

Incentives and Endogenous Risk Taking: A Structural View on Hedge Fund Alphas

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ABSTRACT

This paper studies the link between optimal portfolio choice when the manager is subject to non-linear performance incentives and ex-ante performance attribution measures. We study and compare structural versus reduced form measures of alpha and Sharpe ratios and document the existence of a significant bias in traditional reduced-form measures. The empirical estimation of the structural model allows us to use previously unexploited information about conditional second moments to draw inference about genuine risk-adjusted performance. Intuitively, the structural approach allows us to distinguish the effect of endogenous risk taking and skill from past fund performance, thus providing superior forecasts of hedge fund performance. We extend the work of Kojen (2010) on mutual funds by explicitly modelling hedge fund specific contractual features such as (i) incentive options, (ii) equity investor's redemption options and (iii) primer broker contracts that together create option-like payoffs and affect a hedge fund's risk taking. The optimal investment strategy derived from the model reveals that portfolio leverage depends on the distance to high-water mark. The call option creates an incentive to increase leverage while the put option reduces this incentive when distance to high-water mark is above a certain threshold. Out-of-sample, we show that portfolios formed using structural measures outperform portfolios based on reduced-form measures.

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THIS PAPER STUDIES THE IMPLICATIONS that non-linear managerial incentives and funding contracts have on traditional reduced-form tests of performance attribution for hedge funds. We solve the structural optimal portfolio choice problem of a hedge fund investor who is subject to (i) performance fee-based incentives, (ii) funding options by the prime broker, and (iii) equity investor's redemption options, which together create a non-linear payoff structure that affects endogenous hedge funds' risk taking. The resulting optimal portfolio choice is state-dependent due to the time-varying endogenous incentives perceived by the manager, depending on the distance of the assets under management from the high-water mark. This implies that optimal leverage and reduced-form alphas fluctuate over time. This is important since it implies that traditional performance regressions with constant coefficients are potentially misspecified. The call option-like performance fee incentive motivates the manager to use more leverage, while put option-like features (together with the concern about the future value of the incentive options) induce the manager to reduce leverage, when her fund underperforms below a given threshold. We use the results of the model to estimate, using a large panel dataset of hedge fund returns, the difference between reduced-form alpha and model-implied true managerial skill, which corrects for endogenous risk taking. The empirical method allows us to use previously unexploited information about endogenous risk preference to draw inference about genuine risk-adjusted performance.

Although we use hedge funds in our empirical application, our results have a much broader economic motivation. Separating the effect of risk taking and skill, based on observed investment performance, is a fundamental problem that not only affects investors in alternative investment funds, but also investors in (and regulators of) levered financial institutions such as investment banks which employ incentive contracts. A Financial Times article in 2009 quotes a Bank of England official as saying that "The superior performance of the financial services sector in the years leading up to the credit crisis was almost entirely due to luck rather than skill – and banks increasingly gambled on luck in an effort to keep up with their peers. [...] Good luck and good management need to be better distinguished".¹ This quote illustrates not only the risk management challenges that policy makers face but also the performance attribution problem that investors face. Using traditional tools it is difficult to distinguish performance that is due to true investment skill from that attributable to excessive risk taking.

An extensive literature has investigated the return characteristics of hedge funds. The seminal

¹'Bank profits were due to 'luck, not skill', By Norma Cohen (July 1, 2009)

papers of Fung and Hsieh (1997 and 2004) and Agarwal and Naik (2004) highlights the importance of taking into account the ability of hedge fund to invest in derivatives and suggest empirical methods to correct performance attribution measures by option strategies. These extensions are economically and statistically important. A recurrent result, however, is that even after these corrections, reduced-form (after-fees) alphas in hedge funds are large and positive (Fung and Hsieh, for instance, report a monthly alpha of 0.78% for the TASS dataset). This evidence contrasts with the negative (after-fees) reduced-form alphas obtained for other delegated managers such as mutual funds. The result could be due to a misspecification of the time-varying investment opportunity set. Patton and Ramadorai (2011) allow for dynamic risk exposure using threshold conditioning information based on market-wide economic conditions. They find that their dynamic approach succeeds in reproducing realized hedge fund returns with substantial increased accuracy. However, they find that even after controlling for time-varying macroeconomic beta the reduced-form alphas are similar in size to Fung and Hsieh (2004).

In this paper, we argue that reduced-form alphas can be affected by an endogeneity bias: fund performance is the outcome of a portfolio management decision in which the incentive contracts of the manager makes optimal leverage endogenous. Since hedge funds have substantial discretion in selecting their leverage, this bias is potentially significant. In a simple Merton-type economy, in which the incentives of the manager are aligned to those of the investor, the optimal portfolio choice is time-invariant. In this context traditional reduced-form alphas are both unbiased and efficient. When the manager faces non-linear incentives, however, the conflicts of interest between the manager, the prime broker and the investor induce the manager to dynamically adjust the fund's leverage depending on the distance of the asset values from the fund's high-water mark. The resulting estimator for alpha is biased. Reduced-form estimates of alpha make it difficult, therefore, to disentangle whether a manager's performance stems from his genuine skill or if it has just been the result of the interaction of luck and excessive endogenous risk taking. The first advantage of our structural approach is to explicitly take into account this endogeneity and address this potential bias. The second advantage is in terms of the efficiency of the estimator for skill measure. Our structural approach allows us to use information in time-varying second moments, thus improving the statistical efficiency of the estimator of genuine skill. This is potentially important since reduced-form alphas are based on relatively short samples of data and thus likely to be imprecisely estimated. We build on and extend the pioneering work of Kojen (2010), who applies a structural approach to estimate skill and risk preferences of mutual funds. Kojen (2010) suggests a novel approach to recover cross-sectional

information of mutual fund manager’s managerial ability and risk preferences by using the Euler condition of a mutual fund manager. He shows that the restrictions derived from structural portfolio management models can be used to recover both attributes from the data. However, the application of Kojien’s approach to hedge funds is not straightforward since incentive contracts that govern hedge funds are significantly more complex than those of mutual funds.

While both mutual funds and hedge funds are delegated managers, they differ in terms of most other institutional dimensions: legal framework, capital structure, investment mandate and business model. First, hedge funds’ limited partnership structure allows for both asset segregation and liquidity lock-ups imposed on investors, which facilitate prime brokerage contracts.² The private nature of their legal structure grants them the contractual flexibility in setting long-term lock-ups to investors, whose legal rights are those of a (limited) partner, as opposed to a client. Second, investment advisors that manage hedge funds typically receive an asset management fee and a performance fee. The performance fee is typically subject to a high-water mark, which means that the manager receives a compensation only on increases in the net asset value (NAV) of the fund in excess of the highest net asset value it has previously achieved. The high-water mark provisions can be viewed as a call option issued by the investors to the fund manager (Goetzmann, Ingersoll, and Ross (2003)). However, the funding role played by the prime-broker also makes the capital structure of hedge funds potentially fragile: As the 2007-2008 experience shows, when counterparty risk becomes acute during systemic events, prime brokers tend to increase hedge funds’ collateral requirements and mandate haircuts in response to higher perceived counterparty risk, thus inducing forced deleveraging of risky positions. Dai and Sundaresan (2010) note that a hedge fund’s contractual relationships with its equity investors and prime broker can be considered as short option positions. Therefore, these two option-like features affect the hedge fund’s balance sheet, both from the perspective of its assets and its liabilities. Any structural model of hedge fund behavior must account for these features as they affect the objective function of the fund manager.

The first contribution of this paper is to account for contractual features such as (i) incentive options, (ii) equity investor’s redemption options and (iii) prime broker contracts, which jointly creates option-like payoffs and to show how this affect endogenous risk taking and generates a bias in the reduced-form alpha. The solution to the optimal investment problem is a dynamic investment

²The prime broker plays an essential role in the capital structure of a hedge fund. By contrast, most mutual funds, as Almazan et al. (2004) document, are restricted (by government regulations or investor contracts) with respect to using leverage, holding private assets, trading OTC contracts or derivatives, and short-selling; see also Koski and Pontiff (1999), Deli and Varma (2002) and Agarwal, Boyson, and Naik (2009).

strategy in which optimal leverage is endogenous and depends on the distance of the NAV from the high-water mark. The second contribution is to study the implications of the model to a large sample of hedge funds and investigate the difference between structural and reduced-form estimates of alpha. We impose the structural restrictions implied by economic theory to estimate the deep parameters of the model. These restrictions allow us to separately identify true skill by controlling for endogenous risk appetite. We document the differences between reduced-form regression estimates of performance and the estimates based on the structural true skill and investigate the features that contribute to the bias. We find strong direct evidence that leverage is endogenous and state dependent. The functional form is not monotone in performance, but it is bell-shaped and centered just below the high-water-mark. This is consistent with the model and with the fact that hedge funds are both long a call options and short a put option. This creates a bound for the optimal endogenous leverage: the agency contract with the prime broker and the redemption option of the investors are effective in limiting leverage when a fund's value drops below the high-water mark. Finally, we conduct a simple out-of-sample test and compare the performance of portfolios of hedge funds constructed by ranking funds on the basis of different ex-ante performance measures, in particular: (a) traditional reduced-form alpha, and (b) structural estimate of true skill that is insensitive to endogenous risk taking. We find that true skill estimates lead to superior out-of-sample performance and thus dominate reduced-form ones. This is interesting since it shows the practical importance of controlling for managerial incentives and endogenous risk taking and provides a useful methodology that can be used by funds of hedge funds.

1 Literature review

Our work is related to three streams of the literature. First, we build on the work of Carpenter (2000) who examines the dynamic investment problem of a risk averse manager compensated with a call option on the assets he controls. She shows that under the manager's optimal policy, as the asset value goes to zero, volatility goes to infinity. However, the option compensation does not strictly lead to greater risk seeking: sometimes, the manager's optimal volatility is lower with the option than it would be if she were trading her own account, that is without the option. While the importance of convex incentives for the optimal risk-taking behavior of a hedge fund manager has been long noted, its implications are not trivial. Initially, one might think that the option-like character of hedge funds' performance fee contracts could induce the manager to assume extremely risky positions in the hope of huge payoffs, especially as the manager does not share directly any loss of the fund's

assets. However, this does not necessarily hold in the case of performance fee incentives with high-water mark features. Ross (2004) presents necessary and sufficient conditions for a general class of compensation structures and preferences to make a manager more or less risk averse. Panageas and Westerfield (2009) extend the work of Carpenter (2000) to account for high-water mark restrictions and show that while risk-taking may be increasing in the convexity of incentives in the context of a static two period model, in an intertemporal model this may no longer be the case. Since any loss of fund assets reduces the value of the anticipated sequence of future call options, higher risk taking today increases the risk of losing future options. This is due to the fact that the current high-water mark remains as the effective strike price: all later call options must have strike prices higher than the current high-water mark. The authors show that even risk-neutral managers do not take unbounded risk, despite convex payoffs, when the contract horizon is in(de)finite, thus reversing some of the results obtained in static two period models. Hodder and Jackwerth (2007) investigate the incentive effects of a typical hedge-fund contract as a function of the incentive horizon. They find that the risk shifting behavior of the manager can be highly non-linear and they observe that small changes in the compensation structure can have important implications for risk-taking. Continuous instead of discrete resetting of the high-water mark, for example, can have a dramatic impact on optimal risk-taking by the manager. They also show that the possibility that the fund can be shut down in case of poor performance has significant implications for the manager's risk shifting behavior. Another important contribution in this literature is by Guasoni and Obloj (2011) who study the case of a manager with constant relative risk aversion who maximizes the utility from fees over a long horizon. Basak, Pavlova, and Shapiro (2007) study the implications of a non-linear relationship between mutual fund flows and performance on the manager's risk taking behavior. Even though the mutual fund fee structure is linear in assets under management, the incentive to attract more funds creates convexities in the manager's objective function which leads to risk-shifting. In this paper, we build on these earlier papers and further develop the analysis of optimal risk taking by considering not only the payoff convexity related to the performance fee structure, but also the capital structure fragility induced by the possibility of investors withdrawing capital and prime-brokers forcing deleveraging. In general, this additional ingredient does not allow for closed-form solutions. However, by restricting the incentive contract to a two-period problem we can obtain an expression which is in closed-form up to the solution of a differential equation. This allows us to implement a maximum likelihood estimation of the model, which we use to study empirically the difference between reduced-form and structural alphas. An important difference with the previous literature is that the optimal portfolio

choice is not constant, but it is state dependent and it is a function of the distance to high-water mark. The interaction of capital structure fragility and high-water mark restrictions gives rise to important non-linearities in the endogenous risk-taking behavior.

A second stream of the literature, studies the role of incentive arrangement and discretion in hedge fund performance. Agarwal, Daniel, and Naik (2009) conclude that the level of managerial incentives affects hedge fund returns. In particular, funds with better managerial incentives (such as higher option deltas, greater managerial ownership and the presence of high-water mark provision) delivers better performance. Aragon and Nanda (2012) empirically analyze risk shifting by poorly performing hedge funds and test predictions on the extent to which risk choices are related to the fund's incentive contract, risk of fund closure and dissemination of performance information. Both papers compute measures of risk-adjusted performance using reduced-form regressions. In our paper, we investigate how convex incentives affect endogenous risk taking behavior in the context of a structural model. We document how the bias varies across different fund strategies. Then, we document the value of explicitly accounting for these agency effects by studying the ex-post performance of funds selected based on their high reduced-form alpha versus structural alpha. We find that selecting funds based on structural measures of skill leads to superior out-of-sample performance.

Third, several studies shed light on the fragility of the capital structure of levered financial institutions, such as hedge funds (Dai and Sundaresan (2010), Liu and Mello (2011) and Brunnermeier and Pedersen (2009)). We build on their insights and incorporate some of these features into our structural model.

These results are useful to hedge fund investors and regulators. It can provide investors with a framework to better assess hedge fund performance and the endogenous effects of their underlying contractual relationship. It can assist regulators in accounting for the effects of agency contracts in an attempt to limit a manager's excessive risk taking behavior while providing them the flexibility to deploy their managerial abilities.

The paper is structured as follows. In Section 2 we develop a structural model of the hedge fund manager's objective function. Section 3 presents the estimation methodology. Section 4 describes the data that we use in this study. Section 5 and 6 presents empirical results for the application of the structural model to hedge fund returns. Section 7 concludes.

2 Structural Model and Manager's Investment Problem

In this section we describe our structural model by first defining the investment opportunity set of a hedge fund manager. Then we provide the optimal investment solution of the manager's portfolio problem. According to the optimal portfolio solution we derive restrictions on traditional $\hat{\alpha}_{OLS}$, $\hat{\beta}_{OLS}$, $\hat{\sigma}_{\varepsilon, OLS}$ which are estimated from reduced-form regression. Finally we study the model implications for reduced form performance and compare them to "true" managerial ability.

2.1 Trading technology

We assume that the hedge fund manager can trade in a money market account with a constant interest rate r :

$$dS_t^0 = S_t^0 r dt. \quad (1)$$

She can also invest in a strategy-specific benchmark asset which follows

$$dS_t^B = S_t^B (r + \sigma_B \lambda_B) dt + S_t^B \sigma_B dZ_t^B. \quad (2)$$

The first two assets are commonly available to all managers within the same investment style. However, individual fund managers differ in terms of their pure alpha generating asset, which is assumed to be fund specific.³ We interpret the asset as an unleveraged return generating process which is orthogonal to the strategy-wide exposure, S_t^B . The pure alpha investment technology can be summarized by a process S_t^A which follows

$$dS_t^A = S_t^A (r + \alpha^*) dt + S_t^A \sigma_A dZ_t^A, \quad \text{with } E(dZ_t^A dZ_t^B) = 0. \quad (3)$$

This asset is fund specific. The larger α^* , the better the true alpha generating skill. Since one can write $\alpha^* \equiv \sigma_A \lambda_A$, therefore λ_A can also be interpreted as the Sharpe ratio of the alpha generating asset.

³To keep notation to a minimum, in this section we avoid specifying that (a) the return processes as a style- j dependent, with $dS_t^{j,B} = S_t^{j,B} (r + \sigma_B^j \lambda_B^j) dt + S_t^{j,B} \sigma_B^j dZ_t^{j,B}$ for $j = 1, \dots, J$ and (b) the alpha-technology is fund- i specific within style j , with $dS_t^{ij,A} = S_t^{ij,A} (r + \alpha_B^{ij}) dt + S_t^{ij,A} \sigma_B^{ij} dZ_t^{ij,A}$ for $i = 1, \dots, I$. These consideration will become relevant in the empirical section.

2.2 Manager's Investment Problem

The hedge fund manager decides the proportion of her fund's wealth, θ_t , to invest in the investment opportunity set. Thus, the fund asset value evolves as:

$$dX_t = X_t(r + \theta_t' \mu)dt + X_t \theta_t' \Sigma dZ_t \quad (4)$$

where $\mu \equiv (\sigma_A \lambda_A, \sigma_B \lambda_B)'$, $\bar{\sigma}_A \equiv (\sigma_A, 0)'$, $\bar{\sigma}_B \equiv (0, \sigma_B)'$, $\Sigma \equiv (\bar{\sigma}_A, \bar{\sigma}_B)$ and $Z_t = (Z_t^A, Z_t^B)'$

The manager's investment problem is to decide on an optimal trading strategy, θ_t , to maximize the following objective function

$$\max_{(\theta_s)_{s \in [0, T]}} E_0[U(p(X_T - HWM)^+ + mX_T - c(K - X_T)^+)] \quad (5)$$

subject to the budget constraint (4) and the restrictions that $0 \leq X_t$, $\forall t \in [0, T]$ and $K \leq HWM$. The parameter p denotes performance fee, m denotes management fee and c denotes the level of concern regarding the short put option positions (the funding and redemption options). In general, the high-water mark level is defined as $HWM_t = \max_{s \leq t} X_s$. This implies that future high-water mark values are affected by managerial decisions, so that the manager faces a sequence of call options that depend on past actions. Examples of studies that explicitly account for this effect include Panageas and Westerfield (2009) and Guasoni and Obloj (2011). Using different settings, they discuss how the risk-seeking incentives of option-type compensation contracts affect an infinite horizon investor. In their contexts, the optimal portfolio solution can be derived in closed-form: the optimal portfolio weights are constant and independent of the distance between fund value, X_t , and the high-water mark, HWM .

In this paper, we are interested in studying the implications of the finite horizon problem since in this case the risk-shifting behavior can become state dependent. On the one hand, to retain tractability, we make the simplifying assumption that the high-water mark HWM is fixed and pre-specified at the beginning of the contracts' evaluation period. On the other hand, we allow for the hedge fund manager to derive her utility from three components. The first two components, the management and performance fee, are common in most hedge fund compensation contracts. The first component is a claim to a fraction p of a call option-like payoff with the fund's high-water mark as its strike price. The level of the high-water mark is the running maximum process of the fund's historical net asset value and is known at the beginning of every evaluation periods. The second

component is a linear claim on fund value at the end of the evaluation period. The management fee constitutes a fraction m of the fund value. The last component is intended to capture the nature of the relationship of the hedge fund with its prime broker. Typically, a hedge fund's contract with its prime broker gives the prime broker the right to reduce the supply of funding or to increase margin calls in adverse economic states or when the counterparty risk of the hedge fund is considered dangerously high. According to Dai and Sundaresan (2010), the relationship between a hedge fund and its prime broker can be thought as a short put option position. We use the parameter c to capture the deadweight costs induced by the forced deleveraging imposed by the prime broker. A similar situation is also induced by the contractual arrangement of the hedge fund with its investors, which gives investors the option to redeem their shares upon request. Hodder and Jackwerth (2007) study a scenario that allows for the possibility of an endogenous fund closure decision by the hedge fund manager and they solve the problem numerically.⁴ In contrast, we focus on the implication of options embedded in the capital structure of a hedge fund. The options are put options given to investors and prime brokers of the hedge fund. As a result, the manager does not have control over the exercise of the options. Jurek and Stafford (2011) find supporting evidence that the risk profile of hedge funds resembles that of a short index put option strategy during negative systematic shocks.

2.3 Model's solution

If markets are complete, one can use the martingale approach developed in Cox and Huang (1989) to solve the optimal investment problem. This allows us to solve an easier static problem, instead of solving the dynamic investment problem. Let φ_t be the state price process, which follows the process:

$$\frac{d\varphi_t}{\varphi_t} = -r dt - \lambda' dZ_t, \quad (6)$$

where $\lambda \equiv (\lambda_A, \lambda_B)'$. When markets are complete, there exists a unique non-negative state price process φ_t such that the solution of the optimal investment problem is such that:

$$X_T = \arg \max E_0[U(p(X_T - HWM)^+ + mX_T - c(K - X_T)^+)], \quad (7)$$

subject to

$$E_0[\varphi_T X_T] = X_0 \quad (8)$$

⁴The shutdown decision represents an option given to the manager to liquidate her fund and accept outside work opportunities or to continue managing the fund when the fund performs poorly.

and $0 \leq X_T$. The utility function is defined as

$$U(X_T, HWM, K, p, m, c) = \begin{cases} U(p(X_T - HWM) + mX_T) & \text{for } X_T > HWM \\ U(mX_T) & \text{for } K < X_T \leq HWM \\ U(mX_T - c(K - X_T)) & \text{for } b < X_T \leq K \end{cases}, \quad (9)$$

we make the technical assumption that $U(X_T) = -\infty$ for $X_T \leq b$, where b is the fund value at which the hedge fund manager would otherwise begin to receive negative utility. This is illustrated in Figure 1 and the utility function is of the following CRRA form: $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$.

[Insert Figure 1 here]

Notice that because of the non-linear managerial incentives, the objective function $U(X_T, HWM, K, p, m, c)$ is not globally concave in X_T . It is rational for the manager, therefore, to randomize her investment strategies to achieve higher expected returns. To solve the problem, we use the concavification techniques discussed and used in Carpenter (2000) and Basak, Pavlova, and Shapiro (2007).

Condition 1 \hat{X}_{HWM} and \hat{X}_K exist and satisfy the following conditions

1. $\hat{X}_{HWM} > HWM$ and $\hat{X}_K < K$
2. $U'(p(\hat{X}_{HWM} - HWM) + m\hat{X}_{HWM}) = U'(m\hat{X}_K - c(K - \hat{X}_K))$
3. $U(p(\hat{X}_{HWM} - HWM) + m\hat{X}_{HWM}) = U(m\hat{X}_K - c(K - \hat{X}_K)) + U'(m\hat{X}_K - c(K - \hat{X}_K))(\hat{X}_{HWM} - \hat{X}_K)$
4. $U(p(X_T - HWM) + mX_T) \leq U(m\hat{X}_K - c(K - \hat{X}_K)) + U'(m\hat{X}_K - c(K - \hat{X}_K))(X_T - \hat{X}_K) : \forall X_T \in [HWM, \hat{X}_{HWM}]$

Under Condition 1, the concavification ensures the continuity of the manager's objective function at the concavification points. Figure 2 displays the concavified objective function under Condition 1. In this figure, the original terminal utility function between \hat{X}_K and \hat{X}_{HWM} is dominated ex-ante by the convex combination illustrated by the dotted line, which can be interpreted as a randomization of the initial investment strategies.

In general Condition 1 is satisfied when risk aversion, γ , is low. However, if Condition 1 is violated, one of the concavification points has to occur at $\hat{X}_K = K$, so that the problem admits a closed form solution for high levels of risk aversion, γ . This causes a discontinuity of the concavified objective function which results in a corner solution (Basak, Pavlova, and Shapiro (2007)). The solution to this case is similar to Proposition 1 but with a constraint $\hat{X}_K = K$. Basak, Pavlova, and Shapiro (2007) argue that it is possible to adapt the martingale representation and convex-duality techniques to the non-concave problem.

[Insert Figure 2 here]

Therefore, the concavified objective function reads:

$$\tilde{U}(X_T, HWM, K, \alpha, m, c) = \begin{cases} U(p(X_T - HWM) + mX_T) & \text{for } X_T > \hat{X}_{HWM} \\ U(m\hat{X}_K - c(K - \hat{X}_K)) + \nu(X_T - \hat{X}_K) & \text{for } \hat{X}_K < X_T \leq \hat{X}_{HWM} \\ U(mX_T - c(K - X_T)) & \text{for } b < X_T \leq \hat{X}_K \end{cases}, \quad (10)$$

where $\nu = \frac{U(p(\hat{X}_{HWM} - HWM) + m\hat{X}_{HWM}) - U(m\hat{X}_K - c(K - \hat{X}_K))}{\hat{X}_{HWM} - \hat{X}_K}$

We then solve the problem of a hedge fund manager described in function (7) with the concavified objective function (10). In terms of the new objective function, the manager solves $\max_{X_T} E_0[\tilde{U}(X_T, HWM, K, \alpha, m, c)]$ subject to $E_0[\varphi_T X_T] = X_0$. The first-order conditions of the optimization problem require, firstly, that the manager's marginal utility is proportional to the level of the state price at the terminal time and, secondly, that the budget constraint is satisfied. These conditions uniquely determine the optimal terminal fund value $X_T^* \equiv I(\zeta\varphi_T)$, where ζ is the Lagrange multiplier corresponding to the static constraint, $I \equiv (\tilde{U}')^{-1}$. Then the stochastic process of the optimal fund value can be determined by $X_t^* = E_t(\frac{\varphi_T}{\varphi_t} X_T^*)$. Using Ito's lemma, we can derive the dynamics of dX_t^* and compare the coefficients of its diffusion term to equation (4) to obtain the optimal allocation θ_t^* . The result is summarized in Proposition 1. We simplify the derivation of $E_t(\frac{\varphi_T}{\varphi_t} X_T^*)$ by mapping optimal terminal fund values to terminal state prices as in Carpenter (2000). Appendix A shows the derivation in details.

Proposition 1 *Under Condition 1, the optimal allocation of a hedge fund manager facing non-linear*

incentives is given by

$$\theta_t^* = -\frac{\partial X^*(t, \varphi_t)}{\partial \varphi_t} \frac{\varphi_t}{X_t^*} (\Sigma \Sigma')^{-1} \Sigma \lambda \quad (11)$$

where

$$\begin{aligned} \frac{\partial X^*(t, \varphi_t)}{\partial \varphi} = & -\frac{\zeta^{-1/\gamma} G_t}{\varphi_t^{\frac{1+\gamma}{\gamma}} (p+m)^{1-1/\gamma}} \left(\frac{N(d_{2,t}^{z_{HWM}})}{\gamma} + \frac{N'(d_{2,t}^{z_{HWM}})}{\|\lambda\| \sqrt{T-t}} \right) - \frac{\alpha HWM}{\varphi_t(\alpha+m)} \frac{H_t N'(d_{1,t}^{z_{HWM}})}{\|\lambda\| \sqrt{T-t}} \\ & - \frac{\hat{X}_{HWM} H_t}{\varphi_t \|\lambda\| \sqrt{T-t}} \left[N'(d_{1,t}^{z_K}) - N'(d_{1,t}^{z_{HWM}}) \right] \\ & - \frac{\zeta^{-1/\gamma} G_t}{\varphi_t^{\frac{1+\gamma}{\gamma}} (m+c)^{1-1/\gamma}} \left[\frac{N'(d_{2,t}^{z_b}) - N'(d_{2,t}^{z_K})}{\|\lambda\| \sqrt{T-t}} + \frac{N(d_{2,t}^{z_b}) - N(d_{2,t}^{z_K})}{\gamma} \right] \\ & - \frac{cKH_t}{\varphi_t(m+c) \|\lambda\| \sqrt{T-t}} (N'(d_{1,t}^{z_b}) - N'(d_{1,t}^{z_K})) + b \frac{H_t N(-d_{1,t}^{z_b})}{\varphi_t \|\lambda\| \sqrt{T-t}} \end{aligned} \quad (13)$$

where: $d_{1,t}^{z_i} \equiv \frac{\ln(z_i/\varphi_t) + (r-0.5\lambda'\lambda)(T-t)}{\|\lambda\| \sqrt{T-t}}$, $d_{2,t}^{z_i} \equiv d_{1,t}^{z_i} + \frac{\|\lambda\| \sqrt{T-t}}{\gamma}$, $C_t \equiv \exp(\ln(\varphi_t) - r(T-t))$, $D_t \equiv \exp(\frac{1-\gamma}{\gamma} \ln(\varphi_t) - \frac{1-\gamma}{\gamma} (r+0.5\lambda'\lambda)(T-t) + 0.5 \left(\frac{1-\gamma}{\gamma} \right)^2 \|\lambda\|^2 (T-t))$, $G_t \equiv \exp(-\frac{1-\gamma}{\gamma} (r+0.5\lambda'\lambda)(T-t) + 0.5 \left(\frac{1-\gamma}{\gamma} \right)^2 \|\lambda\|^2 (T-t))$, $H_t \equiv \exp(-r(T-t))$ and $N(x)$ denotes the cumulative Normal distribution function. The characterization for X_t^* is made explicit in Appendix A.

Although the optimal allocation described in equation (11) might look complex, it can be rewritten as

$$\theta_t^* = \frac{(\Sigma \Sigma')^{-1} \Sigma \lambda}{\tilde{\gamma}_t} \quad (14)$$

where $\tilde{\gamma}_t$ can be interpreted as the effective risk aversion. The key result of Proposition 1 is that $\tilde{\gamma}_t$ is endogenous and state-dependent: $\tilde{\gamma}_t(\gamma, \lambda, X_t, HWM, K, p, m, c) \equiv -\frac{\partial X^*(t, \varphi_t)}{\partial \varphi_t} \frac{\varphi_t}{X_t^*}$. Equation (14) reveals that, even if the investment opportunity set were constant (as in Merton), agency contracts make the optimal allocation state dependent. The optimal allocation is not only driven by the characteristics of the investment opportunity set and the specific risk aversion of the hedge fund manager, but also by the contractual parameters such as fee rates (p , m and c), the distance of fund value from HWM , the existence of short put option-like positions and its perceived strike value K .

Figure 3 displays the results implied by (14). When fund value exceeds the high-water mark level, the manager allocates a lower amount of her funds to the risky asset than would be implied by Merton (1969)⁵ benchmark solution. When her call option starts being in the money, the manager has an incentive to reduce leverage, thus reducing her fund volatility, to lock-in her performance

⁵The Merton (1969) constant allocation is $\frac{\mu}{\gamma \sigma^2}$

fee. However, the results suggest that the manager begins to increase her leverage with respect to Merton’s allocation when her fund value is considerably higher than the high-water mark. The results in this region are similar to the analytical result of the problem studied in Carpenter (2000) as well as the numerical solution to the problem in Hodder and Jackwerth (2007). However, our result implies different dynamics when the fund value is less than the high-water mark. Unlike the manager who is maximizing only performance fees (Figure 4, dashed line), the fund manager in our set up doesn’t monotonically increase her allocation to the risky asset as fund value decreases, but instead reduces her leverage as fund value decreases toward the put’s option strike. The presence of put options induces the manager to reduce leverage quite aggressively. The optimal leverage can fall below what is implied by the Merton benchmark solution (Figure 4, solid line) and the manager continues reducing leverage until a level at which the manager is about to receive negative utility. The manager then increases leverage again when fund value falls below that point.⁶

[Insert Figure 3 and 4 here]

Thus, a simple calibration of the model shows that the economic magnitude of the difference in endogenous leverage induced by realistic parameters for the agency contracts can be significant. The empirical implications of these effects and their role in performance attribution measures are the questions that we study next.

2.4 Model’s Implied Restrictions and Bias in Reduced-Form Regression Alpha

The closed-form solution in equation (11) is reminiscent of the classical Merton (1969) optimal portfolio choice solution. It depends, however, on the fund specific structural parameters that describe the agency contracts and the realization of the state variable, the distance between net-asset-value and high-water mark. We use this solution to derive explicit restrictions and study the link between the parameters obtained from a standard reduced form regression and the structural parameters in the same way as in Koijen (2010). Consider the reduced form regression with constant coefficients, where we regress fund performance on the benchmark excess return:

$$\frac{dX_t}{X_t} - rdt = \hat{\alpha}_{OLS}dt + \hat{\beta}_{OLS}\left(\frac{dS_t^B}{S_t^B} - rdt\right) + \hat{\sigma}_{\varepsilon,OLS}dZ_t^A. \quad (15)$$

⁶This is similar to the numerical results shown in Hodder and Jackwerth (2007)

If we substitute the return process of the benchmark, the standard reduced form regression becomes:

$$\frac{dX_t}{X_t} = (r + \hat{\alpha}_{OLS} + \hat{\beta}_{OLS}\sigma_B\lambda_B)dt + \hat{\beta}_{OLS}\sigma_B dZ_t^B + \hat{\sigma}_{\varepsilon,OLS}dZ_t^A. \quad (16)$$

where $\hat{\alpha}_{OLS}$ denotes reduced-form alpha, $\hat{\beta}_{OLS}$ denotes reduced-form beta and $\hat{\sigma}_{\varepsilon,OLS}$ is the standard deviation. According to the fund value process (4) and its optimal investment (14), the optimal fund value evolves according to:

$$\frac{dX_t^*}{X_t^*} = (r + \frac{\alpha^{*2}}{\tilde{\gamma}_t\sigma_A^2} + \frac{\lambda_B^2}{\tilde{\gamma}_t})dt + \frac{\lambda_B}{\tilde{\gamma}_t}dZ_t^B + \frac{\alpha^*}{\tilde{\gamma}_t\sigma_A}dZ_t^A. \quad (17)$$

Let $\theta_{B,t}$ and $\theta_{A,t}$ be the portfolio allocations to the investment opportunities described in (2) and (3). By matching the drift and diffusion terms, the cross restrictions implied by the structural model are:

$$\hat{\alpha}_{OLS} = \theta_{At}^*\alpha^* = \frac{\alpha^{*2}}{\tilde{\gamma}_t\sigma_A^2} = \frac{\lambda_A^2}{\tilde{\gamma}_t} \quad (18)$$

$$\hat{\beta}_{OLS} = \theta_{Bt}^* = \frac{\lambda_B}{\tilde{\gamma}_t\sigma_B}, \quad (19)$$

$$\hat{\sigma}_{OLS,\varepsilon} = \theta_{At}^*\sigma_A = \frac{\alpha^*}{\tilde{\gamma}_t\sigma_A} = \frac{\lambda_A}{\tilde{\gamma}_t} \quad (20)$$

Restriction (18) provides a link between traditional reduced-form alpha, $\hat{\alpha}_{OLS}$, and true alpha, α^* . The link reveals that typical reduced-form alpha, $\hat{\alpha}_{OLS}$, is proportional to true alpha, α^* , in a non-linear way. The non-linear relationship arises from the state-dependent allocation, θ_{At}^* , which is endogenously determined by the incentives perceived by the manager. A question which naturally arises from this restriction is how well the reduced-form alpha measures true managerial skill of a hedge fund manager. The answer depends on the determinants of the optimal allocation made by the manager. If the optimal allocation is constant and determined exclusively by the risk and return characteristics of the investment opportunity set (as in a traditional Merton model without agency distortions), then reduced-form alpha is an unbiased estimate of managerial skill. However, if the optimal allocation is influenced by non-linear agency contracts, then reduced-form alpha is a biased inference of true skill. For instance, a high reduced-form alpha can be the fortunate result of the excessive use of leverage when the managers aim to maximize their incentive options. Obviously, high leverage not only increases expected return to the manager (because of the call option) but it also increases the probability of large negative returns.

These non linearity in the incentives have important implications for reduced-form alphas. We document some preliminary evidence of this effect in Figure 5. If one focuses on the cross-sectional dispersion of OLS alphas, it is possible to see that these reach their highest dispersion around the high-water mark. In this case OLS alphas range from range -15% to +20%. The cross-section dispersion of OLS alphas are much smaller for fund values well below the high-water mark (they range between -0.5% and 5%). In this case, it is optimal for the managers decrease leverage. This can be explained by managers' concern about the impact of their actions on the value of funding and redemption options. The manager reduces the volatility of the portfolio to prevent investors and prime brokers from exercising their redemption and funding options. This structural effect is missed by OLS alphas.

[Insert Figure 5 here]

The restriction (18) also reveals that reduced-form alpha varies over time, since effective (endogenous) risk aversion, $\tilde{\gamma}_t$, is state dependent. This implies that reduced-form specifications with constant coefficients are potentially misspecified and motivates the use of a structural approach that uses fund-specific information to help identify $\tilde{\gamma}_t$. To better illustrate the state-dependent nature of reduced-form alpha (i.e. the link between fund's value in relation to its high-water mark), we plot the model-implied reduced form alpha, α_{OLS} , and true managerial skill, α^* , against different fund values.

[Insert Figure 6 here]

Figure 6 illustrates that typical reduced-form alpha, $\hat{\alpha}_{OLS}$, significantly overestimates true active managerial skill, α^* when the fund value is about 20% below the high-water mark. In contrast, the standard reduced-form alpha, $\hat{\alpha}_{OLS}$, underestimates true active managerial skill, α^* , in the region where the fund value is about 50% below the put option's strike price, K , and fund value is about 18% above high-water mark. The standard reduced-form alpha, $\hat{\alpha}_{OLS}$, overestimates true alpha when the short option positions are deep out-of-the-money. In the region where fund value is far above high-water mark, the call option is deep in-the-money and the bias is reduced. In general, the bias is very significant exactly when a hedge fund's assets under management spend significant time close to the high-water mark, which is the most common situation. This shows the importance of adopting a methodology that explicitly takes into account the endogenous effects generated by non-linear incentive contracts.

3 Estimation Methodology

When estimating a manager's true alpha per unit of her investment technology's volatility, one notices that fund performance (17) is driven by $\frac{\alpha^*}{\sigma_A}$ rather than each of the two parameters separately; for this reason, we proceed by estimating the ratio, λ_A , and use it as our primitive measure of true skill (i.e. the Sharpe Ratio of the manager's alpha technology which is insensitive to endogenous risk-taking arising from the agency contracts). Next, we estimate true managerial skill using a structural approach and compare it to reduced-form values. First, we investigate the results in the context of a simulated economy, then we apply the methodology to a very comprehensive panel data of hedge funds.

3.1 Structural Estimation

The structural estimation procedure is articulated in two-steps. First we estimate the set of a benchmark asset parameters $\hat{\Theta}_B \equiv \{\hat{\sigma}_B, \hat{\lambda}_B\}$. Then, since the fund's conditional return generating process satisfies

$$\frac{dX_t^*}{X_t^*} = (r + \frac{\alpha^{*2}}{\tilde{\gamma}_t \sigma_A^2} + \frac{1}{\tilde{\gamma}_t} \hat{\lambda}_B^2)dt + \frac{1}{\tilde{\gamma}_t} \hat{\lambda}_B dZ_t^B + \frac{\alpha^*}{\tilde{\gamma}_t \sigma_A} dZ_t^A, \quad (21)$$

we proceed to estimate the Sharpe Ratio of the manager's investment technology, λ_A , where $\lambda_A \equiv \frac{\alpha^*}{\sigma_A}$, and use this as our proxy for the manager's true managerial skill. Assuming that asset prices are log-normal, we estimate the structural parameters by maximizing log-likelihood, as $\arg \max_{\lambda_A, \hat{\Theta}_B} \sum_{t=h}^{T/h} \ell(r_t^X | r_t^B; \lambda_A, \hat{\Theta}_B)$. Under the assumption of log-normality, the joint dynamics of benchmark and asset returns can be written in discrete time as

$$r_t^B = (\bar{r} + \hat{\sigma}_B \hat{\lambda}_B - \frac{1}{2} \hat{\sigma}_B^2)h + \hat{\sigma}_B \Delta Z_t^B, \quad (22)$$

$$r_t^X = (\bar{r} + \frac{\hat{\lambda}_B^2}{\tilde{\gamma}_t} + \frac{\lambda_A^2}{\tilde{\gamma}_t} - \frac{1}{2} \frac{\hat{\lambda}_B^2}{\tilde{\gamma}_t^2} - \frac{1}{2} \frac{\lambda_A^2}{\tilde{\gamma}_t^2})h + \frac{\hat{\lambda}_B}{\tilde{\gamma}_t} dZ_t^B + \frac{\lambda_A}{\tilde{\gamma}_t} \Delta Z_t^A, \quad (23)$$

where $\begin{pmatrix} \Delta Z_t^B \\ \Delta Z_t^A \end{pmatrix} \sim N(0, hI)$. Thus, the distribution of r_t^A satisfies :

$$r_t^A | r_t^B \sim N(\mu_t, \sigma_t^2), \quad (24)$$

where

$$\mu_t \equiv (\bar{r} + \frac{\hat{\lambda}_B^2}{\tilde{\gamma}_t} + \frac{\lambda_A^2}{\tilde{\gamma}_t} - \frac{1}{2} \frac{\hat{\lambda}_B^2}{\tilde{\gamma}_t^2} - \frac{1}{2} \frac{\lambda_A^2}{\tilde{\gamma}_t^2})h + \frac{\hat{\lambda}_B}{\tilde{\gamma}_t \hat{\sigma}_B} (r_t^B - (\bar{r} + \hat{\sigma}_B \hat{\lambda}_B - \frac{1}{2} \hat{\sigma}_B^2)h), \quad (25)$$

$$\sigma_t^2 = \frac{\lambda_A^2}{\tilde{\gamma}_t^2} h. \quad (26)$$

Since we know the solution of $\tilde{\gamma}_t(\gamma, \lambda, X_t, HWM, K, p, m, c)$ in closed-form (see Proposition 1), we not only use fund returns when drawing inference about true managerial skill but also fund specific information about their current high-water mark and their contractual arrangements. Applying the above estimation approach to a large hedge fund data set is very computationally intensive due to the complex functional form of $\tilde{\gamma}_t$. The reason is that one needs to calculate the Lagrange multiplier ζ^* , which is not available in closed-form. Moreover, ζ^* is also a function of a key parameter we aim to estimate, λ_A . As a result, the econometrician faces numerical iterations for solving ζ^* inside the maximizing log-likelihood iteration, which in turn significantly slows the estimation process.

We address this numerical issue using a two steps procedure. First, we solve for the exact functional form of $\tilde{\gamma}_t$ as a function of the structural parameters and the state variables, i.e. $\Theta_f \equiv \{X_t/HWM_t, \lambda, \gamma\}$. The volatility parameters are held fixed at their sample values. We use semi-parametric methods to estimate the exponential polynomial function that minimizes the distance to the exact functional form $\frac{1}{\tilde{\gamma}_t}$:

$$\frac{1}{\tilde{\gamma}_t} = \frac{a}{\gamma} + \frac{b * e^{-(c * \frac{X_t}{HWM_t} - d)^2 / h^2}}{\frac{X_t}{HWM_t} * \lambda * \gamma} + \frac{e^{-(g \frac{X_t}{HWM_t})^2}}{\frac{X_t}{HWM_t} * \lambda * k} \quad (27)$$

Figure 7 displays the exact functional form $\frac{1}{\tilde{\gamma}_t}$. It is noticeable that the functional form is relatively simple and almost Gaussian in the region between the put and call strike prices. We find that a second order exponential polynomial provides sufficient accuracy and is able to capture the functional dependence of $\frac{1}{\tilde{\gamma}_t}$ with respect to the variables of interest. Figure 8 shows the behavior of the estimated function (27).

[Insert Figure 7 and Figure 8 here]

3.2 Reduced-Form and Structural Estimation in a Simulated Economy

To investigate both the precision of the estimation method and the differences between the reduced-form alpha, α_{OLS} , estimated from OLS regression with constant coefficients and the one implied from the theoretical restriction, $\alpha_{t,OLS} = \lambda_A^2 / \tilde{\gamma}_t$, which varies over time, we simulate the economy and compute the model implied estimates. First, the innovation terms ΔZ_t^A and ΔZ_t^B are independently

simulated 2,500 times. Each simulation is then used to calculate 36 monthly returns. The simulated value of ΔZ_t^B is used to calculate r_t^B according to equation (22). We calibrate the parameters to the following values: $r = 0.05$, $\sigma_B = 0.05$, $\lambda_B = 0.2$ and $\gamma = 5$. Similarly, r_t^X is calculated according to equation (23) where the true skill per unit of volatility, $\frac{\alpha^*}{\sigma_A}$, is assumed to be $\lambda_A \in \{0.5, 1, 1.5\}$ and $\sigma_A = 0.05$. The variable λ_A is then estimated by means of maximum likelihood and then compared to the true values. Finally, we compute $\tilde{\gamma}_t$ by using the estimation of true skill, $\hat{\lambda}_A$, and information about current fund values, X_t , and its high-water marks, HWM_t . The current high-water mark is defined as $HWM_t = \max_{s \leq t} X_s$ and updated on an annual basis. We assume an initial investment of one dollar.

The distribution of the estimated parameters implied by the structural model are summarized in (24). Table 1 reports the estimation results. Panel A displays the mean and standard deviation (in parentheses) of the 2,500 reduced-form alpha estimates, $\hat{\alpha}_{OLS}$, computed from OLS regressions. Panel B shows the mean and standard deviation (in parentheses) of the structural skill and alpha estimates, $\hat{\lambda}_A$ and $\hat{\alpha}_{t,OLS}$ respectively, implied from the structural restriction (20).

[Insert Table 1 here]

As Table 1 Panel B shows, structural estimation accurately recovers the true value of λ_A : the means of the estimates are close to their corresponding true values and their standard deviations are low. This shows that the estimation methodology performs well even when using discretely sampled (monthly) data. The structural estimation of alpha in the last column of Panel B also confirms that alpha is time varying, despite a constant investment opportunity set, λ_A . The time varying component is caused by a hedge fund manager's time varying effective risk aversion, $\tilde{\gamma}_t$. This occurs even if the hedge fund manager utility function has constant γ : the agency incentives perceived by the manager endogenously cause the indirect risk aversion to changes over time.

The second result relates to the efficiency of the estimators. The structural estimator of alpha, $\hat{\alpha}_{t,OLS}$, has a lower standard deviation than the reduced-form estimator of alpha, $\hat{\alpha}_{OLS}$, in finite samples. The result is robust to different levels of true skill as the standard deviations of cross-sectional estimates suggests. For instance, the standard deviation of $\hat{\alpha}_{OLS}$ is 18.20 when λ_A is 1.5, while the standard deviation of the mean of $\hat{\alpha}_{t,OLS}$ is 9.28. The reason for the improved efficiency is that the structural estimator of alpha uses additional information about skill based on the second moments of fund returns, (20).

4 Data and Benchmark assets

We analyze the performance of hedge funds using monthly net-of-fee returns of live and dead hedge funds reported in BarclayHedge database from January 1994 until December 2010.

There are several reasons why we use the BarclayHedge database for our analysis. A recent comprehensive study of the main commercial hedge fund databases by Joenvaara, Kosowski, and Tolonen (2012), abbreviated JKT (2012)) compares five databases (the BarclayHedge, TASS, HFR, EurekaHedge and Morningstar databases) and finds that Barclayhedge has the largest number of funds (9719), compared to 8220 funds in the TASS database. Moreover, BarclayHedge has the highest percentage of dead/defunct funds (65 percent), thus making it least likely to suffer from survivorship bias. The BarclayHedge database accounts for the largest contribution to the aggregate database that JKT(2012) create. The authors also note that BarclayHedge is superior in the terms of AuM coverage, since it has the longest AuM time-series (57 percent) suggesting different behavior when aggregate returns are calculated on a value-weighted basis. The amount of missing AuM observations varies significantly across data vendors, being lowest for BarclayHedge (11 percent) and HFR (19%) and significantly higher for EurekaHedge (37%), TASS (34%), and Morningstar (32%). JKT (2012) do find, however, that economic inferences based on the Barclayhedge and TASS databases are similar in a number of dimensions. For instance, BarclayHedge, HFR and TASS show economically significant performance persistence for the equal-weighted portfolios at semi-annual horizons, whereas using EurekaHedge and Morningstar databases they find limited evidence of performance persistence.

A key distinguishing feature of this database is its detailed cross-sectional information on hedge fund characteristics. Importantly, the database also includes monthly assets under management as well as information about fund’s high-water mark provisions, which are key determinants of manager’s incentives, performance and risk. Our initial fund universe contains more than 16,000 live and dead funds. To ensure that we have a sufficient number of observations to precisely estimate our model we exclude hedge funds with less than 36 monthly return observations.⁷ These restrictions lead to a sample of 4,828 hedge funds.

We group funds into 11 categories according to their investment objectives: CTA, Convertible Arbitrage, Emerging Markets, Equity Long/Short, Equity Market Neutral, Equity Short Bias, Event Driven, Fixed Income Arbitrage, Global Macro, Multi Strategies and Others. Table 2 displays

⁷In unreported results we show that our results do not qualitatively change when we exclude funds with less than 24 monthly observations.

summary statistics for the sample of funds including medians as well as the first and third quartile of the first four moments of the funds' excess returns. The average excess return across all funds is 6.38 percent per year. Emerging Markets funds and Equity Short Bias funds exhibit the highest volatility while having the highest and the lowest average return, respectively.

[Insert Table 2 here]

We also collect hedge fund investment style index benchmark returns compiled by Dow Jones Credit Suisse and use them as hedge fund benchmark returns in our estimation. According to our assumption about the investment opportunity set, although we assume that all hedge fund managers within the same investment style differ in terms of their alpha generating asset, S^A , they invest in the same strategy specific asset, S^B . Since the strategy-specific benchmark assets are unobservable, we proxy S^B by using the style index returns, adjusting for any potential alpha with respect to seven Fung and Hsieh factors. The strategy-specific proxy can be seen as a portfolio of basis assets in the investment opportunity set of hedge funds in the same investment style. Thus, first we regress each style index' returns on the seven Fung and Hsieh factors including an intercept. Then we use only statistically significant betas from the regression to compute our proxy for the benchmark asset S_t^B .

Table 3 reports summary statistics for hedge fund investment style index returns as well as their maximum drawdowns. The Equity Short Bias index has the largest drawdown among all the benchmarks while the Event Driven index exhibits the lowest maximum drawdown. The Equity Market Neutral index exhibits extreme values for higher order moments because of a big loss in November 2008 when this strategy lost about 40% in a single month.

[Insert Table 3 here]

Table 4 reports the regression results of style index returns on the seven Fung and Hsieh factors by investment objectives. One can notice the large positive Fung-Hsieh reduced-form alphas discussed in the Introduction. These are 4.33 (Global Macro), 4.34 percent (Equity Long/Short), and 4.11 percent (Event Driven). Equity Short Bias exhibits the lowest Fung and Hsieh alpha. Table 5 reports summary statistics of the proxies we use for the benchmark asset returns for each investment objective.

[Insert Table 4 and 5 here]

5 In sample Analysis

5.1 Evidence of Risk Shifting

What is the extent to which one observes direct evidence of hedge funds' risk-shifting in the data? The model predicts a bell-shape link between (endogenous) leverage and distance from high-water mark (Figure 3). We address this question by investigating, for each hedge fund, the relationship between current distance to high-water mark, defined as $(X_t - HWM_t)/HWM_t$, and the subsequent change in 12-month realized volatility. We sample the data at a monthly frequency and produce scatter plots controlling for investment objective (see Figure 9-10). The figures exhibit a bell shape distribution which is consistent with the risk shifting profile suggested in Figure 3.

[Insert Figure 9 to 10 here]

Despite the resemblance to Figure 3, however, Figures 9 and 10 do not unambiguously provide evidence of risk shifting since most observations are concentrated in the middle of each plot. It is possible that these observations are generated by funds which on average have relatively high constant realized volatility regardless of the current distance to high-water mark and the fund asset values happen to be near their high-water marks. These funds seem to provide limited information about risk shifting as a function of their distance to high-water mark. In order to control for this potential source of ambiguity, therefore, we condition on those funds that have experienced large negative or positive distance to high-water mark at some stage in the overall sample. We set the deviation threshold for inclusion to be 15 percent below or above their high-water mark. Then, we recalculate scatter plots for the relative change in realized volatilities as a function of funds' current distance to their high-water mark. We define the relative change in realized volatility as the ratio between subsequent and prior 12-month realized volatility. Finally, we use non-parametric kernel fitting to visualize the risk shifting. Figure 11 and 12 displays the results for Fixed Income Arbitrage and Event Driven funds (the results for other investment objective are similar). The evidence of risk shifting near the high-water mark is striking. Moreover, the figures are consistent with the interpretation that the fund managers do not only consider the call option component of her incentive contract but also the put option component. If only the call option component were active, the link between the increase in volatility and distance to high-water mark would be monotone and decreasing (see Carpenter (2000)). Finally, the results are consistent with fund managers having finite investment horizons. Otherwise, the fitted line should be flat as suggested by Panageas and Westerfield (2009)

and Guasoni and Obloj (2011).

[Insert Figure 11 to 12 here]

We run a panel regression controlling for firm-fixed effect to test the hypothesis of risk shifting. We define risk shifting by the change in volatility between periods $[t, t + T]$ and $[t - T, t]$:

$$\sigma_{i,[t,t+T]} - \sigma_{i,[t-T,t]} = u_i + \beta_1 \times D_{\{Dist2HWM_{i,t} > 0\}} \times Dist2HWM_{i,t} \quad (28)$$

$$+ \beta_2 \times D_{\{Dist2HWM_{i,t} < 0\}} \times Dist2HWM_{i,t} \quad (29)$$

$$+ \beta_3 \times (AVGVIX_{[t,t+T]} - AVGVIX_{[t-lag,t]}) + \varepsilon_{i,t} \quad (30)$$

where $Dist2HWM_t \equiv (X_t - HWM_t)/HWM_t$ and $D_{\{X\}}$ is an indicator variable equal to 1 if $X = true$ and equal to 0 if $X = false$. In the regression we also control for market volatility proxied by the difference in the average VIX index over the period $[t, t + T]$ and $[t - T, t]$. According to the bell shape function of the optimal portfolio volatility around the high-water mark, we should expect that funds with a negative (positive) distance to high-water mark experience an increase (decrease) in volatility. In terms of regression (28), there is evidence of risk shifting if $H_0 : \beta_1 < 0$ and $\beta_2 > 0$. On the other hand, a Merton type investor should have β_1 and β_2 coefficients which are not significantly different from zero, i.e. $H_0 : \beta_1 = 0$ and $\beta_2 = 0$. This is also the optimal allocation of the hedge fund managers studied in Panageas and Westerfield (2009) and Guasoni and Obloj (2011). The manager studied in Carpenter (2000) would exhibit $H_1 : \beta_2 < 0$ and $\beta_1 < 0$.

Since it is possible that the realized volatility of hedge fund returns are affected by market-wide volatility, we control for this potential effect by including the change in VIX index over $[t, t + T]$ and $[t - T, t]$ in the regression. We further distinguish between funds that have experienced a deviation of at least 15 percent below or above the high-water mark from the overall sample. Panel A of Table 7 summarizes the regression results when we use the difference between subsequent 12-month and prior 12-month realized volatility, $T = 12$, as the proxy for risk-shifting; Panel B reports the equivalent results when one considers the difference between subsequent 6-month and prior 6-month realized volatility, $T = 6$. We find that the results are robust to different volatility proxies. In both cases, hedge funds on average exhibit risk shifting around high-water mark as our theory predicts. When we condition the regression on hedge fund investment categories, the results in Panel A show strong evidence of risk shifting at a 12 month frequency for almost all investment categories with the exception of Equity Market Neutral. While the results are marginally weaker when we use 6-month

windows (Panel B), Convertible Arbitrage, Equity Long/Short, Equity Short Bias, Event Driven, Fixed Income Arbitrage, Multi Strategy funds and Others exhibit risk-shifting.

[Insert Table 7 here]

5.2 In sample estimation

In this section we first discuss the implementation of the estimation methodology described in Section 3 and then discuss the estimation results, which are summarized in Table 6.

According to Section 3, one of the inputs we need in the estimation is the fund-specific time-varying moneyness of the agency contracts, $\frac{X_t}{HWM_t}$. The quantity is not simple to compute since the high-water mark of a hedge fund is unobservable as it depends on the specific timing of the fund flows at the individual investor level. We carefully build an empirical proxy for the current high-water mark for each fund by using fund-specific fund flow information.⁸ At each month, when the fund receives net positive inflows, we assume the creation of a new share class with a high-water mark equal to the current value of the fund. The total high-water mark of the fund manager is then the value weighted average of high-water marks of all existing and new share classes. When there is negative fund flow, we assume that the outflows occur from investors holding the average vintage share class.⁹

Besides high-water mark, the strike price of put options is also unobservable and it is non-trivial to proxy accurately. According to a sensitivity analysis of the effective risk-taking with respect to the strike price of put options, the strike price of put options affects the width and height of the bell shape profile. We estimate the width and height of bell shape of the function (27) through parameter h and b in such a way that it maximizes average in-sample likelihood, by investment objectives. See Appendix C for the calibration details.

Using these fund-specific characteristics, we then estimate using panel information structural skill measures, λ_A , the OLS alpha, α_{OLS} , and the OLS alpha implied from the structural model restriction, (18), $\alpha_{t,OLS}$. Table 6 summarizes the main results by investment objectives with the results across all investment objectives in the bottom row. In each column we report the cross-sectional median of the skill measures as well as their first and third quartiles in parentheses. Since structural alpha

⁸See Agarwal, Daniel and Naik (2009) for the fund flows calculation.

⁹To improve this last approximation, we further optimize by calibrating the position of the peak of the function (27) through the parameter d to maximize the average in-sample likelihood, by investment objectives. Refer Appendix C for the calibration details.

estimate, $\alpha_{t,OLS}$, varies over time due to the time varying effective risk aversion, $\tilde{\gamma}_t$, we report the mean and standard deviation in the fourth and fifth column respectively. First we observe that OLS alpha, which is assumed to be constant by construction in reduced-form regressions, differs from the mean of the time varying structural alpha, $\alpha_{t,OLS}$. The overall median of the OLS alpha is greater than the overall median of the structural alpha. When we look at the investment objective level, the same remark still applies with the exception of two investment strategies, Equity Short Bias and Multi Strategy. The relative ranking of the OLS and structural alpha is also different. The OLS alpha measure implies that Emerging Markets funds have the highest active skill while the structural alpha measure suggests that Equity Short Bias funds offer the highest alpha. Fixed Income Arbitrage funds are the least skillful by the OLS alpha measure; in contrast, the structural measure suggests Convertible Arbitrage funds (see Table 6, second column). This is due to the fact that the true skill measure is different from OLS alpha in the sense that it is not affected by time varying effective risk aversion, $\tilde{\gamma}_t$, through the hedge fund manager's portfolio allocation. The time varying risk aversion could cause OLS alpha to be misleadingly high. For instance, a low skill hedge fund manager which observes her fund value to be below the high-water mark could temporarily increase her risk-taking level. This way she could increase the volatility of her fund in the short term to increase the probability that her fund value can get above the high-water mark and hence receive performance fees. If the gamble turns out to be successful, this could result in an artificially high value of alpha due to pure luck rather than genuine skill. A direct way to investigate these properties is to conduct an out-of-sample analysis of the different skill measures, which is the focus of the following Section.

[Insert Table 6 here]

6 Out of sample Analysis

6.1 Portfolio sorted by true skill measure

The implication of Section 5.2 is that a true skill measure, λ_A , has better in-sample properties than reduced-form estimates both because of a reduction in bias and an improvement in efficiency. This is due to the fact that it controls for endogenous leverage and uses information from the second moments of fund returns in the estimation of skill. In this section, we test this hypothesis by comparing the out-of-sample performance of a portfolio of hedge funds formed on these two measures. Every year

we sort all hedge funds according to (a) their structural true skill measure, λ_A^i , and (b) their reduced form alpha, α_{OLS}^i into deciles. Then we form decile portfolios. The portfolios are rebalanced every January from 2001 until 2010 based on the skill measures estimated using hedge fund monthly net-of-fee returns and high-water marks in the 36-month window preceding portfolio rebalancing month. The portfolios are equally weighted on a monthly basis, so that the weights are readjusted whenever a fund disappears¹⁰. Finally, we compute the out-of-sample performance of these portfolios.

We report two tables with the out-of-sample results conditional on different skill measures. Table 8 reports the results based on reduced form alpha while Table 9 is based on a true skill measure. Both tables report several portfolio performance measures including alphas based on the Fung-Hsieh 7-factor model.

Table 8 shows that OLS alpha, α_{OLS}^i , doesn't perform well in distinguishing good funds from bad funds. The out-of-sample alpha performance is only 1.85 (=6.28-4.43) percent higher for the top decile portfolio (6.28) than the bottom decile portfolio (4.43) as the second column of Table 8 demonstrates. We test the significance of the difference in alpha by computing Fung-Hsieh 7-factor alpha of the spread returns between top and bottom decile portfolio. The alpha performance of the portfolio is -0.46 and insignificantly different from zero (with a t -statistics of -0.14) as the results show in the last row of the table. Some performance measures such as portfolio average return ('Mean Ret') and growth of one dollar investment ('1\$ growth') show that the top decile portfolio slightly outperform the bottom decile portfolio. In contrast, the information ratio performance (IR) suggests that bottom decile portfolio outperforms top decile portfolio and both portfolios perform equally well when considering the Sharpe Ratio (SR). These show that the out-of-sample performances of OLS alpha are mixed and very sensitive to the performance measure. Based on this evidence, OLS alpha is an unreliable measure of managerial skill when the agent faces non-linear agency contracts.

[Insert Table 8 here]

Table 9 displays the out-of-sample performances of true skill, λ_A^i , obtained from structural estimates. The results show that structural skill measures perform well in distinguishing good hedge funds from bad hedge funds as the top decile portfolio outperforms the bottom decile portfolio by 11.01 (=11.44-0.43) percent in term of Fung-Hsieh 7-factor alpha. The difference is statistically significant as the alpha of the spread portfolio, reported in the last row of the table, is 8.70 percent

¹⁰We follow the same portfolio construction procedure as described in Carhart (1997) and Kosowski, Naik, and Teo (2007).

and significantly different from zero at 5 percent confidence level. Other performance measures also suggest that the top decile portfolio significantly outperforms the bottom decile portfolio. For instance, a hedge fund investor investing one dollar in the top decile portfolio ranked by structural skill, λ_A^i , in January 2001, would receive 4.32 dollars at the end of the sample period. This compares to 1.46 dollars by investing in bottom decile portfolio. The Sharpe Ratio of the top decile portfolio over the 10-year investment is 0.51 (=1.05-0.54) higher than the bottom decile portfolio. According to this evidence, structural skill measure performs well in distinguishing good and bad hedge funds.

[Insert Table 9 here]

Table 10 reports the Fung-Hsieh 7-factor alphas of the decile portfolio returns conditional on hedge fund investment objectives and skill measures. Similar to the results reported in Table 8 and 9, the results suggest that the structural skill measure is better than OLS alpha in term of distinguishing good and bad hedge fund managers for eight out of eleven investment objectives as the differences between Fung-Hsieh 7-factor alphas of top and bottom decile portfolios between the two measures suggest. For instance, the difference between the alphas of Decile 1 and Decile 10 portfolios formed by structural skill is 10.93 percent while it is only 0.89 percent for the portfolios formed by OLS alpha as the last column suggests. The difference is even more significant for fixed income arbitrage and global macro funds: the bottom decile portfolios of the two strategies outperform the top decile portfolios by 6.98 percent (fixed income arbitrage) and 5.37 percent (global macro). In contrast, the decile portfolios of fixed income arbitrage funds and global macro funds sorted by the structural skill measure outperform their bottom decile portfolio by 8.74 and 17.59 percent.

[Insert Table 10 here]

An additional result should be highlighted. The top decile portfolio across *all* hedge funds, formed on the basis of structural skill, delivers Fung-Hsieh 7-factor alpha of 11.44 percent while the portfolio formed by OLS alpha provides the risk-adjusted measure of 6.28 percent (see Table 10, last row). This result holds also *within strategies* in eight out of eleven strategies. For instance, the top decile portfolio of Fixed Income Arbitrage funds formed by structural skill measure is 11.33 (12.87-1.54) percent higher than the portfolio formed by OLS alpha. This large outperformance is likely related to the tendency of this group of funds to use leverage dynamically. When risk aversion is low and leverage is high, then reduced-form performance can be driven by low risk aversion and not by

true skill, possibly explaining why ranking these funds based on the structural skill estimate leads to out-of-sample superior performance. In addition to Fixed Income Arbitrage, another investment objective that shows similar results is Global Macro. Funds in this strategy are often viewed as a directional bets on macro economic themes. As a result, Global Macro funds have more potential for discretionary variations in leverage than systematic investment funds such as CTA whose leverage is (supposedly) decided independently of the difference between NAV and high-water mark.

Figure 13-15 display the growth of 1 dollar invested in the top decile portfolio based on OLS alpha and structural skill measure for all hedge funds and each investment objective. The first figure shows that investing in the top 10 percent of the hedge funds selected by structural skill measure would have generated a greater final period wealth than the OLS alpha (January 2001- December 2010), as the "ALL" sub-figure demonstrates. The same observation applies to six out of eleven investment objectives when we condition the analysis by hedge fund strategies.

[Insert Figure 13-15 here]

It is also interesting to note that despite the similarity between the performance of the two skill measures for Convertible Arbitrage funds prior the second half of 2008, the structural skill measure significantly outperforms OLS alpha after 2008. This is likely due to the fact that, unlike the structural skill measure, the OLS alpha is sensitive to the dynamics of leverage before, during, and after the 2008 Crisis. An example is offered by convertible funds. Managers in this strategy experienced very severe volatility in their NAV due to the prime-broker crisis and forced deleveraging, making them very sensitive to accounting for endogeneity issues. Conditioning on their structural skills, these hedge fund managers perform better as the "Convertible Arbitrage" sub-figure demonstrates. We also see a similar phenomenon for All, Emerging Markets, Long/Short Equity, Fixed Income Arbitrage and Global Macro funds.

7 Conclusion

In this paper we develop a comprehensive structural approach to better measure and predict the performance of leveraged financial institutions with complex incentive contracts. The empirical application of the structural model allows us to use previously unexploited information in second moments in a novel way to draw inference about the structural risk-adjusted performance of investment funds with option-like contractual features. Intuitively, we are able to better distinguish the

effect of risk taking and skill on past fund performance. In the structural model, we carefully motivate our assumptions about the manager’s objective and trading technology, and derive the optimal investment strategy and the implied dynamics of assets under management. We build on and extend the pioneering work of Kojien (2010) on mutual funds by explicitly modelling hedge fund specific contractual features such as (i) high-water marks, (ii) equity investors’ redemption options and (iii) primer broker contracts that together create option-like payoffs and affect hedge funds’ risk taking.

Our second contribution is to apply the model to a large sample of hedge funds. We impose the structural restrictions implied by economic theory to estimate the deep parameters of the model. These restrictions allow us to separately identify true skills from endogenous risk appetite. We document differences between in-sample reduced form regression estimates of performance and the estimates based on the structural model.

Our third contribution is to present empirical evidence of hedge funds’ risk-shifting, as suggested by our theoretical model. The findings reject the hypothesis of constant risk taking implied by models with infinite horizon or models with a monotonically decreasing optimal leverage in distance to high-water mark. They emphasize the importance of accounting for short option positions in reducing leverage when fund value is below high-water mark; these options act as a disciplining device for the manager, who would otherwise increase unboundedly leverage as fund value decreases.

Finally, we document the economic impact of using structural versus reduced form estimates. Estimates of managerial ability based on our structural model are shown in-sample to be more accurate than those based on reduced form models. Out-of-sample portfolios of hedge funds formed using a structural measures of skill outperform portfolios based on reduced-form alphas.

Although we use hedge funds in our empirical application, our results have broader economic implications. Separating the effect of risk aversion and skill on investment performance is a fundamental problem that not only affects investors in alternative investment funds, but also investors (and regulators of) levered financial institutions such as banks which employ incentive contracts.

A Appendix A: Proof of Proposition 1

Under Condition 1, a hedge fund manager facing non-linear incentives maximizes the following concavified objective function:

$$\max_{X_T} E_0[\tilde{U}(X_T, HWM, K, p, m, c)], \quad (31)$$

subject to

$$E_0[\varphi_T X_T] = X_0, \quad (32)$$

where $\tilde{U}(X_T, HWM, K, p, m, c)$ is defined as:

$$\tilde{U}(X_T, HWM, K, \alpha, m, c) = \begin{cases} U(p(X_T - HWM) + mX_T) & \text{for } X_T > \hat{X}_{HWM} \\ U(m\hat{X}_K - c(K - \hat{X}_K)) + \nu(X_T - \hat{X}_K) & \text{for } \hat{X}_K < X_T \leq \hat{X}_{HWM} \\ U(mX_T - c(K - X_T)) & \text{for } b < X_T \leq \hat{X}_K \end{cases}, \quad (33)$$

We then proceed to define the Lagrangian problem, which can be written in terms of the optimal control X_T applied to the concavified utility function (33). Let ζ be the Lagrange multiplier applied to the constraint (32), the Lagrangian reads:

$$\max_{X_T} \tilde{U}(X_T, HWM, K, p, m, c) - \zeta \varphi_T X_T. \quad (34)$$

From the first order conditions, it is possible to show that optimal terminal fund value is given by:

$$\begin{aligned} X_T^* = & \frac{\left(\frac{\zeta \varphi_T}{p+m}\right)^{-1/\gamma} + p HWM}{p+m} 1_{\{X_T > \hat{X}_{HWM}\}} + \hat{X}_{HWM} 1_{\{\hat{X}_K < X_T \leq \hat{X}_{HWM}\}} \\ & + \frac{\left(\frac{\zeta \varphi_T}{m+c}\right)^{-1/\gamma} + cK}{m+c} 1_{\{b < X_T < \hat{X}_K\}} + b 1_{\{X_T \leq b\}}, \end{aligned} \quad (35)$$

where ζ solves $E[\varphi_T X_T^*] = X_0$ and $1_F = 1$ when F occurs and $1_F = 0$ otherwise.

We can simplify the derivation of the expectation of $\varphi_T X_T^*$ by mapping the set of terminal fund value to the state price. This is similar to the solution method used in Carpenter (2000). Therefore equation (35) is equivalent to :

$$\begin{aligned}
X_T^* = & \frac{\left(\frac{\zeta\varphi_T}{p+m}\right)^{-1/\gamma} + pHWM}{p+m} 1_{\{\varphi_T < z_{HWM}\}} + \widehat{X}_{HWM} 1_{\{z_K > \varphi_T > z_{HWM}\}} \\
& + \frac{\left(\frac{\zeta\varphi_T}{m+c}\right)^{-1/\gamma} + cK}{m+c} 1_{\{z_b > \varphi_T > z_K\}} + b 1_{\{\varphi_T > z_b\}},
\end{aligned} \tag{36}$$

where state price, z_i , solves $\widetilde{U}'(X^i, HWM, K, p, m, c) = \zeta z_i$. This is the first order condition of the Lagrangian. As a results, critical value for the state prices are :

$$z_{HWM} = (p(\widehat{X}_{HWM} - HWM) + m\widehat{X}_{HWM})^{-\gamma}(p+m)/\zeta \tag{37}$$

$$z_K = (m\widehat{X}_K - c(K - \widehat{X}_K))^{-\gamma}(m+c)/\zeta \tag{38}$$

$$z_b = (mb - c(K - b))^{-\gamma}(m+c)/\zeta \tag{39}$$

Notice that there is an inverse relation between the state price and fund value. This is intuitive since the state price is closely related to marginal utility which is a decreasing function of the level of consumption. According to the martingale property, the optimal fund value is the process:

$$X_t^* = E_t\left(\frac{\varphi_T}{\varphi_t} X_T^*\right) \tag{40}$$

This can be analytically derived: the optimal fund value is given by:

$$\begin{aligned}
X_t^* = & \frac{\zeta^{-1/\gamma}}{\varphi_t(p+m)^{1-1/\gamma}} I_{1,t} + \frac{pHWM}{\varphi_t(p+m)} I_{2,t} + \frac{\widehat{X}_{HWM}}{\varphi_t} I_{5,t} \\
& + \frac{\zeta^{-1/\gamma}}{\varphi_t(m+c)^{1-1/\gamma}} I_{3,t} + \frac{cK}{\varphi_t(m+c)} I_{4,t} + \frac{b}{\varphi_t} I_{6,t}
\end{aligned} \tag{41}$$

where: $I_{1,t} \equiv E_t(\varphi_T^{1-1/\gamma} 1_{\{\varphi_T < z_{HWM}\}}) = D_t N(d_{2,t}^{z_{HWM}})$, $I_{2,t} \equiv E_t(\varphi_T 1_{\{\varphi_T < z_{HWM}\}}) = C_t N(d_{1,t}^{z_{HWM}})$, $I_{3,t} \equiv E_t(\varphi_T^{1-1/\gamma} 1_{\{z_K \leq \varphi_T < z_b\}}) = D_t (N(d_{2,t}^{z_b}) - N(d_{2,t}^{z_K}))$, $I_{4,t} \equiv E_t(\varphi_T 1_{\{z_K \leq \varphi_T < z_b\}}) = C_t (N(d_{1,t}^{z_b}) - N(d_{1,t}^{z_K}))$, $I_{5,t} \equiv E_t(\varphi_T 1_{\{z_{HWM} \leq \varphi_T < z_K\}}) = C_t (N(d_{1,t}^{z_K}) - N(d_{1,t}^{z_{HWM}}))$, $I_{6,t} \equiv E_t(\varphi_T 1_{\{z_b \leq \varphi_T\}}) = C_t N(-d_{1,t}^{z_b})$. Where $N(x)$ denotes the cdf function, with $d_{1,t}^{z_i} \equiv \frac{\ln(z_i/\varphi_t) + (r-0.5\lambda'\lambda)(T-t)}{\|\lambda\|\sqrt{T-t}}$, $d_{2,t}^{z_i} \equiv d_{1,t}^{z_i} + \frac{\|\lambda\|\sqrt{T-t}}{\gamma}$, and $C_t \equiv \exp(\ln(\varphi_t) - r(T-t))$, $D_t \equiv \exp(\frac{1-\gamma}{\gamma} \ln(\varphi_t) - \frac{1-\gamma}{\gamma} (r + 0.5\lambda'\lambda)(T-t) + 0.5 \left(\frac{1-\gamma}{\gamma}\right)^2 \|\lambda\|^2 (T-t))$.

According to the optimal fund value in equation (41), we can use Ito's lemma to derive the

dynamic of $dX^*(t, \varphi_t)$ and then compare a coefficient of its diffusion term to equation (4) to obtain the following optimal allocation:

$$\theta_t^* = -\frac{\partial X^*(t, \varphi_t)}{\partial \varphi_t} \frac{\varphi_t}{X_t^*} (\Sigma \Sigma')^{-1} \Sigma \lambda \quad (42)$$

where

$$\begin{aligned} \frac{\partial X^*(t, \varphi_t)}{\partial \varphi} = & -\frac{\zeta^{-1/\gamma} G_t}{\varphi_t^{\frac{1+\gamma}{\gamma}} (p+m)^{1-1/\gamma}} \left(\frac{N(d_{2,t}^{HWM})}{\gamma} + \frac{N'(d_{2,t}^{HWM})}{\|\lambda\| \sqrt{T-t}} \right) - \frac{\alpha HWM}{\varphi_t(\alpha+m)} \frac{H_t N'(d_{1,t}^{HWM})}{\|\lambda\| \sqrt{T-t}} \\ & - \frac{\hat{X}_{HWM} H_t}{\varphi_t \|\lambda\| \sqrt{T-t}} \left[N'(d_{1,t}^{z_K}) - N'(d_{1,t}^{HWM}) \right] \\ & - \frac{\zeta^{-1/\gamma} G_t}{\varphi_t^{\frac{1+\gamma}{\gamma}} (m+c)^{1-1/\gamma}} \left[\frac{N'(d_{2,t}^{z_b}) - N'(d_{2,t}^{z_K})}{\|\lambda\| \sqrt{T-t}} + \frac{N(d_{2,t}^{z_b}) - N(d_{2,t}^{z_K})}{\gamma} \right] \\ & - \frac{cKH_t}{\varphi_t(m+c) \|\lambda\| \sqrt{T-t}} (N'(d_{1,t}^{z_b}) - N'(d_{1,t}^{z_K})) + b \frac{H_t N(-d_{1,t}^{z_b})}{\varphi_t \|\lambda\| \sqrt{T-t}} \end{aligned} \quad (43)$$

with $G_t \equiv \exp(-\frac{1-\gamma}{\gamma}(r+0.5\lambda'\lambda)(T-t) + 0.5\left(\frac{1-\gamma}{\gamma}\right)^2 \|\lambda\|^2 (T-t))$ and $H_t \equiv \exp(-r(T-t))$.

B Appendix B: Concavification

In this Appendix we discuss concavification technique and role of Condition 1. The objective function which we study is not globally concave, therefore we transform the utility function with a tangent line superimposed on the convex region of the function to solve the optimization problem.

The concavification of this problem is to solve for the concavification point \hat{X}_K and \hat{X}_{HWM} and the tangent line between the two points which dominates the convex region. Condition 1 guarantees the continuity of manager's objective function at concavification points.

In general \hat{X}_K and \hat{X}_{HWM} under Condition 1 exist when level of risk aversion, γ , is low. However, Condition 1 could be insatiable when risk aversion, γ , is high. Panel (a) of Figure 16 depicts this case. When risk aversion, γ , is high, for a set of other parameters, it is possible that even the point with lowest slope between (b, K) cannot provide a tangent line which satisfy Condition 1.

Similar to Basak, Pavlova, and Shapiro (2007) we solve the optimization problem with discontinuous objective function by putting a constraint: $\hat{X}_K = K$. The concavified objective function with the discontinuous point at $\hat{X}_K = K$ is displayed in Panel (b) of Figure 16. See Basak, Pavlova,

and Shapiro (2007) . for the proof that shows it is possible to “adapts the martingale representation and convex-duality techniques to a non-concave problem.”

[Insert Figure 16 here]

C Appendix C: Effective risk aversion function

In this Appendix we discuss the importance of simplifying and calibrating the state-dependant effective risk aversion function, $\tilde{\gamma}_t$, when estimating the model with real hedge fund data. First as we discuss in Section 3.1, even though we know the full functional form of $\tilde{\gamma}_t$, estimating the model on a large hedge fund data set is infeasible due to its extensive computational time. The reason is that one needs to calculate the Lagrange multiplier ζ^* , which is not available in closed-form. Moreover, ζ^* is also a function of a key parameter we aim to estimate, λ_A . As a result, one is faced with numerical iterations for solving ζ^* inside the maximizing log-likelihood iteration, which in turn significantly slows the estimation process.

To circumvent the numerical issue we use a two steps procedure. First, we solve for the exact functional form of $\tilde{\gamma}_t$ as a function of the structural parameters and the state variables, i.e. $\Theta_f \equiv \{X_t/HWM_t, \lambda, \gamma\}$. The volatility parameters are held fixed at their sample values. We use semi-parametric methods to estimate the exponential polynomial function that minimize the distance to the exact functional form $\frac{1}{\tilde{\gamma}_t}$:

$$\frac{1}{\tilde{\gamma}_t} = \frac{a}{\gamma} + \frac{b * e^{(-(c * \frac{X_t}{HWM_t} - d)^2 / h^2)}}{\frac{X_t}{HWM_t} * \lambda * \gamma} + \frac{e^{(-(g * \frac{X_t}{HWM_t})^2)}}{\frac{X_t}{HWM_t} * \lambda * k} \quad (45)$$

Figure 8 shows the behavior of the estimated function (45). We find that a second order exponential function provides sufficient accuracy.

Nevertheless, the empirical implementation of the estimation using function (45) with hedge fund data is non-trivial for two reasons. First we cannot observe the true current high-water mark perceived by a hedge fund manager due to the share classes structure of hedge funds. Although, we try to compute the true high-water mark of a hedge fund manager by using fund flow information, we cannot completely control for the share classes structure, as we discuss in section 5.2. To address the problem we allow function (45) to adapt slightly to different values of the high-water mark implied

by the data. This can be done by calibrating the position of the peak of the function (45) slightly further toward the left or right depending on the hedge fund data. For each investment objective, we calibrate this through parameter d in such away that it maximizes the average in-sample likelihood.

In addition to the computational issue related to the high-water mark, the strike price of put options is also unobservable and it is non-trivial to proxy it. According Figure 17, a sensitivity analysis of the effective risk-taking with respect to the strike price of put options, the strike price of put options affects the width and height of the bell shape profile. Based on this finding we account for the strike price of put options into the model by calibrating the width and height of bell shape of the function (27) through parameter h and b in such away that it maximizes average in-sample likelihood, by investment objectives.

Therefore instead of just estimating variable λ , we also calibrate centrality parameter d , width parameter h and height parameter b in such away that it maximizes average in-sample likelihood, by investment objectives. Figure 18 shows the profile of the function (45) with different values of the parameters.

[Insert Figure 17 and 18 here]

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Table 1: Reduced-Form and Structural Skill Estimates in Simulated Economies. This table displays means and standard deviations (in parentheses) of skill estimates from 2,500 simulations of 36 monthly returns. Panel A shows the estimate of reduced-form alpha, α_{OLS} , while Panel B shows the estimates of true skill measure, λ_A , mean and standard deviation of time varying reduced-form alpha, $\alpha_{t,OLS}$, which is implied from the theoretical restriction: $\alpha_{t,OLS} = \lambda_A^2 / \tilde{\gamma}_t$. The 36-month returns are simulated with the following parameters; $\lambda_A \in \{0.5, 1, 1.5\}$, $\gamma = 5$, $r = 0.05$, $\lambda_B = 0.2$, $\sigma_B = 0.05$ and $\sigma_A = 0.05$. Where λ_A denotes true skill, γ denotes constant coefficient of relative risk aversion, r denotes risk free rate, λ_B denotes Sharpe Ratio of benchmark asset B and σ_B denotes volatility of benchmark asset B.

True parameter	Panel A: Reduced-form Estimation	Panel B: Structural Estimation		
λ_A	$\hat{\alpha}_{OLS}$	$\hat{\lambda}_A$	Mean ($\hat{\alpha}_{t,OLS}$)	Std ($\hat{\alpha}_{t,OLS}$)
0.5	5.93 (10.15)	0.49 (0.09)	5.39 (2.44)	3.12 (1.49)
1	19.67 (14.99)	0.99 (0.15)	19.12 (5.95)	8.05 (2.60)
1.5	39.82 (18.20)	1.48 (0.19)	38.80 (9.28)	13.68 (3.13)

Table 2: Summary Statistics for Hedge Funds Returns. This table displays, for each investment category, the number of funds (in parentheses) and fund cross-sectional median as well as first and third quartiles (in parentheses) of the annualised excess mean returns over risk free rate , standard deviation , skewness and kurtosis. The statistics are computed by using monthly net-of-fee returns of live and dead hedge funds reported in the BarclayHedge database between January 1994 and December 2010.

Investment Objectives	Mean (pct/ann.)	Std (pct/ann.)	Skewness	Kurtosis
CTA (1655)	5.80 (1.33 , 12.06)	15.28 (9.94 , 24.57)	0.34 (-0.12 , 0.86)	4.28 (3.41 , 6.03)
Convertible Arbitrage (155)	4.97 (2.34 , 6.49)	6.98 (4.22 , 10.55)	-0.72 (-1.45 , -0.08)	6.13 (4.37 , 10.97)
Emerging Markets (512)	9.20 (3.98 , 16.04)	19.40 (10.97 , 29.33)	-0.30 (-0.94 , 0.25)	5.14 (3.84 , 7.92)
Equity Long/Short(807)	6.57 (2.74 , 11.21)	11.73 (8.43 , 17.54)	-0.01 (-0.53 , 0.54)	4.52 (3.56 , 6.24)
Equity Market Neutral (154)	2.64 (-0.11 , 5.64)	7.27 (5.28 , 10.13)	-0.10 (-0.42 , 0.22)	4.14 (3.10 , 5.58)
Equity Short Bias (34)	1.40 (-3.76 , 6.40)	21.57 (13.40 , 30.79)	0.30 (-0.08 , 0.66)	4.32 (3.56 , 5.46)
Event Driven (211)	7.10 (4.34 , 10.65)	10.84 (7.19 , 14.86)	-0.43 (-1.19 , 0.32)	5.91 (4.53 , 9.79)
Fixed Income Arbitrage (93)	3.57 (0.31 , 8.53)	9.02 (5.52 , 12.99)	-0.77 (-2.45 , 0.06)	8.53 (4.68 , 15.20)
Global Macro (185)	5.79 (2.55 , 10.43)	12.35 (8.58 , 17.08)	0.09 (-0.43 , 0.49)	4.33 (3.56 , 6.24)
Multi Strategy (196)	5.23 (1.22 , 8.96)	9.85 (6.04 , 15.13)	-0.38 (-1.21 , 0.34)	5.35 (3.60 , 9.36)
Others (826)	8.23 (3.66 , 14.15)	13.91 (8.44 , 23.29)	-0.03 (-0.65 , 0.59)	4.90 (3.60 , 7.00)
All Funds (4828)	6.38 (2.26 , 11.99)	13.33 (8.29 , 21.68)	0.05 (-0.56 , 0.62)	4.70 (3.58 , 6.94)

Table 3: Summary Statistics for Hedge Fund Style Index Returns. This table displays, by investment category, maximum drawdown, risk premium over risk free rate, Sharpe Ratio, mean, median, 1st and 3rd quartiles, standard deviation, skewness and kurtosis of the index returns . The statistics are computed by using monthly hedge fund style index returns reported in the Dow Jones Credit Suisse between January 1994 and December 2010.

Index	Max. Draw- down (%)	Risk Pre- mium (Ann.)	Sharpe Ratio	Mean (Ann.)	Median (Ann.)	1st Quartile (Ann.)	3rd Quartile (Ann.)	Std (Ann.)	Skewness	Kurtosis
CTA	17.74	3.70	0.31	7.13	4.74	-21.78	36.18	11.81	0.00	2.98
Convertible Arbitrage	32.86	4.42	0.62	7.86	12.36	1.02	17.94	7.08	-2.74	18.75
Emerging Markets	45.15	5.66	0.37	9.10	16.26	-16.26	34.50	15.22	-0.78	7.88
Equity Long/Short	21.97	6.85	0.69	10.29	10.38	-9.96	28.44	9.97	-0.01	6.35
Equity Market Neutral	45.11	2.29	0.22	5.73	8.34	2.64	15.42	10.64	-11.71	156.21
Equity Short Bias	58.64	-5.88	-0.35	-2.45	-8.28	-41.70	35.46	17.05	0.69	4.45
Event Driven	19.15	6.67	1.09	10.11	12.30	1.86	22.68	6.11	-2.42	15.93
Fixed Income Arbitrage	29.03	1.88	0.32	5.32	8.88	2.34	14.46	5.93	-4.29	31.27
Global Macro	26.78	8.88	0.88	12.31	13.08	-1.68	27.36	10.03	-0.03	6.44
Multi Strategy	24.75	4.63	0.85	8.07	9.96	1.26	19.74	5.45	-1.76	9.06
All	19.67	5.89	0.77	9.33	9.78	-1.98	23.10	7.70	-0.22	5.38

Table 4: Regression Results of Hedge Fund Style Index Returns on Fung and Hsieh 7-factor Benchmark. This table reports Fung and Hsieh 7 factors alpha, beta coefficients and their Newey-West t-statistics (in parentheses) of hedge fund index returns by investment objectives. The estimates are computed by using monthly hedge fund style index returns reported in the DowJones Credit Suisse, Fung and Hsieh (2004) risk factors reported in David A. Hsieh's hedge fund data library. Bond factors are duration adjusted, the bond durations data are from Datastream. All data are between January 1994 and December 2010

Hedge fund index	Alpha (pct/ann.)	PTFSBD	PTFSFX	PTFSCOM	SNPMRF	SCMLC	BD10RET	BAAMTSY	Adj. R2
CTA	4.05 (1.63)	0.03 (1.92)	0.04 (2.67)	0.04 (2.52)	0.01 (0.14)	0.04 (0.66)	0.24 (1.97)	0.01 (0.08)	15.43
Convertible Arbitrage	2.12 (1.06)	-0.02 (-2.07)	0.00 (-0.88)	0.00 (-0.43)	0.04 (1.39)	0.01 (0.40)	0.08 (1.52)	0.60 (5.54)	44.45
Emerging Markets	1.26 (0.32)	-0.04 (-1.42)	-0.01 (-0.59)	0.01 (0.73)	0.42 (6.60)	0.24 (3.43)	0.07 (0.68)	0.35 (3.81)	35.38
Equity Long/Short	3.50 (1.94)	-0.02 (-1.55)	0.01 (1.04)	0.01 (1.02)	0.39 (8.16)	0.32 (3.63)	0.14 (2.43)	0.09 (1.51)	58.24
Equity Market Neutral	1.68 (0.66)	-0.03 (-1.10)	0.02 (1.69)	0.02 (1.52)	0.14 (2.68)	0.02 (0.52)	-0.31 (-1.15)	0.12 (1.56)	15.03
Equity Short Bias	-1.49 (-0.52)	0.00 (0.27)	0.00 (-0.40)	-0.02 (-1.02)	-0.86 (-10.14)	-0.50 (-8.89)	0.06 (0.79)	0.24 (1.53)	69.62
Event Driven	4.57 (3.21)	-0.03 (-2.07)	0.00 (1.11)	0.01 (1.03)	0.18 (7.17)	0.10 (3.86)	-0.03 (-0.65)	0.23 (5.39)	54.93
Fixed Income Arbitrage	-0.10 (-0.06)	-0.02 (-2.52)	-0.01 (-1.72)	0.01 (0.71)	0.02 (0.57)	0.00 (0.14)	0.10 (2.07)	0.51 (4.53)	45.02
Global Macro	6.18 (2.53)	-0.03 (-1.77)	0.01 (0.94)	0.02 (0.90)	0.13 (1.78)	0.04 (0.50)	0.34 (4.26)	0.26 (2.41)	12.58
Multi Strategy	3.33 (2.42)	-0.01 (-1.05)	0.01 (1.08)	0.00 (0.52)	0.05 (1.92)	0.02 (0.83)	-0.03 (-0.53)	0.37 (5.21)	32.32
All	3.10 (2.11)	-0.03 (-2.50)	0.01 (1.48)	0.02 (1.51)	0.23 (5.72)	0.14 (2.33)	0.14 (2.16)	0.24 (4.56)	43.78

Table 5: Summary Statistics for Proxies of Benchmark Assets. This table displays, by investment objectives, benchmark asset returns maximum drawdown, risk premium over risk free rate, Sharpe Ratio, mean, median, 1st and 3rd quartiles, standard deviation, skewness and kurtosis. The benchmark asset for each investment objective is hedge fund style index adjusted for potential Fung and Hsieh alpha. The statistics are computed by using monthly benchmark asset returns which are computed from hedge fund style index returns reported in the Dow Jones Credit Suisse and Fung and Hsieh 7 risk factors reported in David A. Hsieh's hedge fund data library. Bond factors are duration adjusted, the bond durations data are from Datastream. All data are between January 1994 and December 2010

Benchmark asset	Max. Draw- down (%)	Risk Pre- mium (Ann.)	Sharpe Ratio	Mean (Ann.)	Median (Ann.)	1st Quartile (Ann.)	3rd Quartile (Ann.)	Std (Ann.)	Skewness	Kurtosis
CTA	12.60	0.23	0.05	3.67	1.75	-7.18	12.14	4.42	0.68	3.75
Convertible Arbitrage	18.29	1.83	0.41	5.26	5.92	-0.08	11.45	4.42	-1.71	16.61
Emerging Markets	31.24	3.45	0.39	6.88	12.88	-9.18	25.17	8.74	-1.15	7.00
Equity Long/Short	20.97	2.88	0.40	6.31	9.17	-11.18	22.90	7.28	-0.67	4.22
Equity Market Neutral	7.44	0.80	0.34	4.24	5.23	-0.42	10.02	2.34	-0.76	3.96
Equity Short Bias	47.17	-5.25	-0.35	-1.81	-9.67	-37.33	32.11	14.99	0.70	4.10
Event Driven	16.23	2.23	0.48	5.67	8.34	-1.97	15.12	4.63	-1.44	8.29
Fixed Income Arbitrage	14.27	1.88	0.51	5.31	6.20	0.78	11.00	3.65	-1.98	19.61
Global Macro	5.71	1.56	0.56	5.00	5.79	-0.57	10.93	2.76	-0.84	8.37
Multi Strategy	10.48	0.93	0.35	4.36	4.88	1.71	7.94	2.62	-1.99	19.23
All	17.66	2.96	0.55	6.40	9.05	-3.03	18.41	5.34	-1.39	8.14

Table 6: In-sample Skill Estimates. This tables displays, by investment objectives, the in-sample estimation of true skill, λ_A , typical OLS alpha, α_{OLS} , and structural alpha, $\alpha_{t,OLS}$, implied from theoretical restriction. Each column reports the cross-sectional median of the estimates as well as their first and third quartiles in parentheses. Since structural alpha estimate varies overtime, we report the mean and standard deviation of the measure in fourth and fifth column respectively. The estimates are computed using monthly net-of-fee returns of live and dead hedge funds reported in the BarclayHedge database between January 1994 and December 2010.

Investment Objectives	True Skill λ_A	OLS Alpha (pct/ann.) α_{OLS}	Mean Structural Alpha (pct/ann.) Mean($\alpha_{t,OLS}$)	Std Structural Alpha (pct/ann.) Std($\alpha_{t,OLS}$)
CTA (1655)	0.55 (0.33, 0.93)	5.58 (1.48 , 11.60)	3.14 (1.57, 6.53)	2.16 (0.93, 4.24)
Convertible Arbitrage (155)	0.00 (0.00, 0.26)	3.19 (1.03 , 5.60)	0.00 (0.00, 4.58)	0.00 (0.00, 0.56)
Emerging Markets (512)	0.70 (0.37, 1.01)	6.09 (1.60 , 12.07)	3.29 (1.03, 7.46)	3.96 (1.18, 7.07)
Equity Long/Short(807)	0.44 (0.31, 0.67)	4.34 (0.82 , 8.44)	1.23 (0.59, 2.75)	1.24 (0.49, 2.54)
Equity Market Neutral (154)	0.12 (0.00, 0.26)	2.00 (-0.99 , 4.53)	1.06 (0.00, 1.86)	0.40 (0.00, 1.08)
Equity Short Bias (34)	0.47 (0.25, 1.15)	3.20 (0.02 , 9.18)	6.90 (2.95, 15.77)	2.82 (1.46, 7.61)
Event Driven (211)	0.41 (0.30, 0.59)	4.11 (0.58 , 8.34)	1.11 (0.45, 2.20)	1.31 (0.12, 2.86)
Fixed Income Arbitrage (93)	0.17 (0.10, 0.33)	1.63 (-2.08 , 5.26)	0.65 (0.27, 1.31)	0.51 (0.18, 1.42)
Global Macro (185)	0.61 (0.46, 0.80)	4.33 (1.13 , 8.83)	1.89 (1.14, 2.82)	1.56 (0.75, 2.55)
Multi Strategy (196)	0.41 (0.04, 0.74)	5.99 (3.39 , 9.20)	6.37 (0.66, 11.40)	2.22 (0.20, 4.82)
Others (826)	0.53 (0.34, 0.88)	5.15 (0.74 , 10.44)	1.58 (0.51, 4.87)	1.19 (0.12, 4.76)
All Funds (4828)	0.49 (0.29, 0.82)	4.85 (1.09 , 9.87)	2.08 (0.75, 5.24)	1.62 (0.44, 3.96)

Table 7: Evidence on Risk Shifting. This table reports, by investment category, regression coefficients and adjusted R square. In Panel A and B, we report evidence on risk shifting based funds that have experienced at least a 15 percent deviation of assets value from their high water mark. In Panel A we use the difference between subsequent 12-month and prior 12-month realized volatility as the dependence variable while Panel B we use the difference between subsequent 6-month and prior 6-month realized volatility as the dependence variable in the regression. The hedge fund data are monthly net-of-fee returns reported in BarclayHedge database between January 1994 and December 2010

Investment Objectives	Panel A : $\sigma_{t+12mth} - \sigma_{t-12mth}$				Panel B: $\sigma_{t+6mth} - \sigma_{t-6mth}$			
	Intercept	Beta1	Beta2	Beta3	Intercept	Beta1	Beta2	Beta3
				Adj.R ²				Adj.R ²
All Strategies	0.009***	-0.063***	0.137***	0.556***	-0.004***	-0.038***	0.087***	0.528***
CTA	0.001	-0.096***	0.025***	0.304***	-0.010***	-0.067***	-0.057***	0.303***
Convertible Arbitrage	0.009***	-0.067***	0.092***	1.116***	-0.001***	-0.078***	0.124***	0.997***
Emerging Markets	0.006***	-0.015**	0.127***	0.913***	-0.012***	0.020***	0.098***	0.839***
Equity Long/Short	0.005***	-0.019***	0.118***	0.408***	-0.005***	-0.005	0.085***	0.412***
Equity Market Neutral	-0.003***	0.056***	0.029**	0.224***	-0.009***	0.067***	-0.009	0.208***
Equity Short Bias	0.052***	-0.150***	0.196***	0.418***	0.009	-0.120**	0.028	0.620***
Event Driven	0.008***	-0.101***	0.101***	0.782***	0.001	-0.108***	0.127***	0.706***
Fixed Income Arbitrage	0.019***	-0.173***	0.339***	0.833***	0.014***	-0.195***	0.377***	0.758***
Global Macro	0.004**	-0.054***	0.036**	0.457***	-0.011***	0.027**	-0.054***	0.411***
Multi Strategy	0.009***	-0.065***	0.167***	0.669***	0.003***	-0.069***	0.206***	0.604***
Others	0.014***	-0.040***	0.185***	0.642***	0.000	-0.016**	0.149***	0.631***

Table 8: Performance persistence tests - Ranking funds on OLS alphas. This table displays out-of-sample performances of decile portfolios ranked by OLS alphas relative to empirical proxies for benchmark assets. The hedge funds are sorted every year into deciles on the 1st January (from 2001 until 2010) based on the OLS alpha estimation. The monthly net-of-fee returns of individual hedge funds, considered all investment categories, in the 36-month rolling window preceding every 1st January are used to evaluate the OLS alphas. The portfolios are equal weighted on monthly basis, therefore the weights are readjusted whenever a fund disappears. Decile 1 comprises funds with the highest OLS alphas, while decile 10 comprises the lowest. The last row represents the spread returns between 1st and 10th decile portfolios, Decile 1 - Decile 10. Column two reports Fung and Hsieh (2004) OLS alphas based on out-of-sample portfolios returns between January 2001 and December 2010. Column three and four report t-statistics and p-values of the Fung and Hsieh (2004) OLS alphas. Column five reports the adjusted R^2 (Adj R^2) of the OLS regression of out-of-sample portfolio returns on Fung and Hsieh 7 factors. The rest of the columns report annualised percentage mean returns (Mean Ret), growth of 1 dollar investment (\$1 growth), annualised information ratio (IR), tracking error (TE), and Sharpe Ratio (SR) of out-of-sample portfolios returns.

Portfolio	FH Alpha (pct/ann.)	t -stat	p -value	Adj R^2	Mean Ret (pct/ann.)	\$1 growth	IR	TE	SR
Decile 1	6.28	2.13	0.04	40.37	11.25	2.90	0.78	8.00	0.85
Decile 2	3.14	1.77	0.08	40.43	7.03	1.96	0.58	5.45	0.66
Decile 3	3.36	2.21	0.03	33.04	7.44	2.07	0.77	4.36	0.95
Decile 4	3.46	3.18	0.00	40.37	6.72	1.93	0.92	3.77	0.90
Decile 5	3.38	2.86	0.01	42.46	7.15	2.02	1.01	3.34	1.09
Decile 6	2.79	3.00	0.00	49.97	6.30	1.86	0.94	2.98	0.95
Decile 7	2.40	2.58	0.01	47.21	6.01	1.81	0.85	2.81	0.95
Decile 8	2.79	3.54	0.00	47.55	6.13	1.83	1.05	2.65	1.05
Decile 9	3.23	2.82	0.01	52.86	6.69	1.93	1.03	3.14	0.96
Decile 10	4.43	2.71	0.01	56.30	8.05	2.18	1.00	4.44	0.85
Spread (Decile1-10)	-0.46	-0.14	0.89	12.83	3.20	1.33	-0.06	7.41	0.12

Table 9: Performance persistence tests - Ranking funds on True skill measure. This table displays out-of-sample performances of decile portfolios ranked by true skill measure, λ_A , relative to empirical proxies for benchmark assets. The hedge funds are sorted every year into deciles on the 1st January (from 2001 until 2010) based on the true skill measure, λ_A , estimation. The monthly net-of-fee returns of individual hedge funds, considered all investment categories, in the 36-month rolling window preceding every 1st January are used to evaluate the true skill measure, λ_A . The portfolios are equal weighted on monthly basis, therefore the weights are readjusted whenever a fund disappears. Decile 1 comprises funds with the highest true skill measure, while decile 10 comprises the lowest. The last row represents the spread returns between 1st and 10th decile portfolios, Decile 1 - Decile 10. Column two reports Fung and Hsieh (2004) OLS alphas based on out-of-sample portfolios returns between January 2001 and December 2010. Column three and four report t-statistics and p-values of the Fung and Hsieh (2004) OLS alphas. Column five reports the adjusted R^2 (Adj R^2) of the OLS regression of out-of-sample portfolio returns on Fung and Hsieh 7 factors. The rest of the columns report annualised percentage mean returns (Mean Ret), growth of 1 dollar investment (\$1 growth), annualised information ratio (IR), tracking error (TE), and Sharpe Ratio (SR) of out-of-sample portfolios returns.

Portfolio	Alpha (pct/ann.)	t -stat	p -value	Adj R^2	Mean Ret (pct/ann.)	\$1 growth	IR	TE	SR
Decile 1	11.44	3.55	0.00	41.73	15.50	4.32	1.22	9.41	1.05
Decile 2	5.58	3.15	0.00	52.14	9.36	2.45	0.98	5.69	0.85
Decile 3	5.09	2.89	0.00	42.92	9.34	2.47	0.95	5.38	0.98
Decile 4	2.25	1.36	0.18	42.24	6.85	1.95	0.51	4.37	0.78
Decile 5	2.87	2.35	0.02	43.72	6.56	1.90	0.76	3.79	0.83
Decile 6	2.21	1.89	0.06	41.86	5.96	1.79	0.64	3.43	0.81
Decile 7	2.35	2.50	0.01	47.30	5.76	1.76	0.86	2.75	0.91
Decile 8	1.45	1.63	0.11	48.58	4.91	1.62	0.56	2.60	0.72
Decile 9	1.58	2.32	0.02	27.06	4.65	1.58	0.73	2.16	0.91
Decile 10	0.43	0.38	0.71	39.20	3.87	1.46	0.19	2.25	0.54
Spread (Decile1-10)	8.70	2.43	0.02	35.29	11.64	2.98	0.94	9.26	0.80

Table 10: Performance persistence tests - Conditional on investment objectives and skill measures. This tables displays, by investment objectives, out-of-sample Fung and Hsieh (2004) OLS alphas of decile portfolios ranked by OLS alpha and true skill measure, $\lambda_{A,relative}$ to empirical proxies for benchmark assets. The hedge funds are sorted every year into deciles on the 1st January (from 2001 until 2010) based on the OLS alpha and true skill measure, λ_A , estimation The monthly net-of-fee returns of individual hedge funds in the 36-month rolling window preceding every 1st January are used to evaluate the OLS alpha and true skill measure, λ_A . The portfolios are equal weighted on monthly basis, therefore the weights are readjusted whenever a fund disappears. Decile 1 comprises funds with the highest true skill measure, while decile 10 comprises the lowest. The last column reports the difference between Fung and Hsieh (2004) OLS alphas of Decile 1 and Decile 10 portfolio.

Investment Objectives	Portfolio	Decile 1	2	3	4	5	6	7	8	9	Decile 10	Decile 1-10
CTA	OLS alpha	10.71	7.48	4.84	3.37	4.51	3.39	3.73	4.14	2.91	1.41	9.31
	True skill	12.52	5.50	6.63	4.88	5.13	4.37	2.62	3.11	0.06	1.55	10.96
Convertible Arbitrage	OLS alpha	-2.39	8.83	4.47	5.56	1.81	-1.67	1.15	2.91	5.43	-2.72	0.33
	True skill	3.97	5.63	3.65	2.35	0.11	1.62	3.96	3.53	-0.54	1.11	2.86
Emerging Markets	OLS alpha	11.65	11.13	8.96	6.39	4.16	5.39	7.18	8.69	8.95	10.75	0.89
	True skill	17.51	12.28	5.77	12.78	8.39	4.41	4.83	6.53	3.26	6.58	10.93
Long/Short Equity	OLS alpha	2.49	-0.63	2.79	2.30	1.12	2.11	1.85	2.91	4.19	3.97	-1.48
	True skill	6.09	1.68	3.55	1.86	1.80	2.08	0.89	2.49	0.75	1.66	4.43
Equity Market Neutral	OLS alpha	1.26	3.69	4.74	-1.10	1.04	-1.01	-0.82	1.43	0.57	-0.29	1.54
	True skill	2.17	1.32	-0.82	0.43	2.19	-0.60	0.11	-0.68	2.59	1.21	0.97
Equity Short Bias	OLS alpha	-1.26	-7.51	2.02	-1.56	3.05	4.17	-2.00	-2.74	-5.16	-1.71	0.44
	True skill	0.38	5.34	6.36	4.72	-0.23	3.36	-9.09	-0.94	-0.83	-6.92	7.30

Table 10 (continued)

Investment Objectives	Portfolio/Decile	Decile 1	2	3	4	5	6	7	8	9	Decile 10	Decile 1- 10
Event Driven	OLS alpha	9.41	1.98	3.97	1.24	2.86	3.17	0.84	3.48	4.56	3.99	5.42
	True skill	8.76	6.39	2.93	5.55	2.44	2.37	-2.06	2.56	1.30	5.62	3.14
Fixed Income Arbitrage	OLS alpha	1.54	-1.95	-3.17	-0.50	2.64	0.91	-0.10	0.62	-1.91	8.52	-6.98
	True skill	12.87	-3.21	3.09	0.79	2.81	0.67	-6.93	-1.15	-6.08	4.13	8.74
Global Macro	OLS alpha	3.70	8.87	3.98	3.87	3.44	0.84	7.12	5.68	2.61	9.06	-5.37
	True skill	17.34	9.52	2.02	1.87	7.58	2.03	1.37	1.19	5.24	-0.25	17.59
Multi-Strategy	OLS alpha	7.27	2.02	5.32	5.31	5.71	3.08	0.54	3.85	2.21	1.04	6.23
	True skill	5.78	2.59	9.32	4.98	3.64	1.87	3.77	1.82	2.27	1.63	4.15
Other (Hedge Fund Index)	OLS alpha	4.16	2.44	2.67	-0.63	2.63	-0.81	1.15	-0.92	4.58	2.77	1.39
	True skill	3.75	3.03	2.79	-0.86	2.73	4.06	2.03	-1.77	0.28	1.41	2.34
All	OLS alpha	6.28	3.14	3.36	3.46	3.38	2.79	2.40	2.79	3.23	4.43	1.85
	True skill	11.44	5.58	5.09	2.25	2.87	2.21	2.35	1.45	1.58	0.43	11.01

Figure 1: Payoff Function. This figure displays the payoff to a hedge fund manager plotted against terminal fund value. The payoff is derived from performance fee, management fee and concern about short put positions. The underlying parameters are as follows $p = 0.2$, $m = 0.02$ and $c = 0.02$.

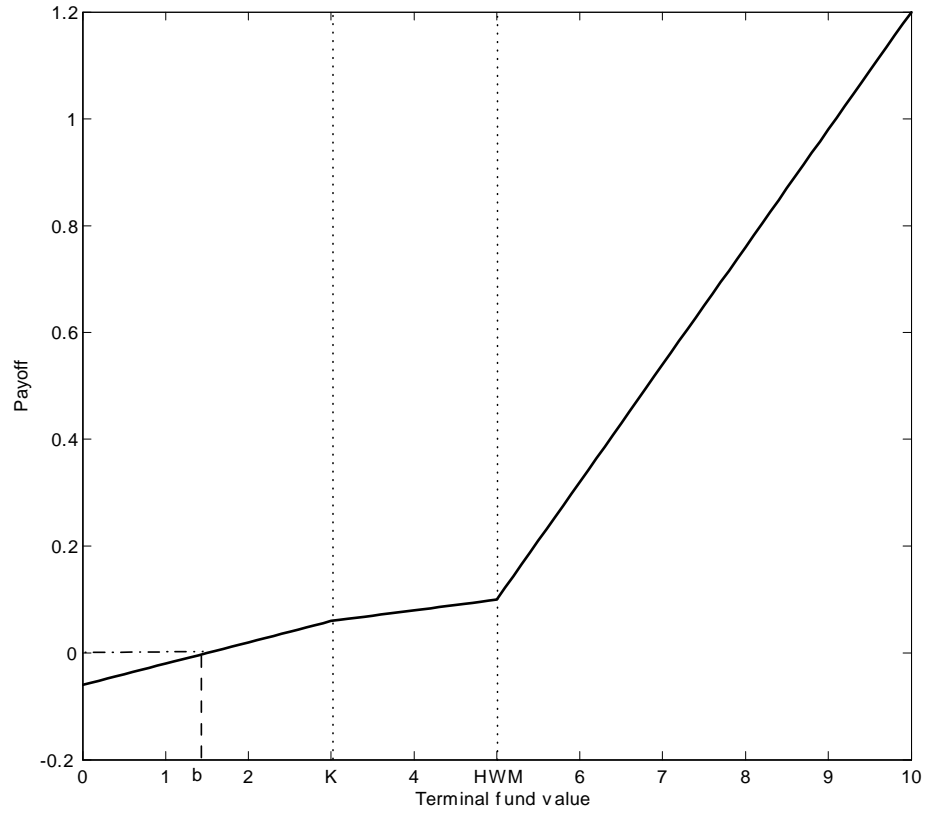


Figure 2: Concavified Utility Function. This figure displays the concavified utility function at different levels of terminal fund value. The solid line is the original utility function which is superimposed with a dotted line between X_K and X_{HWM} .

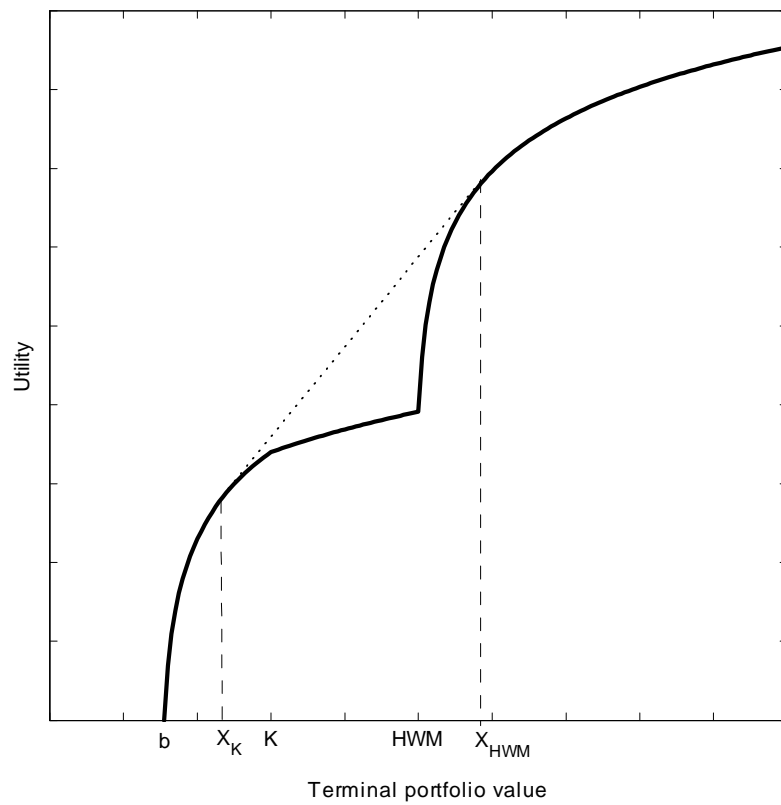


Figure 3: Optimal Allocation. This figure displays the optimal allocation to the risky asset against current fund value. The manager considers an incentive fee, management fee and the presence of put options when making her investment decision. The underlying parameters are as follows: $p = 0.2, m = 0.02, c = 0.02, \gamma = 2, \lambda = 0.4, r = 0$ and $t = 0$. Where p denotes the performance fee, m denotes the management fee, c denotes the concern level on short put option positions, γ denotes the level of risk aversion, λ denotes the Sharpe Ratio and r denotes the risk free rate.

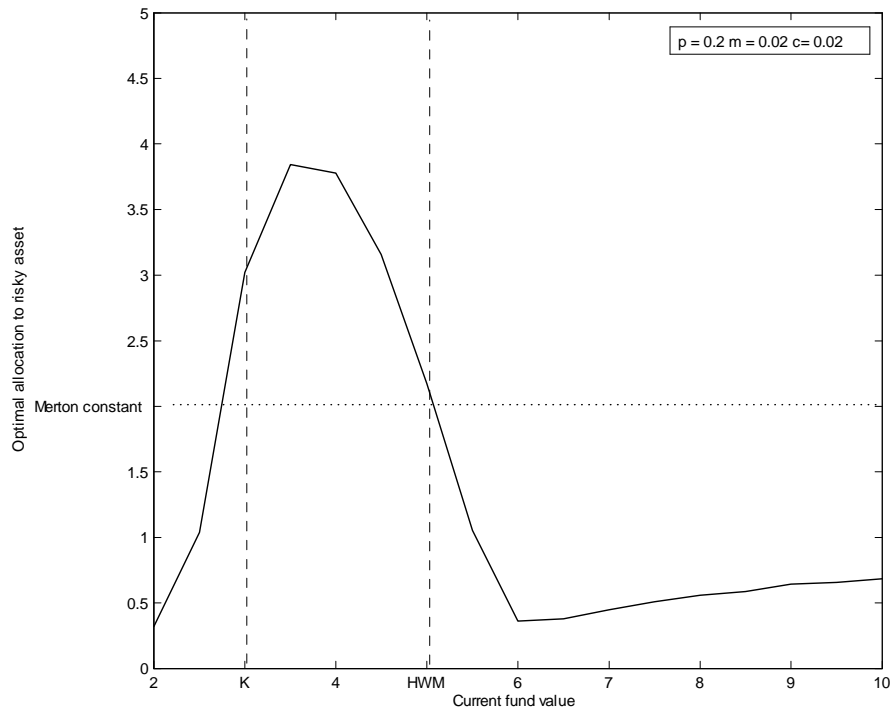


Figure 4: Sensitivity Analysis of Optimal Allocation. This figure displays the optimal allocation to a risky asset against current fund value. The dashed line represents the decision made by a manager who considers only an incentive fee, while the solid line represents a manager who considers both incentive fee and management fee when making her investment decision. The underlying parameters are as follows: $p = 0.2$, $m = [0, 0.02]$, $c = 0$, $\gamma = 2$, $\lambda = 0.4$, $r = 0$ and $t = 0$. Where p denotes the performance fee, m denotes the management fee, c denotes the concern level on short put oppositions, γ denotes level of the risk aversion, λ denotes the Sharpe Ratio and r denotes the risk free rate .

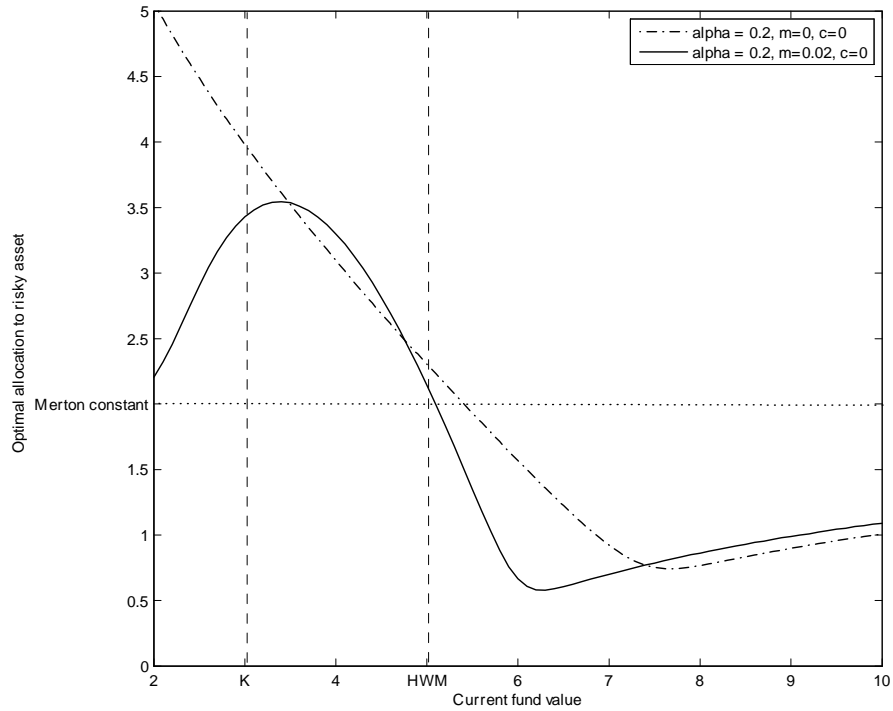


Figure 5: Scatter Plot of Fung-Hsieh Alpha. This figure displays the scatter plot of OLS Fung and Hsieh alpha on the y-axis and the ratio of current fund value to fund high water mark on the x-axis. Each dot corresponds to the estimated alpha of a fund over 36-month rolling estimation window. The hedge fund data is monthly net-of-fee returns of live and dead hedge funds reported in the BarclayHedge database . We exclude funds with less than 36 monthly observations from our fund universe.

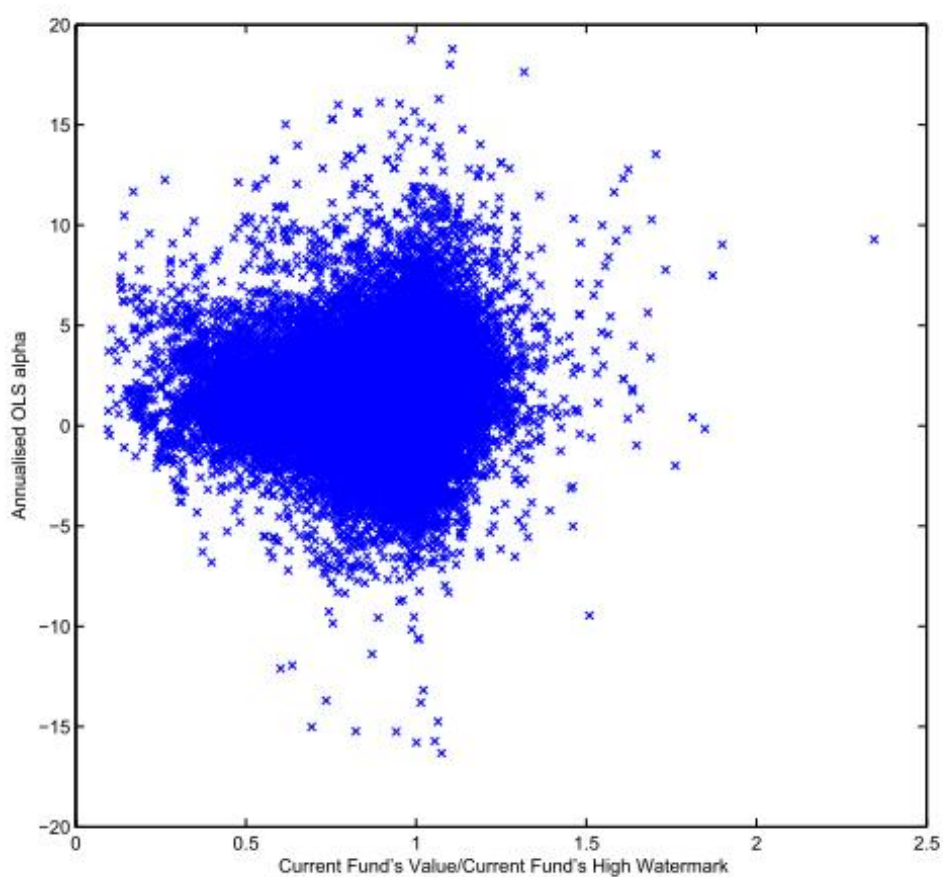


Figure 6: Difference between Reduced Form and True Alpha. This figure displays the difference between typical OLS alpha and true alpha against current fund value. The underlying parameters are as follows: $\alpha^* = 0.05$, $\sigma = 0.1$, $c = 0.02$, $\gamma = 1.5$, $\lambda = 0.5$, and $r = 0.05$. Where α^* denotes true alpha, σ denotes volatility of alpha generating process, c denotes concern level about short put option positions, γ denotes level of risk aversion, λ denotes Sharpe Ratio and r denotes the risk free rate.

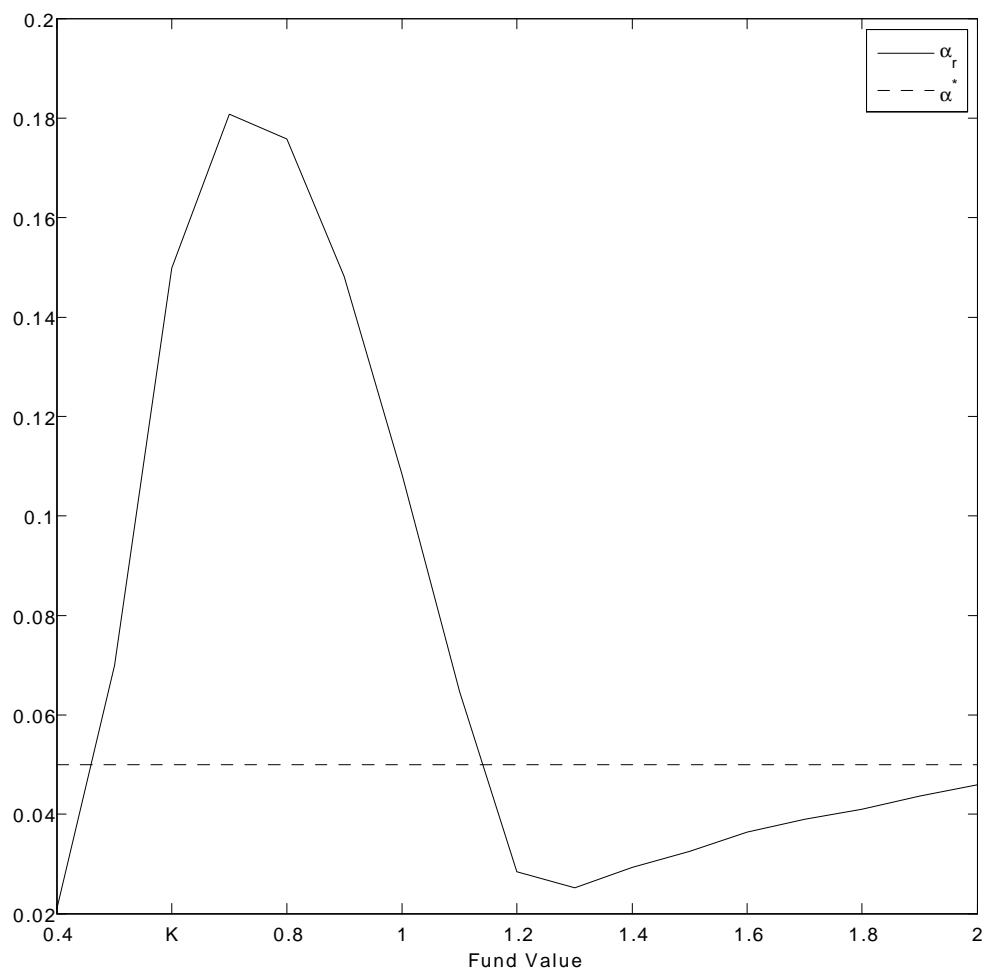


Figure 7: Risk Taking Plots. This figure displays the value of $\frac{1}{\gamma_t}$ as function of current fund value (X_t) normalised by current level of high water mark (HWM_t) at different value of λ_A . Other parameters are calibrated to the following values: $K = 0.6 * HWM_t$, $\gamma = 1.5$, $\rho = 20\%$, $m = 2\%$, $c = 2\%$.

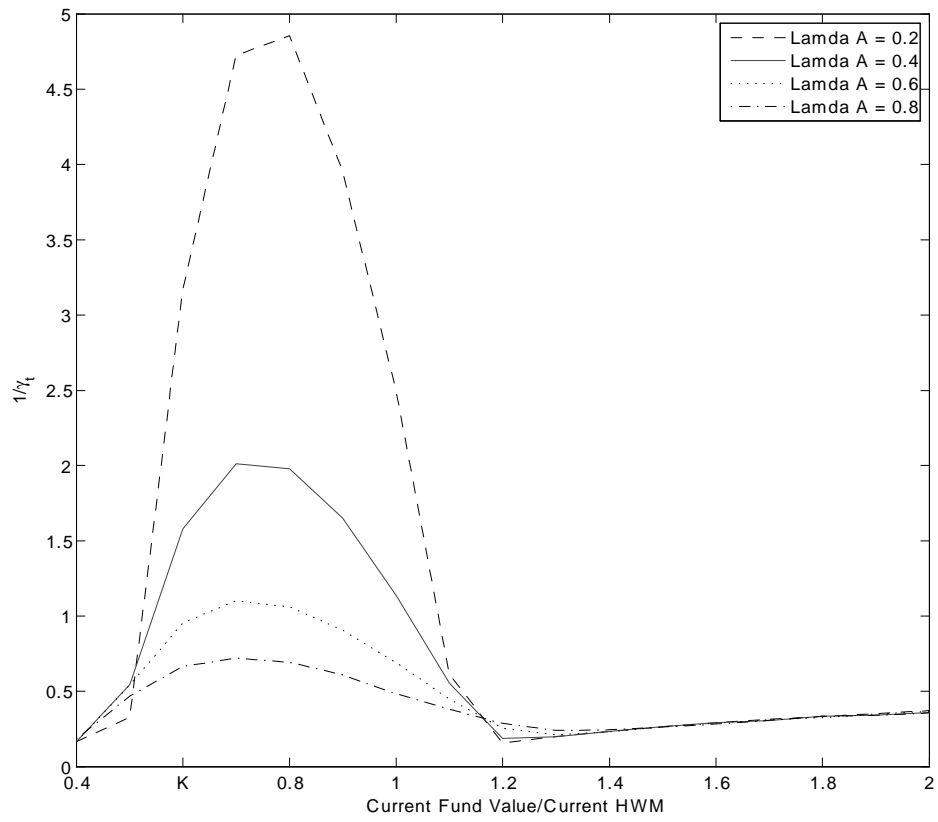


Figure 8: Risk Taking Plots from Simplified Functional Form. This figure displays the value of $\frac{1}{\gamma_t}$ implied from simplified function. The value of $\frac{1}{\gamma_t}$ is plotted against the current fund value (X_t) normalised by current level of high water mark (HWM_t) at different value of λ_A . The calibrated parameters in the function are; $a = 0.413, b = 0.221, c = 4.307, d = 3.483, g = 4.646$ and $h = 0.090$.

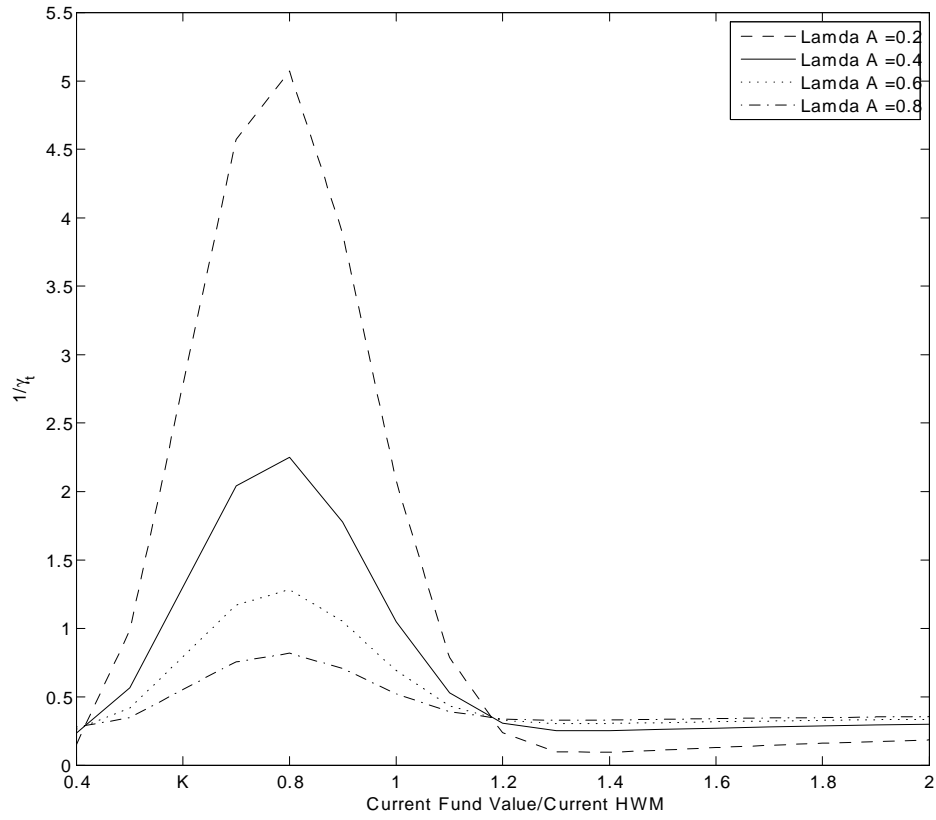


Figure 9: Evidence of Risk Shifting. This figure displays scatter plots, by investment category, between the current distances to high water marks, defined as $(X_t - HWM_t)/HWM_t$, of all hedge funds and their corresponding subsequent 12-month realized volatility between December 1999 and December 2010.

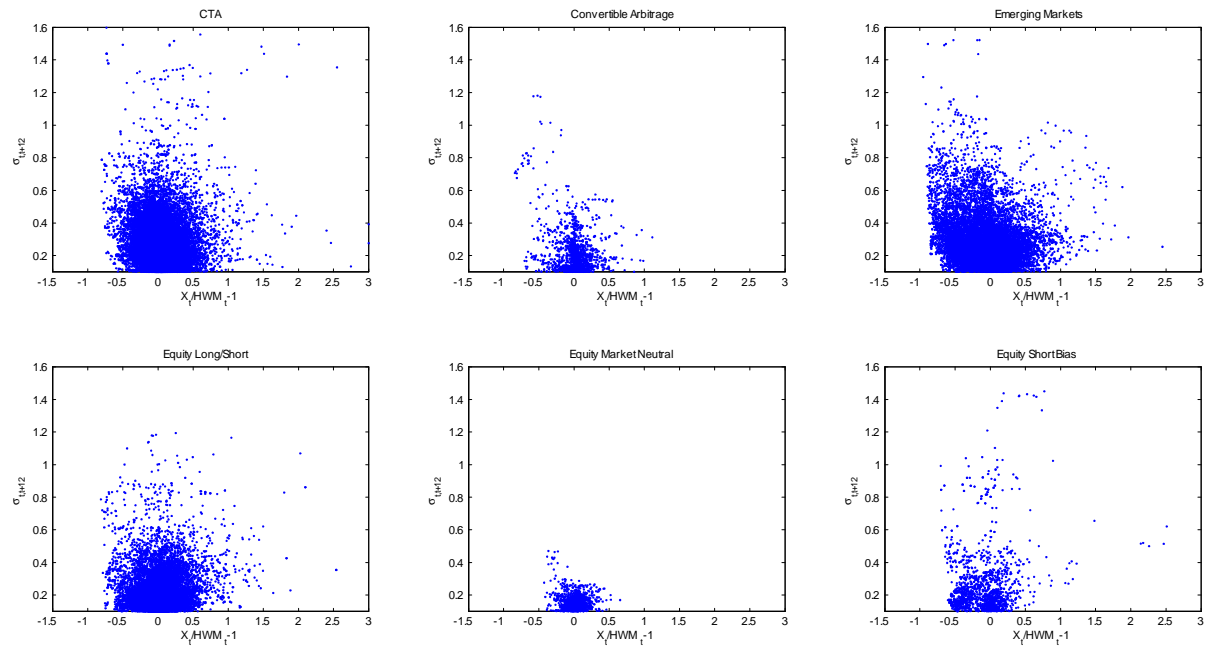


Figure 10: Evidence of Risk Shifting (cont). This figure displays scatter plots, by investment category, between the current distances to high water marks, defined as $(X_t - HWM_t)/HWM_t$, of all hedge funds and their corresponding subsequent 12-month realized volatility between December 1999 and December 2010.

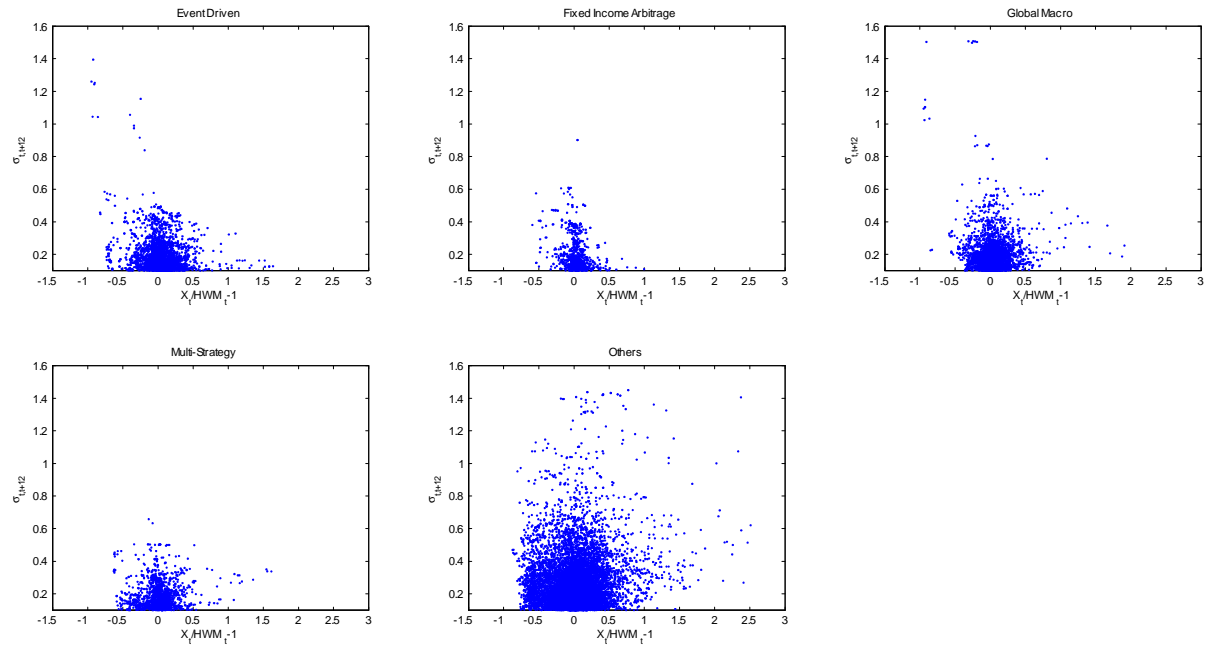


Figure 11: Evidence of Risk Shifting - Kernel fit, Fixed Income Arbitrage. This figure displays the scatter plot and its kernel fit between distance to high water mark and relative change in subsequent 12 month realized volatility of Fixed Income Arbitrage funds. The observation is between December 1999 and December 2010.

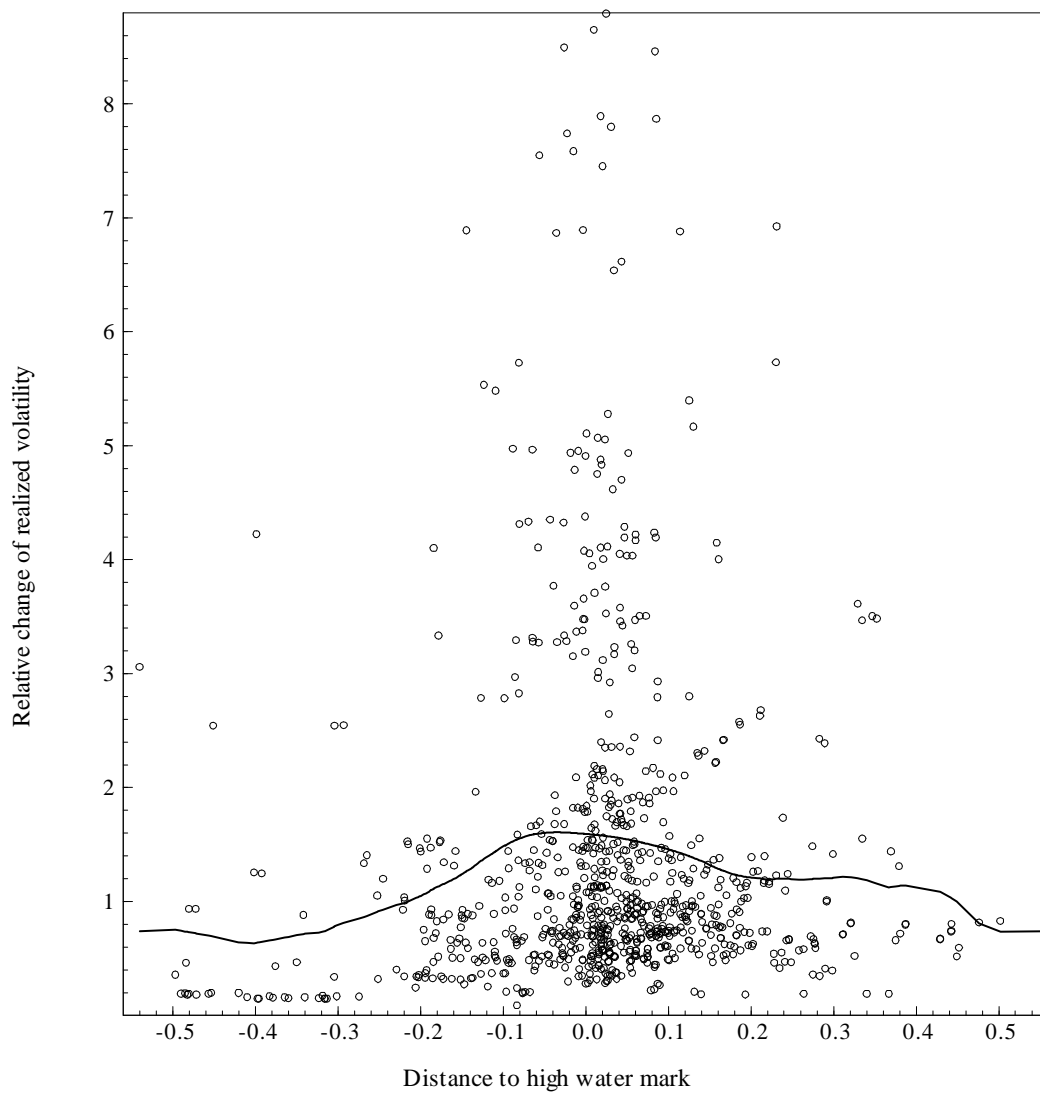


Figure 12: Evidence of Risk Shifting - Kernel fit, Event Driven. This figure displays the scatter plot and its kernel fit between distance to high water mark and relative change in subsequent 12 month realized volatility of Event Driven funds. The observation is between December 1999 and December 2010.

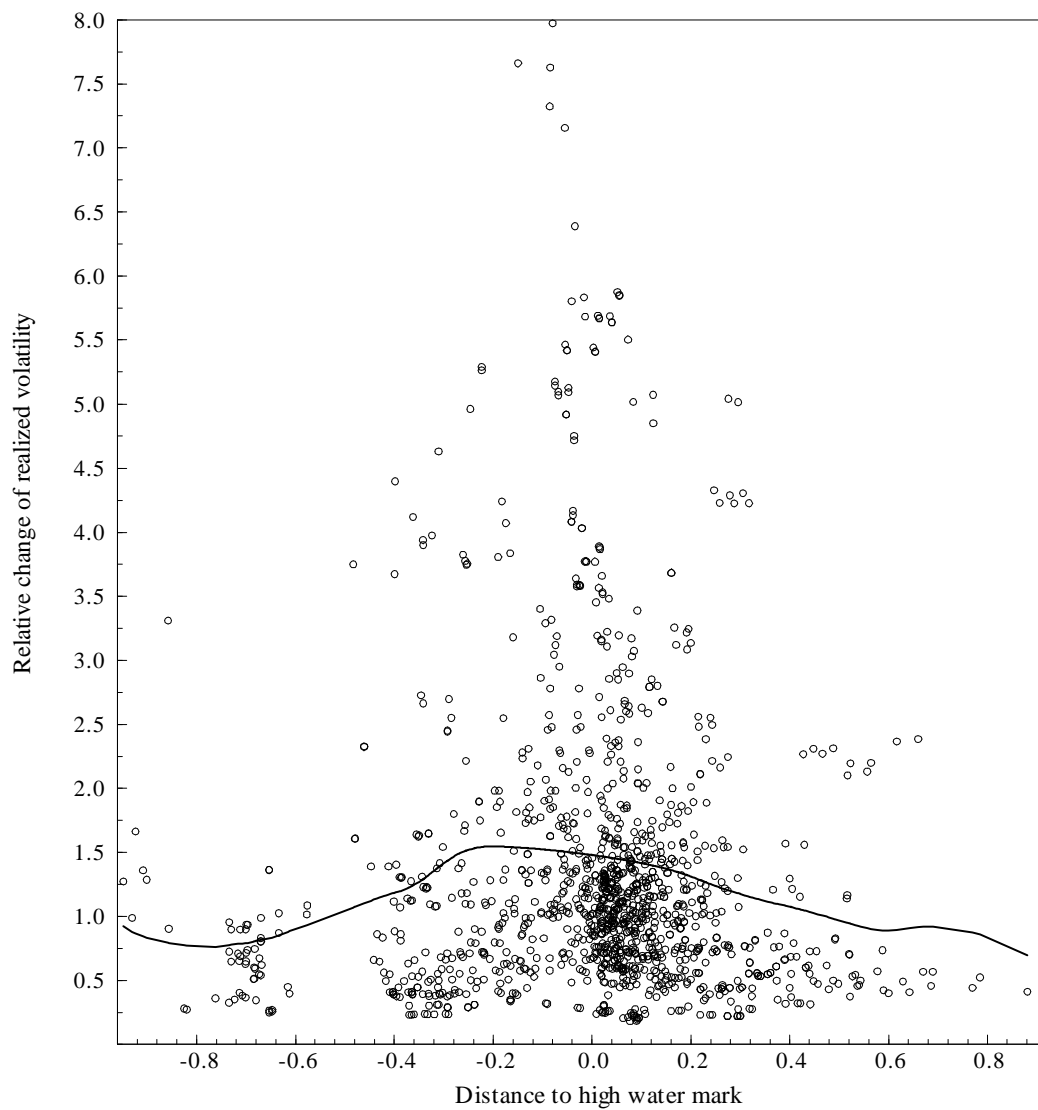


Figure 13: Performance persistence test - Cumulative returns of 1 dollar investment. This figure displays plots, for all hedge funds and by investment objectives, of cumulative returns of 1 dollar investment in top decile portfolios sorted by true skill measure and typical OLS alpha between January 2001 and December 2010.

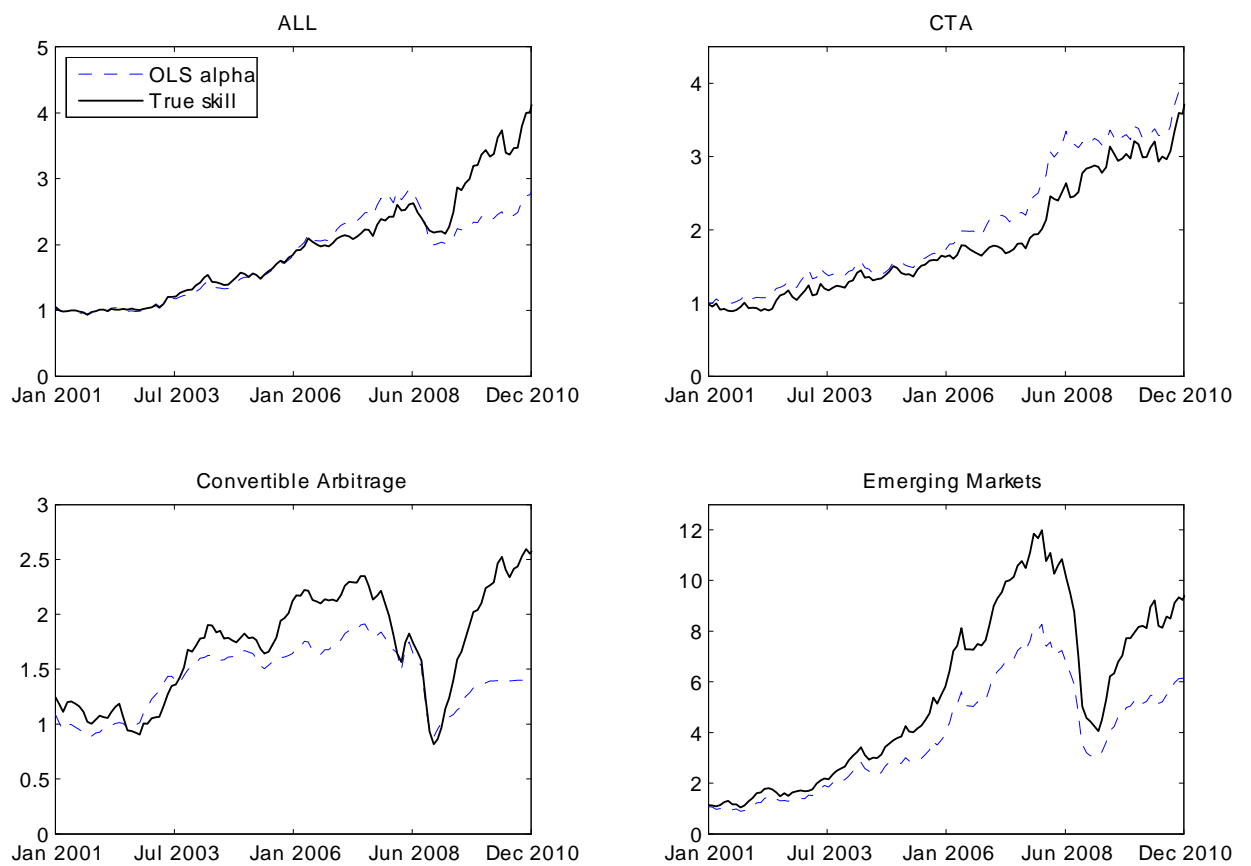


Figure 14: Performance persistence test - Cumulative returns of 1 dollar (cont) investment. This figure displays plots, for all hedge funds and by investment objectives, of cumulative returns of 1 dollar investment in top decile portfolios sorted by true skill measure and typical OLS alpha between January 2001 and December 2010.

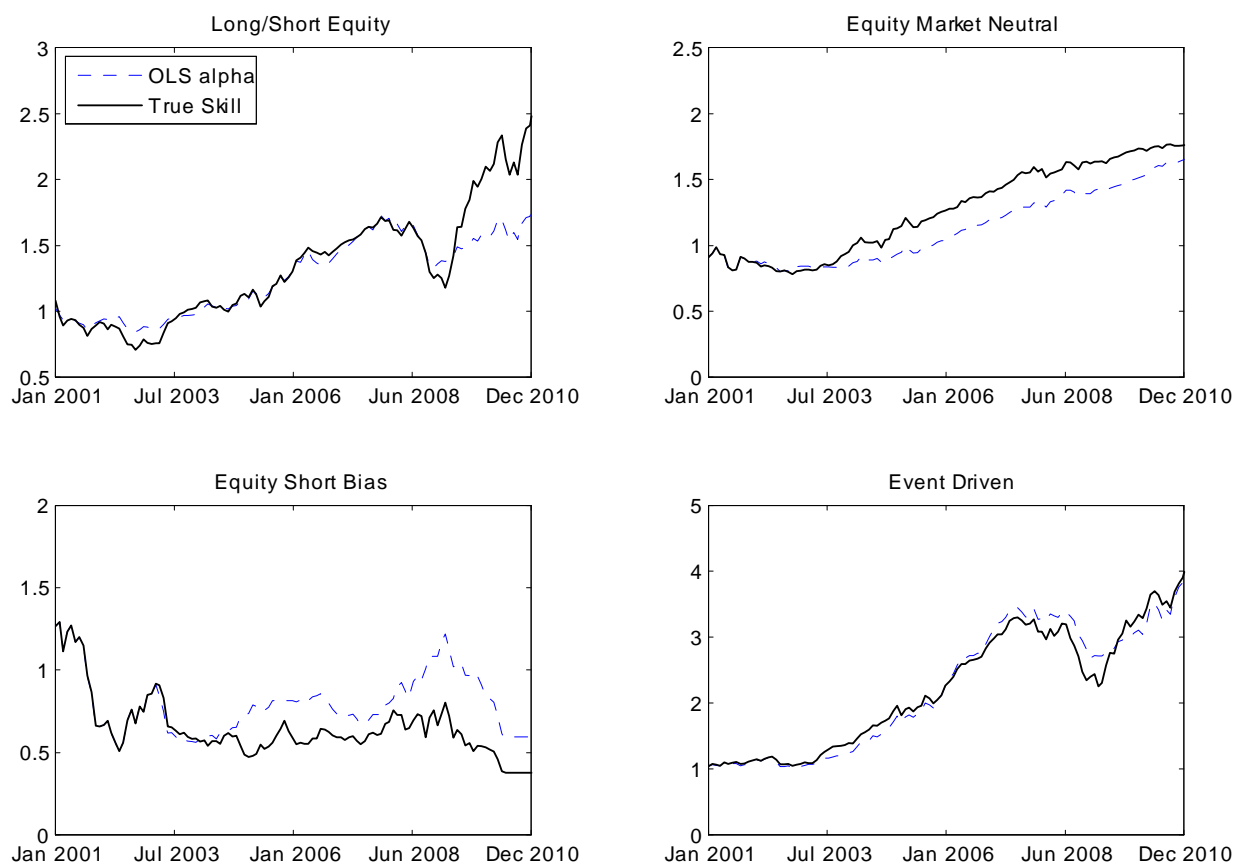


Figure 15: Performance persistence test - Cumulative returns of 1 dollar investment (cont). This figure displays plots, for all hedge funds and by investment objectives, of cumulative returns of 1 dollar investment in top decile portfolios sorted by true skill measure and typical OLS alpha between January 2001 and December 2010.

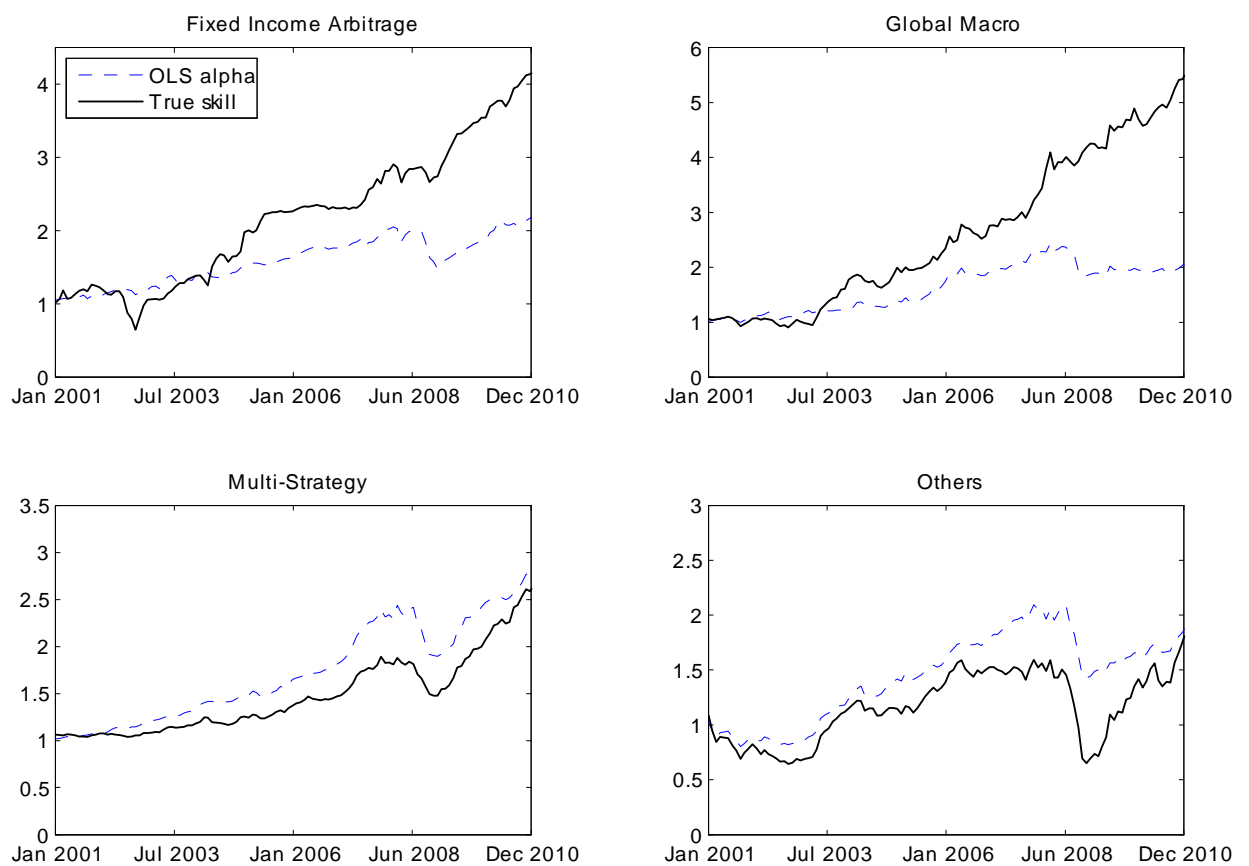


Figure 16: Nonconcave Utility Function. This figure displays a nonconcave utility function with a set of parameters which violates Condition 1. Panel (a) displays the nonconcave utility function with a tangent line which cannot satisfy Condition 1. Panel (b) displays the nonconcave utility function with a tangent line which satisfies Condition 1 when constraint X_K equal to K .

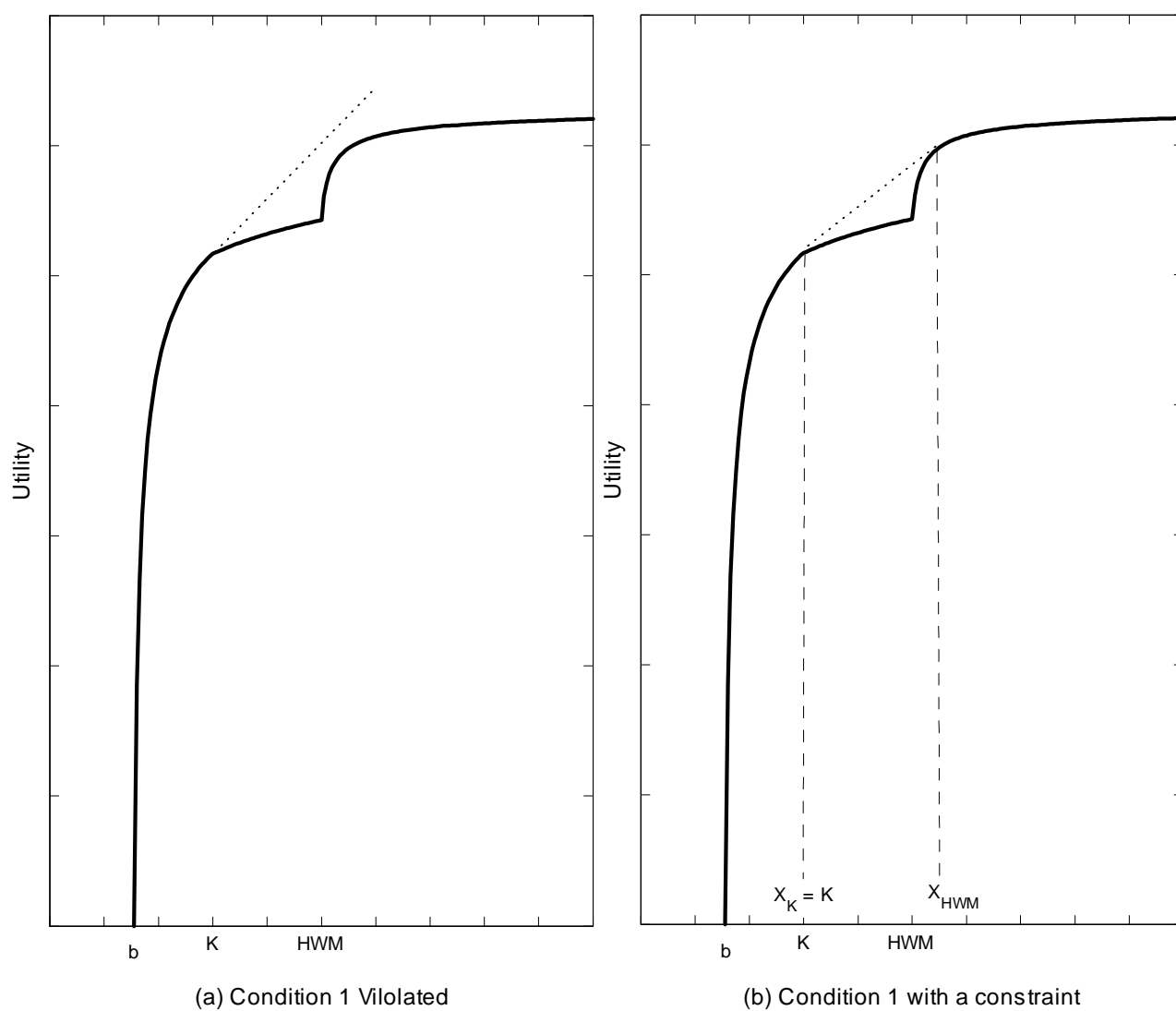


Figure 17: Sensitivity of risk taking profile with respect to put strike. This figure displays the sensitivity analysis of risk taking, an inverse of risk aversion, with respect to strike price of a put option.

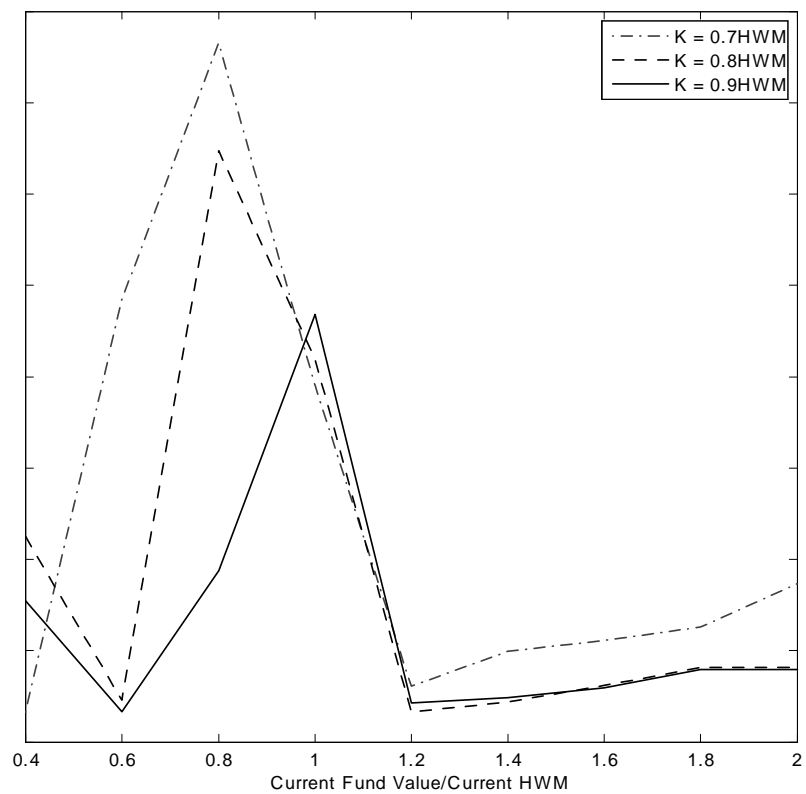


Figure 18: Calibration of risk taking function. This figure display the profile of risk taking profile, an inverse of risk aversion, with respect to different value of height, width and centrality parameters. The height parameter takes value (2,4,6), the width parameter takes value (0.5,0.8,1.1) and centrality parameter takes value (3.2,3.5,4).

