greater than $\frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K}$ .

Regulator impose the minimum requirement of liquidity buffer $\alpha^*$ such that any

$$\alpha^* \in \left(\max\left\{\frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K},\ 0\right\},\ 1\right)$$

, in safe assets, and insurees invest $\beta^*$where

$$\beta^* = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha^* K$$

and the opitmal insurance contracts are

$$d^L = \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f}{1 - \beta^*}$$

$$d^H = \frac{(1+K)M^H + (\beta+\alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f}{1-\beta^*}$$

Now we derive the optimal taxation scheme.

- in bad aggregate state,
  - successful insurer's payoff:

$$\big(1+K-(\beta^*+\alpha^* K)\big)R_H + T_S^L - d^L(1-\beta^*)$$

  - failed insurer's payoff:

$$\big(1+K-(\beta^*+\alpha^* K)\big)R_L + T_F^L - d^L(1-\beta^*)$$

  - each insurer doesn't bear idiosycratic risk:

$$\begin{aligned} & \big(1+K-(\beta^*+\alpha^* K)\big)R_H + T_S^L - d_L(1-\beta^*) \\ = \quad & \big(1+K-(\beta^*+\alpha^* K)\big)R_L + T_F^L - d_L(1-\beta^*) \\ & = C^{L*} = -\alpha^* K R_f \end{aligned}$$

$$\Rightarrow \begin{cases} T_S^L & = d_L(1-\beta^*) - \alpha^* K R_f - \big(1+K-(\beta^*+\alpha^* K)\big)R_H \\ & = (1+K-(\beta^*+\alpha^* K))(M^L - R_H) < 0 \\ T_F^L & = d_L(1-\beta^*) - \alpha^* K R_f - \big(1+K-(\beta^*+\alpha^* K)\big)R_L \\ & = (1+K-(\beta^*+\alpha^* K))(M^L - R_L) > 0 \end{cases}$$

- in good aggregate state,
  - successful insurer's payoff:

$$\big(1+K-(\beta^*+\alpha^* K)\big)R_H + T_S^H - d^H(1-\beta^*)$$

– failed insurer's payoff:

$$\big(1 + K - (\beta^* + \alpha^* K)\big)R_L + T_F^H - d^H(1 - \beta^*)$$

– each insurer doesn't bear idiosycratic risk:

$$\begin{aligned} &\big(1 + K - (\beta^* + \alpha^* K)\big)R_H + T_S^H - d_H(1 - \beta^*) \\ = &\big(1 + K - (\beta^* + \alpha^* K)\big)R_L + T_F^H - d_H(1 - \beta^*) \\ = &C^{H^*} = \tfrac{ER \cdot K}{1-q} - \alpha K \cdot R_f \end{aligned}$$

$$\begin{cases} T_S^H & = d^H(1 - \beta^*) + \frac{ER \cdot K}{1-q} - \alpha K \cdot R_f - \big(1 + K - (\beta^* + \alpha^* K)\big)R_H \\ & = \big(1 + K - (\beta^* + \alpha^* K)\big)(M^H - R_H) < 0 \\ T_F^H & = d^H(1 - \beta^*) + \frac{ER \cdot K}{1-q} - \alpha K \cdot R_f - \big(1 + K - (\beta^* + \alpha^* K)\big)R_L \\ & = \big(1 + K - (\beta^* + \alpha^* K)\big)(M^H - R_L) > 0 \end{cases}$$

The tax budget balance constraints:

- in bad aggregate state
  – the proportion of failed insurers: $q(1 - \tau) + \tau$
  – the proportion of successful insurers: $(1 - q)(1 - \tau)$
  – taxation balance:

$$(1 - q)(1 - \tau) \cdot T_S^L + \big(q(1 - \tau) + \tau\big) \cdot T_F^L = (1 + K - (\beta^* + \alpha^* K))(M^L - M^L) = 0$$

- in good aggregate state
  – the proportion of failed insurers: $q(1 - \tau)$
  – the proportion of successful insurers: $(1 - q)(1 - \tau) + \tau$

– taxation balance:

$$q(1-\tau)\cdot T_F^H + \big((1-q)(1-\tau)+\tau\big)\cdot T_S^H = (1+K-(\beta^*+\alpha^* K))(M^H - M^H) = 0$$

Thus we have:

$$\begin{cases} T_S^L & = (1+K-(\beta^*+\alpha^* K))(M^L - R_H) \\ T_F^L & = (1+K-(\beta^*+\alpha^* K))(M^L - R_L) \\ T_S^H & = \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_H) \\ T_F^H & = \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_L) \end{cases}$$

■

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