Dual Criteria Decisions

by

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February 2013

Forthcoming, *Journal of Economic Psychology*

Abstract. The most popular models of decision making use a single criterion to evaluate projects or lotteries. However, decision makers may actually consider multiple criteria when evaluating projects. We consider a dual criteria model from psychology. This model integrates the familiar tradeoffs between risk and utility that economists traditionally assume, allowance for rank-dependent decision weights, and consideration of income thresholds. We examine the issues involved in full maximum likelihood estimation of the model using observed choice data. We propose a general method for integrating the multiple criteria, using the logic of mixture models, which we believe is attractive from a decision-theoretic and statistical perspective. The model is applied to observed choices from a major natural experiment involving intrinsically dynamic choices over highly skewed outcomes. The evidence points to the clear role that income thresholds play in such decision making, but does not rule out a role for tradeoffs between risk and utility or probability weighting.

Keywords: risk, multiple criteria, individual decision making, natural experiment

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When decisions are being made about risky investments, do decision makers boil all of the facets of the prospect down to one criterion, which is then used to rank alternatives and guide choice, or do they use multiple criteria? The prevailing approach of economists to this problem is to generally assume a single criterion, whether it reflects the decision processes of standard expected utility theory (EUT), rank-dependent utility (RDU) theory, or prospect theory (PT). In each case the risky prospect is reduced to some scalar, representing the preferences, framing and budget constraints of the decision-maker, and then that scalar is used to rank alternatives. Many other disciplines assume the use of decision-making models or heuristics with multiple criteria. And often one encounters frustration that it is not possible to encapsulate all aspects of a decision into one of the popular single-criterion models.

In the psychology literature the expression "dual process" is not just about modeling two or more decision *criteria*. It is more specifically about modeling the interaction of impulsive decision processes with more sophisticated decision processes, often referred to as the Impulsive system and the Reflective system. Different academic disciplines will reasonably define the notions of "impulsive" and "more sophisticated" in different ways. As economists we interpret one of the two processes considered here as less sophisticated and the other as more sophisticated, but understand that psychologists might view both as more sophisticated and reflective. Our methodological objective is to demonstrate to economists and psychologists how multiple processes can be rigorously modeled and econometrically estimated from appropriate data, whichever "system" they come from.

We consider the dual criteria approach by means of an extraordinarily rich case study: the television game show *Deal Or No Deal*. Behavior in this show provides a wonderful opportunity to examine dynamic choice under uncertainty in a controlled manner with substantial stakes. The show has many of the features of a controlled natural experiment: contestants are presented with well-defined dynamic choices where the stakes are real and sizeable, and the tasks are repeated in the same manner from contestant to contestant.[1] However, we do not know how the producers of the game show select the contestants, and have no information about their personal characteristics apart from sex, and in some cases their age, marital status and region where they live.

The game involves each contestant deciding in a given round whether to accept a deterministic cash offer or to continue to play the game. It therefore represents a non-strategic game of timing, and is often presented to contestants as exactly that by the host. If the subject chooses "No Deal," and continues to play the game, then the outcome is uncertain. The sequence of choices is intrinsically dynamic because the deterministic cash offer evolves in a relatively simple manner as time passes. Apart from adding drama to the show, this temporal connection makes the choices particularly interesting and, arguably, more relevant to the types of decisions one expects in naturally occurring environments. For example, one can decide to cash in on an investment in light of new information or continue with the investment for another period to time. Cubitt and Sugden [2001] make this point explicitly, contrasting the static, one-shot nature of the choice tasks typically encountered in laboratory experiments with the sequential, dynamic choices that theory is supposed to be applied to in the field. It is also clearly stated in Thaler and Johnson [1990; p. 643], who recognize that the issues raised by considering dynamic sequences of choices are "quite general since decisions are rarely made in temporal isolation." We explain the format of the show in section 2, and discuss this temporal connection.

We examine two modeling approaches to these data. One is the single-criterion RDU model, which can be viewed as a generalization of EUT to allow for non-linear decision weights. The other is a specific dual-criteria model from psychology which could have been built with this task domain in mind: the SP/A theory of Lopes [1995]. The SP/A model departs from EUT, RDU and PT in one major respect: it is a dual criteria model. Each of the single criterion models, even if they have a number of psychological components to their evaluation stage, boil down to a scalar index for each

[1] Game shows are increasingly recognized as a valuable source of replicable data on decision-making with large stakes. Andersen, Harrison, Lau and Rutström [2008b] review the applications to choice under uncertainty, including many recent applications of data from *DOND*. All of these studies consider single-criterion models.

lottery. The SP/A model instead explicitly posits two distinct but simultaneous ways in which the same subject might evaluate a given lottery. One is the SP part, for a process that weights the "security" and "potential" of the lottery in ways that are similar to RDU. The other is the A part, which focuses on the "aspirations" of the decision-maker. In many settings these two parts appear to be in conflict, which means that one must be precise as to how that conflict is resolved. We discuss each part, and then how the two parts may be jointly estimated in section 3.

As economists, we view the A part of the models as far less sophisticated and reflective than the SP part. Indeed, a special case of the SP part is the familiar, reflective workhouse of economists, EUT. Thus our approach should be of particular interest to economists seeking ways of formally integrating into their analysis non-standard, psychological processes that do not reflect the aversion to variability of outcomes underlying EUT.

Apart from using a systematic maximum-likelihood approach to the estimation of the SP/A model, we propose a natural decision-theoretic and statistical framework to resolve the potential conflict between the two criteria. This is the notion of a mixture of latent decision-making processes. Rather than view the observed data as generated by a single decision-making process, such as EUT, RDU *or* PT, one could easily imagine the data from a sample being generated by some mixture of these processes. Harrison and Rutström [2009], for example, allowed (laboratory lottery) choices to be made by EUT *and* PT, with a statistical mixture model being used to estimate the fraction of choices better characterized by EUT and the fraction better characterized by PT. In our case we simply extend this mixture notion to the two criteria of one model, rather than the two criteria of two models. We discuss this approach, and its interpretation, in section 4. We argue that mixture models provide a natural formalization, in theory and applied work, of multiple-criterion models.

We present empirical results in section 5, estimating an RDU model and then an SP/A model with data drawn from the UK version of *Deal Or No Deal*. We employ data covering 2,317 choices by 461 contestants over prizes ranging from 1 penny to £250,000. This prize range is roughly equivalent to US $0.02 and US $460,000. Average earnings in the game show are £17,737 in our sample. The distribution of earnings is heavily skewed, with relatively few subjects receiving the highest prizes, and median earnings are £13,000.

We find evidence that there is indeed some probability weighting being undertaken by contestants. We also find evidence that "aspiration levels" and "security levels" play a role in decision-making in the SP/A model, which was motivated by psychological findings in task domains that have highly skewed prize distributions. To some extent one can view these aspiration and security levels as similar to reference points and loss aversion, concepts from PT, although the psychological motivation and formal modeling is quite distinct. Thus we conclude that more attention should be paid to the manner in which psychologically-motivated notions of choice in risky behavior are modeled.

In section 1 we discuss the manner in which decision processes relate to decision criteria, from the perspective of economics and psychology. In section 2 we document the game show format and field data we use. In section 3 we describe the SP/A model of the latent decision-making process. In section 4 we review the use and interpretation of mixture specifications in dual criteria models. Section 5 presents empirical results from estimating the SP/A model using the large-stakes game show data, and section 6 examines implications for model comparisons. Finally, section 7 offers conclusions.

1. Decision Criteria and Decision Processes

The different theories that economists have about decision making under risk reflect differences in the decision making processes that are assumed.

Expected Utility Theory (EUT) posits a psychological aversion to variability of final outcomes. This is often stated as an aversion to variance, but can include any variability such as skewness or kurtosis. The process of evaluation under EUT is that the decision maker evaluates alternative lotteries in terms of their variability, and the extent to which that variability trades off with the expected outcome. This process can be represented by a criteria that characterizes the choice: that it reflects the lottery with the highest expected utility, where expected utility is evaluated with a utility function that has appropriate mathematical characteristics (e.g., diminishing marginal utility, prudence, or temperance). Hence the criteria here is just a formal characterization of the *outcome* of the decision-making process.

The Rank Dependent Utility (RDU) theory of Quiggin [1982] posits two psychological processes instead of the one psychological process of EUT. It presumes that the decision maker has an aversion to variability of outcomes, but also exhibits some "probability optimism" or "probability pessimism." The decision maker is said to be pessimistic about probabilities if he assigns lower decision weights to better outcomes and higher decision weights to worse outcomes, where "lower" and "higher" are relative to the objective probabilities. This psychological process of weighting probabilities of outcomes by something other than the objective probability itself can generate an aversion to risk even if there is no aversion to variability of outcomes.[2] But RDU allows for both psychological processes to operate, and indeed that dual process feature of RDU is it's reason for existence: to allow it to explain choice patterns that EUT allegedly cannot. For instance, RDU has an immediate explanation for why the same person would purchase insurance *and* lottery tickets, even if both are actuarially unfair. The explanation is that one process explains the demand for insurance and another process explains the demand for gambling.

As it happens, the two psychological processes of RDU can be represented by one decision criteria. In this case the decision maker is said to behave as if they compare the decision-weighted utility of one lottery with the decision-weighted utility of the other lottery, and choose the lottery with the highest decision-weights. If one replaces "decision weights" with "objective probabilities," then RDU collapses to the special case of EUT, and in that sense can be viewed as a generalization of EUT. Even if it can be formally represented by a single decision criterion, however, that criterion derives from two distinct psychological processes.

The Cumulative Prospect Theory (CPT) of Tversky and Kahneman [1992] adds one further psychological process to RDU, the notion of sign dependence. In this case the decision maker is assumed to represent the outcomes of each lottery as gains or losses relative to some reference

[2] This special case of RDU is known as Dual Theory, and was developed by Yaari [1987].

point, and to treat gains and losses differently in terms of the aversion to variability of each and the probability weighting applied to each. In addition to losses and gains exhibiting potentially different aversions to variability, and different probability weighting, there is a direct utility aversion to losses. Hence there is a complex interaction of processes with CPT. The notion of aversion to variability of outcome is the same as EUT, but it interacts with sign dependence. Similarly, the notion of probability weighting is the same as RDU, but it also interacts with sign dependence. And then there is a separate utility aversion to losses compared to gains. Economists view CPT as positing three psychological decision processes (aversion to variability of outcomes, probability weighting, and sign dependence), but one could also view it as having five distinguishable psychological processes.[3] Remarkably, CPT also boils down formally to just one decision criteria: that the choice again reflect the lottery with the highest probability-weighted utility, but where the probability weights and utility evaluations take into account the sign dependence of the outcomes.[4]

Thus, the models favored by behavioral economists are couched in terms multiple psychological processes, but result in formal characterizations in terms of single decision criteria.

In psychology the notion of dual or multiple processes posits different ways of evaluating lotteries. Chaiken and Trope [1999] and Stanovich and West [2000] provide reviews of the literature, which encompasses a wide array of psychological processes.[5] As summarized by Mukherjee [2010; p.243]:

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[3] These are (1) aversion to variability of outcomes coded as gains, (2) aversion to variability of outcomes coded as losses, (3) probability weighting for outcomes coded as gains, (4) probability weighting for outcomes coded as losses, and (5) direct utility aversion to losses compared to gains.

[4] The Original Prospect Theory of Kahneman and Tversky [1979] contained two clear psychological processes that were not reducible to a single decision criteria. The first was the "editing process" by which lotteries were cognitively represented by decision makers, and the second was the "evaluation process" by which the edited lottery was evaluated with a single criteria. In turn the evaluation process allowed for the three processes of CPT, even if the decision weights were not assumed to be generated in the same manner. In OPT the editing process is assumed to apply first, and then the evaluation process follows lexicographically. In this manner OPT avoided certain unattractive predictions, such as violations of first-order stochastic dominance from probability weighting, since these were assumed to be avoided by choices made during the editing process.

[5] In addition, there is a large literature from neuropsychology and neuroeconomics that purports to identify the physical locations of some of these processes in the human brain. For reasons detailed in Harrison [2008a][2008b], we do not find this evidence compelling, even if we do find the notion of multiple, concurrent processes compelling.

> The dual process paradigm of reasoning states that there are two fundamentally different ways of processing information, one variously labeled as intuitive, automatic, natural, narrative, and experiential and the other seen as analytical, verbal, deliberative, and rational. Stanovich and West [2000] referred to the former system collectively as *System 1* and the latter collectively as *System 2*.

These processes sometimes lead to specific criteria for evaluating lotteries, as illustrated by Mukerjhee [2010], who in fact uses mixtures of the utility functions and probability weighting functions from RDU and CPT.

It is often the case that what is represented in psychology as distinct processes would be viewed by economists as a purely semantic distinction. For instance, Hsee and Rottenstreich [2004] hypothesize that there is a process of "valuation by calculation" and a process of "valuation by feeling." Their experimental design rests on subjects in different treatments being "primed" for one or the other process, and exhibiting different monetary valuations for the same objects. But to an economist the utility function embodies how one feels about a commodity in comparison to another commodity, as a matter of definition of revealed preference. That is, the utility numbers assigned to two objects of choice are *defined* by economists as any numbers that are larger for the object chosen. So if the individual calculated the utility of the two objects and then chose the object with the higher utility, or chose the object that felt better, the individual cannot be grammatically said to be behaving differently. Now this might be seen as just reflecting the semantic paucity of economists, but it is important to be clear on the use of words when one is translating models from psychology to economics, and *vice versa*.

However, the fact that economists would not *view* these as distinct processes in any operationally meaningful sense does not mean that they have an immediate explanation for the experimental data. The point here is not to dismiss the data claim, just to avoid assuming a distinct decision process every time one cannot immediately explain the data. This is a methodological problem with behavioral economics in general, and one that one also wants to avoid when evaluating psychological models of multiple processes.

Our approach has a clear methodological purpose. We want to evaluate how one can formally state and estimate a model in which there are multiple psychological processes, resulting in two distinct decision criteria. We deliberately want to take a model of some currency from cognitive psychology and demonstrate how one interprets and estimates it using the tools of economics.

2. The Naturally Occurring Game Show Data

The version of *Deal Or No Deal* shown in the United Kingdom starts with a contestant being randomly picked from a group of 22 preselected people.[6] They are told that a known list of monetary prizes, ranging from 1p up to £250,000, has been placed in 22 boxes.[7] Each box has a number from 1 to 22 associated with it, and one box has been allocated at random to the contestant before the show. The contestant is informed that the money has been put in the boxes by an independent third party, and in fact it is common that any unopened boxes at the end of play are opened so that the audience can see that all prizes were in play. The picture below shows how the prizes are displayed to the subject, the proto-typically British "Trevor," at the beginning of the game. The contestant keeps his box during the game and opens the other boxes one at a time. As the other boxes are opened the contestant obtains more information about the likely value of his own box. The contestant receives the amount in the box if he or she declines all offers to buy the box during the show.

[6] The 22 boxes are randomly allocated to 22 preselected people, and one person is randomly picked to be the contestant. The remaining 21 participants will open their boxes during the show and reveal the prizes in the boxes, and they will become contestants in later episodes. This format implies that each contestant will have had time to learn and reflect on the game show, and the contestants frequently ask the other participants for advice during the game.

[7] There is no fixed fee for participating in the game show. Harrison, Lau and Rutström [2009] show that the common use of non-stochastic show-up fees may generate samples that are more risk averse than would otherwise have been observed. They do not consider treatments with no show-up fees, and it is likely that the format of the game show attracts people who are less risk averse than the general population.

In round 1 the contestant must pick 5 of the remaining 21 boxes to be opened, so that their prizes can be displayed. A good round for a contestant occurs if the opened prizes are low, and hence the odds increase that his box holds one of the higher prizes. At the end of each round the host is phoned by a "banker" who makes a deterministic cash offer to buy the contestant's box.

The initial offer in early rounds is typically low in comparison to expected offers in later rounds. We document an empirical offer function later, but the qualitative trend is quite clear: the bank offer starts out at roughly 15% of the expected value of the unopened boxes, and increases to roughly 24%, 34%, 42%, 54% and then 73% in rounds 2 though 6. This trend is significant, and serves to keep all but extremely risk averse contestants in the game for several rounds. For this reason it is clear that the contestant's box has an option value in future rounds.[8]

In round 2 the contestant must pick 3 boxes to open, and then there is another bank offer to consider. In rounds 3 through 6 the contestant must open 3 boxes in each round. At the end of round 6 there are only 2 unopened boxes, one of which is the contestant's box.

In round 6 the decision is a relatively simple one from an analyst's perspective: either take the non-stochastic cash offer or take the lottery with a 50% chance of either of the two remaining unopened prizes. We could assume some latent utility function, or non-standard decision function, and directly estimate parameters for that function that best explains the observed binary choices in this round. Unfortunately, relatively few contestants get to this stage, having accepted offers in earlier rounds. In our data, only 39% of contestants reach that point.[9] More serious than the smaller sample size, one naturally expects that risk attitudes would affect those surviving to this round. Thus there would be a serious sample selection bias if one just studied choices in later rounds.

[8] Things become much more complex if the bank offer in any round is statistically informative about the prize in the contestant's box. In that case the contestant has to make some correction for this possibility, and also consider the strategic behavior of the banker's offer. Bombardini and Trebbi [2012] offer clear evidence that this occurs in the Italian version of the show, but there is no evidence that it occurs in the UK version.

[9] This fraction is even smaller in other versions of the game show that are broadcast in other countries, where there are typically 9 rounds. Other versions generally have bank offers that are more generous in later rounds, with most of them approaching 100% of the expected value of the unopened boxes. In some cases the offers exceed 100% of this expected value. In the UK version the generosity of later-round bank offers slowly improved over the seasons of the show, and we allow for this by using a lagged estimate of the empirical distribution of offers.

In round 5 the decision is conceptually much more interesting. Again the contestant can just take the non-stochastic cash offer. But now the decision to continue amounts to opting for one of two potential lotteries: (i) take the offer that will come in round 6 after three more boxes are opened, or (ii) decide in round 5 to reject that offer in round 6, and then play out the final 50/50 lottery. Each of these two options is an uncertain lottery, from the perspective of the contestant in round 5. Choices in earlier rounds involve larger and larger sets of potential lotteries of this form.

Saying "No Deal" in early rounds provides one with the option of being offered a better deal in the future, independently of the expected value of the unopened prizes in future rounds. Thus a forward looking subject may choose to continue to another round since the expected improvement in the bank offer provides some compensation for the additional risk of going into another round. To evaluate the parameters of some latent utility function given observed choices in earlier rounds, we have to mentally play out all possible future paths that the contestant faces.[10] When the subject expects to "Deal" with the banker then depends on the assumed parameter values of the latent utility function, and how far the subject is looking ahead. This corresponds to procedures developed in the finance literature to price path-dependent derivative securities using Monte Carlo simulation (e.g., Campbell, Lo and MacKinlay [1997; §9.4]).

The SP/A model suggested by Lopes [1984] combines two decision criteria that appear to fit the decision-making process well. The first criterion considers the security and potential of the lottery, and it devotes extra attention to relatively small and high outcomes compared to moderate outcomes. Indeed, the host often refers to the "big five" prizes, with particular attention to the two highest prizes of £100,000 and £250,000, and there are often expressions of relief from contestants when the lowest prizes are revealed during the game and therefore cannot be in the contestant's box. The second criterion considers the aspiration level of the subject and can be considered as a reference point that the subject brings to the game show. The host sometimes asks the contestant to

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[10] Or make some *a priori* judgement about the bounded rationality of contestants. For example, one could assume that contestants only look forward one or two rounds, or that they completely ignore bank offers.

privately write down the amount that he or she hopes to win, and the outcome of the game is then evaluated against that aspiration level when the contestant has ended the game. This aspiration level is sometimes revealed, but that is a private decision by the contestant and not recorded in our data.[11]

The show began broadcasting in the United Kingdom in October 2005, and has been showing constantly since. There are normally 6 episodes per week: a daytime episode and a single prime time episode, each roughly 45 minutes in length. Our data are drawn primarily from direct observation of recorded episodes, but we also verify data against those tabulated on the web site http://www.dond.co.uk/. Our data consists of behavior on 461 contestants.

3. Modeling Contestant Behavior

Most models of economics assume that decision-makers use just one criteria for evaluating prospects, even if there are various psychological pathways to that evaluation. For instance, EUT allows for aversion to variability (such as variance, skewness and kurtosis), RDU allows for probability weighting, and CPT further allows for loss aversion. But all end up "boiling" these pathways down into a scalar for each prospect in a choice setting, as explained earlier.

In economics one exception is the class of lexicographic models, although one might view the criterion at each stage as being contemplated simultaneously. For example, Rubinstein [1988] and Leland [1994] consider the use of similarity relations in conjunction with "some other criterion" if the similarity relation does not recommend a choice. In fact, Rubinstein [1988] and Leland [1994] reverse the sequential order in which the two criteria are applied, indicating some sense of uncertainty about the strict sequencing of the application of criterion. Similarly, as noted earlier, the OPT theory of Kahneman and Tversky [1979] considered an editing stage to be followed by an evaluation stage. Another exception in behavioral economics is the class of dual self models. For instance, Benhabib and Bisin [2005] and Fudenberg and Levine [2006] propose models that posits that the decision-maker has two selves. One self has a utility function defined over wealth and

[11] Farber [2008] considers reference-dependent preferences for taxi drivers in New York City and finds that the reference level of income varies substantially from day to day.

another self has a utility function defined over income. In effect, the former self constrains choices actually observed by the latter self. Under some circumstances observed choices will be consistent with either self. Andersen, Harrison, Lau and Rutström [2008a] examine the econometric implications of this framework for experimental data.

Models with multiple criteria are more popular in other disciplines, such as psychology. Quite apart from the specific model from psychology evaluated here, there is a large literature in psychology referenced by Starmer [2000] and Brandstätter, Gigerenzer and Hertwig [2006]. Again, many of these models of the decision process present multiple criteria that might be used in a strict sequence, but which are sometimes viewed as being used simultaneously. In decision sciences the weighted sum model of Fishburn [1967] remains popular, although it could be viewed as a multi-attribute utility model. The analytic hierarchy process model of Saaty [1980] remains very popular in corporate settings, and has gone through numerous revisions and extensions. Popular textbooks on multi-criterion decision making in business schools include Kirkwood [1997] and Liberatore and Nydick [2002]; the emphasis at that level is on alternative software packages that are commercially available.

In some cases models with multiple criteria can be reduced to a single criterion framework, and simply represent a recognition that there may be many attributes or arguments of that criterion. For example, multi-attribute expected utility, reviewed in Keeney and Raiffa [1976] or von Winterfeldt and Edwards [1986; ch.7]. These models may be estimated using traditional econometric methods, assuming that the experimental design allows for identification of all structural parameters as illustrated by Andersen, Harrison, Lau and Rutström [2011]. Or one can seek appropriate single-criterion utility representations of informal dual-criteria decision rules, such as the well-known tradeoff between "risk" and "return" (e.g., Bell [1995]). In some cases these criteria do not lead to crisp scalars derivable by formulae.[12]

We evaluate one popular alternative from psychology, and show how it can be formally

[12] For example, old debates in psychology about when one should use "heads instead of formulas," reviewed by Kleinmutz [1990]. Also see Hogarth [2001] for a related perspective.

defined and estimated using concepts from mixture modeling.

A. Rank-Dependent Preferences

One route of departure from EUT has been to allow preferences to depend on the rank of the final outcome. The idea that one could use non-linear transformations of the probabilities of a lottery when weighting outcomes, *instead* of non-linear transformations of the outcome into utility, was most sharply presented by Yaari [1987]. To illustrate the point clearly, he assumed that one employed a linear utility function, in effect ruling out any risk aversion or risk seeking from the shape of the utility function *per se*. Instead, concave (convex) probability weighting functions would imply risk seeking (risk aversion). It was possible for a given decision maker to have a probability weighting function with both concave and convex components, and the conventional wisdom held that it was concave for smaller probabilities and convex for larger probabilities.

The idea of rank-dependent preferences had two important precursors.[13] In economics Quiggin [1982] had formally presented the general case in which one allowed for subjective probability weighting in a rank-dependent manner and allowed non-linear utility functions. This branch of the family tree of choice models has become known as Rank-Dependent Utility (RDU). The Yaari [1987] model can be seen as a pedagogically important special case, and can be called Rank-Dependent Expected Value (RDEV).

The other precursor, in psychology, is Lopes [1984]. Her concern was motivated by clear preferences that experimental subjects exhibited for lotteries with the same expected value but alternative shapes of probabilities, as well as the verbal protocols those subjects provided as a possible indicator of their latent decision processes. One of the most striking characteristics of *DOND* is that it offers contestants a "long shot," in the sense that there are small probabilities of extremely high prizes, but higher probabilities of lower prizes. We return below to consider a later

[13] Of course, many others recognized the basic point that the distribution of outcomes mattered for choice in some holistic sense. Allais [1979; p.54] was quite clear about this, in a translation of his original 1952 article in French. Similarly, in psychology it is easy to find citations to kindred work in the 1960's and 1970's by Lichtenstein, Coombs and Payne, inter alia.

formalization of the ideas of Lopes [1984].

In the RDU model utility can be defined over money m using a Constant Relative Risk Aversion (CRRA) function

$$u(m) = m^{1-\varrho} / (1-\varrho) \tag{1}$$

where $\varrho \neq 1$ is the RRA coefficient, and $u(m) = \ln(m)$ for $\varrho = 1$. With this parameterization, and assuming EUT, $\varrho = 0$ denotes risk neutral behavior, $\varrho > 0$ denotes risk aversion, and $\varrho < 0$ denotes risk loving. In fact, under RDU there is more to the characterization of risk attitudes than the concavity of the utility function, so we refer instead to ϱ as simply controlling the curvature of the utility function, rather than defining risk attitudes.

Let p_k denote the probability induced by the task for outcome k. To calculate decision weights under RDU one replaces the expected utility of lottery i,

$$EU_i = \sum_{k=1, 20} [p_k \times u_k], \tag{2}$$

with the rank-dependent utility of the lottery,

$$RDU_i = \sum_{k=1, 20} [w_k \times u_k]. \tag{2'}$$

where

$$w_i = \omega(p_i + ... + p_n) - \omega(p_{i+1} + ... + p_n) \tag{3a}$$

for i=1,... , n-1, and

$$w_i = \omega(p_i) \tag{3b}$$

for i=n, where the subscript indicates outcomes ranked from worst to best, and $\omega(p)$ is some probability weighting function.

Picking the right probability weighting function is obviously important for RDU specifications. A weighting function proposed by Tversky and Kahneman [1992] has been widely used. It is assumed to have well-behaved endpoints such that $\omega(0)=0$ and $\omega(1)=1$ and to imply weights

$$\omega(p) = p^{\gamma}/[p^{\gamma} + (1-p)^{\gamma}]^{1/\gamma} \tag{4}$$

for $0<p<1$. The normal assumption, backed by a substantial amount of evidence reviewed by Gonzalez and Wu [1999], is that $0<\gamma<1$. This gives the weighting function an “inverse S-shape,” characterized by a concave section signifying the overweighting of small probabilities up to a crossover-point where $\omega(p)=p$, beyond which there is then a convex section signifying underweighting. Under the RDU assumption about how these *probability* weights get converted into *decision* weights, $\gamma<1$ implies *over*weighting of extreme outcomes. Thus the probability associated with an outcome does not directly inform one about the decision weight of that outcome. If $\gamma>1$ the function takes the less conventional "S-shape," with convexity for smaller probabilities and concavity for larger probabilities.[14] Under RDU $\gamma>1$ implies *under*weighting of extreme outcomes.

B. Rank and Sign-Dependent Preferences: SP/A Theory

Kahneman and Tversky [1979] introduced the notion of sign-dependent preferences, stressing the role of the reference point when evaluating lotteries. The notion of rank-dependent decision weights was incorporated into their sign-dependent PT by Starmer and Sugden [1989], Luce and Fishburn [1991] and Tversky and Kahneman [1992]. Unfortunately, economists tend to view psychological models as monolithic, represented by the variants of PT. In fact there are many alternative models in the literature, although often they have not been developed in a way that would facilitate application and estimation.[15] One that seems unusually well suited to the *DOND* environment is also rank and sign-dependent: the SP/A theory of Lopes [1995].

The SP/A model departs from EUT, RDU and PT in one major respect: it is a dual criteria model. Each of the single criterion models, even if they have a number of components to their evaluation stage, boil down to a scalar index for each lottery such as (2), (2') and (2''). The SP/A model instead explicitly posits two distinct but simultaneous ways in which the same subject might evaluate a given lottery. One is the SP part, for a process that weights the "security" and "potential" of the lottery in ways that are similar to RDEV. The other is the A part, which focuses on the

[14] There are some well-known limitations of the probability weighting function (4). It does not allow independent specification of location and curvature; it has a crossover-point at $p=1/e=0.37$ for $\gamma<1$ and at $p=1-0.37=0.63$ for $\gamma>1$; and it is not increasing in p for small values of γ. There exist two-parameter probability weighting functions that exhibits more flexibility than (4), but for our purposes the standard probability weighting function is adequate.

[15] Starmer [2000] provides a well-balanced review from an economist's perspective.

"aspirations" of the decision-maker. In many settings these two parts appear to be in conflict, which means that one must be precise as to how that conflict is resolved. We discuss each part, and then how the two parts may be jointly estimated.

Although motivated differently, the SP criterion is formally identical to the RDEV criterion reviewed earlier. The decision weights in SP/A theory derive from a process by which the decision-maker balances the security and potential of a lottery. On average, the evidence collected from experiments, such as those described in Lopes [1984], seems to suggest that an inverted-S shape familiar from PT

> ... represents the weighting pattern of the average decision maker. The function is security-minded for low outcomes (i.e., proportionally more attention is devoted to worse outcomes than to moderate outcomes) but there is some overweighting (extra attention) given to the very best outcomes. A person displaying the cautiously hopeful pattern would be basically security-minded but would consider potential when security differences were small. (Lopes [1995; p.186])

The upshot is that the probability weighting function

$$\omega(p) = p^\gamma / [\, p^\gamma + (1-p)^\gamma \,]^{1/\gamma} \tag{4}$$

from RDU would be employed by the *average* subject, with the expectation that $\gamma<1$.[16] However, there is no presumption that any individual subject follow this pattern. Most presentations of the SP/A model assume that subjects use a linear utility function, but this is a convenience more than anything else. Lopes and Oden [1999; p.290] argue that

> Most theorists assume that [utility] is linear without asking whether the monetary range under consideration is wide enough for nonlinearity to be manifest in the data. We believe that [utility] probably does have mild concavity that might be manifest in some cases (as, for example, when someone is considering the huge payouts in state lotteries). But for narrower ranges, we prefer to ignore concavity and let the decumulative weighting function carry the theoretical load.

So the SP part of the SP/A model collapses to be the same as RDEU, although the interpretation of the probability weighting function and decision weights is quite different. Of course, the stakes in *DOND* are huge, so it is appropriate to allow for non-linear utility.

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[16] Lopes and Oden [1999; equation (10), p.290] propose an alternative function which would provide a close approximation to (4). Their function is a weighted average of a convex and concave function, which allows them to interpret the average inverted S-pattern in terms of a weighted mixture of security-minded subjects and potential-minded subjects.

The aspiration part of the SP/A model collapses the indicator of the value of each lottery down to an expression showing the extent to which it satisfies the aspiration level of the contestant. This criterion is sign-dependent in the sense that it defines a threshold for each lottery: if the lottery exceeds that threshold, the subject is more likely to choose it. If there are up to K prizes, then this indicator is given by

$$A_i = \sum_{k=1, K} [\, \eta_k \times p_k \,] \tag{5}$$

where η_k is a number that reflects the degree to which prize m_k satisfies the aspiration level. Although Oden and Lopes [1997] advance an alternative interpretation using fuzzy set theory, so that η_k measures the degree of membership in the set of prizes that are aspired to, we can view this as simply a probability. It could be viewed as a crisp, binary threshold for the *individual* subject, which is consistent with it being modelled as a smooth, probabilistic threshold for a *sample* of subjects, as here.

This concept of aspiration levels is close to the notion of a threshold income level debated by Camerer, Babcock, Loewenstein and Thaler [1997] and Farber [2005][2008]. The concept is also reminiscent of the "safety first" principle proposed by Roy [1952][1956] and the "confidence limit criterion" of Baumol [1963], although in each case these are presented as extensions of an expected utility criterion rather than as alternatives. It is also related to the vast literature on "chance-constrained programming," applied to portfolio issues by Byrne, Charnes, Cooper and Kortanek [1967][1968].

The implication of (5) is that one has to estimate some function mapping prizes into probabilities, to reflect the aspirations of the decision-maker. We use an extremely flexible function for this, the cumulative non-central Beta distribution defined by Johnson, Kotz and Balakrishnan [1995]. This function has three parameters, χ, ξ and ψ. We employ a flexible form simply because we have no *a priori* restrictions on the shape of this function, other than those of a cumulative distribution function, and in the absence of theoretical guidance prefer to let the data determine these values.[17] We want to allow it to be a step function, in case the average decision-maker has some crisp focal point such as £25,000, but the function should then determine the value of the focal point (hence the need for a non-central distribution, given by the parameter ψ). But we also want to allow it to have an inverted S-shape in the same sense that a logistic curve might, or to be convex or concave over the entire domain (hence the two parameters χ and ξ).

Once we have values for η_k it is a simple matter to evaluate A_i using (5). We then construct the likelihood of the data assuming that this criterion was used to explain the observed choices.

4. Mixtures of Decision Criteria

There is a deliberate ambiguity in the manner in which the SP and A criteria are to be combined to predict a specific choice. One reason is a desire to be able to explain evidence of intransitivities, which figures prominently in the psychological literature on choice (e.g., Tversky [1969]). Another reason is the desire to allow context to drive the manner in which the two criteria are combined, to reconcile the model of the choice process with evidence from verbal protocols of decision makers in different contexts. Lopes [1995; p.214] notes the SP/A model can be viewed as a function F of the two criteria, SP and A, and that it

> ... combines two inputs that are logically and mathematically distinct, much as Allais [1979] proposed long ago. Because SP and A provide conceptually independent assessments of a gamble's attractiveness, one possibility is that F is a weighted average in which the relative weights assigned to SP and A reflect their relative importance in the current decision environment. Another possibility is that F is multiplicative. In either version, however, F would yield a unitary value for each gamble, in which case SP/A would be unable to predict the sorts of intransitivities demonstrated by Tversky [1969] and others.

These proposals involve creating a unitary index of the relative attractiveness of one lottery over another.[18]

A more natural formulation is provided by thinking of the SP/A model as a mixture of two

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[17] It also helps that this function can be evaluated as an intrinsic function in advanced statistical packages such as *Stata*.

[18] Lopes and Oden [1999; equation 16, p.302] offer a multiplicative form which has the same implication of creating one unitary index of the relative attractiveness of one lottery over another.

distinct latent, data-generating processes. If we let π^{SP} denote the probability that the SP process is correct, and $\pi^{A} = (1-\pi^{SP})$ denote the probability that the A process is correct, the grand likelihood of the SP/A process as a whole can be written as the probability weighted average of the conditional likelihoods. The literal interpretation of the mixing probabilities is at the level of the observation, which in this instance is the choice between saying "Deal" or "No Deal" to a bank offer. In the case of the SP/A model this is a natural interpretation, reflecting two latent psychological processes for a given contestant and decision.[19]

This approach assumes that any one observation can be generated by both criteria, although it admits of extremes in which one or other criterion wholly generates the observation. One could alternatively define a grand likelihood in which observations or subjects are classified as following one criterion or the other on the basis of the latent probabilities π^{SP} and π^{A}. El-Gamal and Grether [1995] illustrate this approach in the context of identifying behavioral strategies in Bayesian updating experiments. In the case of the SP/A model, it is natural to view the tension between the criteria as reflecting the decisions of a given individual for a given task. Thus we do not believe it would be consistent with the SP/A model to categorize *choices* as wholly driven either of SP or A. These priors also imply that we prefer not to use mixture specifications in which *subjects* are categorized as completely SP or A. The general problem with this approach is that it assumes that there is no effect on the probability of SP and A from task domain. We do not want to impose that assumption, even from a relatively homogenous task design such as ours.

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[19] Byrne et al. [1967; p.19] elegantly view the multiple-criteria problem as characterizing the *objective of the latent decision-maker* as a probability distribution rather than reducing it to a scalar: "Some of the approaches we shall examine are also concerned with choices that maximize a single figure of merit. Others are concerned with developing the relevant combinations of probability distributions so that these may themselves be used as a basis for managerial choice. [...] To avoid misunderstanding it should be said, at this point, that this paper is not concerned with issues such as whether a 'present value' provides a better figure of merit than an 'internal rate of return' via a 'bogey adjustment' or a 'payback period' computation. Indeed it will be one purpose of this paper to suggest that some of these issues might be resolved – or at least placed in a different perspective – if some of the new methodologies can make it possible to avoid insisting on the use of one of these figures to the exclusion of all others." This is completely consistent with our approach, which characterizes the *objective of the econometrician* in terms of a scalar (the log-likelihood of a mixture model) derived from modeling the *objective of the latent decision-maker* in terms of a probability distribution defined over two or more criteria.

5. Results

Table 1 collects estimates of the RDEV and RDU models applied to *DOND* behavior. In each case we find estimates of $\gamma<1$, consistent with the usual expectations from the literature. Figure 1 displays the implied probability weighting function and decision weights. The decision weights are shown for a 2-outcome lottery and then for a 5-outcome lottery, since these reflect the lotteries that a contestant faces in the last two rounds in which a bank offer is made. The rank-dependent specification assigns the greatest weight to the lowest prize, which we indicate by the number 1 even if it could be any of the original 22 prizes in the *DOND* domain. That is, by the time the contestant has reached the last choice round, the lowest prize might be 1 penny or it might be £100,000. In either case the RDU model assigns the greatest decision weight to it. Similarly, for K-outcome lotteries and K>2, a higher weight is given to the top prize compared to the others, although not as high a weight as for the lowest prize. Thus the two extreme outcomes receive relatively higher weight. Ordinal proximity to the extreme prizes slightly increases the weights in this case, but not by much. Again, the actual dollar prizes these decision weights apply to change with the history of each contestant.

There is evidence from the RDU estimates that the RDEV specification can be rejected, since ϱ is estimated to be 0.302 and significantly greater than zero. Thus we infer that there is some evidence of concave utility as well as probability weighting. Constraining the utility function to be linear in the RDEV specification slightly increases the curvature of the probability weighting function, as one might expect.

Table 2 and Figure 2 show the results of estimating the SP/A model. First, we find evidence that the utility function is concave, since $\varrho>0$ and has a 95% confidence interval between 0.60 and 0.70. Hence it would be inappropriate to assume RDEV for the SP part of the SP/A model in this high-stakes domain, exactly as Lopes and Oden [1999; p.290] conjecture.

Second, we find evidence that the SP weighting function is initially concave and then convex in probabilities. In the jargon of the SP psychological processes underlying this weighting function, this indicates that potential-minded attitudes dominate the security-minded attitudes for smaller probabilities, but that this is reversed for higher probabilities. In the *DOND* context, the probability distribution of potential outcomes in each round is uniform, and the probability of any single outcome is significantly less than ½ for round 1 to 5. Hence the predominant attitude implied by $\gamma<1$ is that of potential-minded attitudes.

Third, the estimates of χ, ξ and ψ for the aspiration weighting function imply that it is steadily increasing in the prize level, with the concave shape shown in Figure 2. At a prize level of roughly £44,000 the aspiration threshold is ½. This cumulative distribution function does not assign zero weight to prizes below that level, although the functional form effectively allowed that. If we round prizes to the nearest £10,000, the aspiration weights are 0.09, 0.22, 0.34 and 0.46 for each of the prizes from £0 up to £40,000. A median aspiration level of £44,000 may seem optimistic, given that average earnings in the game show are £17,737 in our sample, and median earnings are £13,000. However, this is just one of two decision criteria that the contestant is assumed to use in making *DOND* decisions. The other criterion combines security and potential considerations, as noted above.

Finally, the two component processes of the SP/A model each receive significant weight overall. We estimate that the weight on the SP component, π^{SP}, is 0.35, with a 95% confidence interval between 0.30 and 0.40. A formal hypothesis test that the two components receive equal weight can be rejected at a *p*-value of less than 0.001, but each component clearly plays a role in decision-making in this domain.

6. Model Comparisons

Mixture models provide a natural way to compare the predictive power of alternative models of behavior, whether or not the models are nested.[20] The estimates for the role of the SP and A criteria within the SP/A model already indicate that decision makers appear to use both the familiar "risk-utility" tradeoffs of traditional economics models, as well as the notion of an income

[20] Harrison and Rutström [2009] point out that the older statistical literature on non-nested hypothesis tests evolved as a "second best" alternative to being able to estimate finite mixture models.

threshold.

To the extent that the SP component is simply a re-statement of the RDU model, this SP/A model nests RDU, so this result provides some evidence in favor of the SP/A notion that one needs two criteria to appropriately characterize behavior in *DOND*. Furthermore, since the RDU model nests at least one parametric version of EUT, these results indicate that behavior is inconsistent with that version of EUT.

We can extend our analysis to consider the possibility that some choices or decision makers are consistent with EUT and some are consistent with RDU. In effect, this hypothesis suggests a nested mixture model: at the top level one allows EUT and SP/A latent decision-making process explain behavior, and at the bottom level *within* the SP/A process one allows latent SP and A processes to explain behavior. This hypothesis is not the same as the hypothesis that the SP criterion (which is the RDU criterion) collapses to EUT. In effect, this hypothesis is that there are *three* latent decision-making processes at work: EUT, RDEU and the A criterion. Assuming a CRRA utility function, we estimate that the EUT process accounts for only 6.3% of observed choices, with the relative weights on the SP/A criterion accounting for the rest. In turn, the relative weights on the SP and A criteria remain essentially the same as when we assumed that none of the choices were generated by EUT decision-makers.

7. Conclusions

We provide a formal statement and application of a model of choice under uncertainty from psychology that has been neglected by economists, but which has many interesting features. First, and foremost, it explicitly employs multiple criteria for the evaluation of prospects, reflecting several assumed decision processes. Moreover, these criteria cannot be collapsed to a single criterion, which is a characteristic of most of the models of choice under risk used by economists.

We demonstrate how one can obtain full maximum likelihood estimates of the SP/A model, and integrate the dual decision criteria in a natural decision-theoretic and statistical manner. These methodological insights extend to applications of the SP/A model to other settings, as well as to other multiple-criteria decision models. Additional decision processes that might be similarly modeled as multiple criteria include appeals to income thresholds, editing processes, the use of similarity relations, and other heuristics from psychology.[21]

We apply the model to a rich domain in which prizes are highly skewed, and where it is plausible to expect individuals to have income thresholds that might affect behavior in addition to familiar utility-risk tradeoffs. Our statistical results allow the data to determine the relative weight of the two criteria, and do not *a priori* constrain behavior to use either or both. We find evidence that both criteria play a role in explaining behavior. Although the specific weights attached to each criterion might be expected to vary from task domain to domain, we find that nearly two-thirds of the weight is on the aspiration criterion.

The results support the SP/A model over the assumption that behavior is characterized *solely* by EUT or *solely* by RDU decision making. These findings have important implications for policy evaluations. Predicted welfare effects from policy changes are always uncertain, partly because policy simulation models use strong assumptions about the decision making process, and imprecise parameter values. Our results suggest that subjects generally are averse to variation in outcomes, with an estimated CRRA coefficient of 0.66, which is comparable to findings in lab and field experiments with considerably lower prizes (e.g., Holt and Laury [2002] and Harrison, Lau and Rutström [2007]). We also find that disproportionate higher weights are given to extreme outcomes, emphasizing the security and potential of the lottery, and that aspiration levels are high compared to the expected value of the lottery and average earnings. The median aspiration level is £44,000 compared to an average prize of £25,712 and average earnings of £17,737. Of course, the advantage of using data from *DOND* is that we can evaluate decision making over very large stakes. The disadvantage is that we have little information about the contestants and how they compare to the general population, and we can not control for sample selection into the game show.

[21] See Starmer [2000] on editing processes, Tversky [1977], Luce [1956], Rubinstein [1988] and Leland [1994] on similarity relations, and Brandstätter, Gigerenzer and Hertwig [2006] on the myriad of heuristics proposed in the broader psychology literature.

Table 1: Estimates for *Deal or No Deal* Game Show Assuming RDU

Parameter	Estimate	Standard Error	Lower 95% Confidence Interval	Upper 95% Confidence Interval
A. RDEV, assuming utility is linear in prizes				
γ	0.342	0.011	0.322	0.363
μ	0.060	0.004	0.053	0.068
B. RDU				
ϱ	0.302	0.014	0.274	0.330
γ	0.550	0.022	0.506	0.595
μ	0.085	0.004	0.077	0.092

Figure 1: Decision Weights under RDU

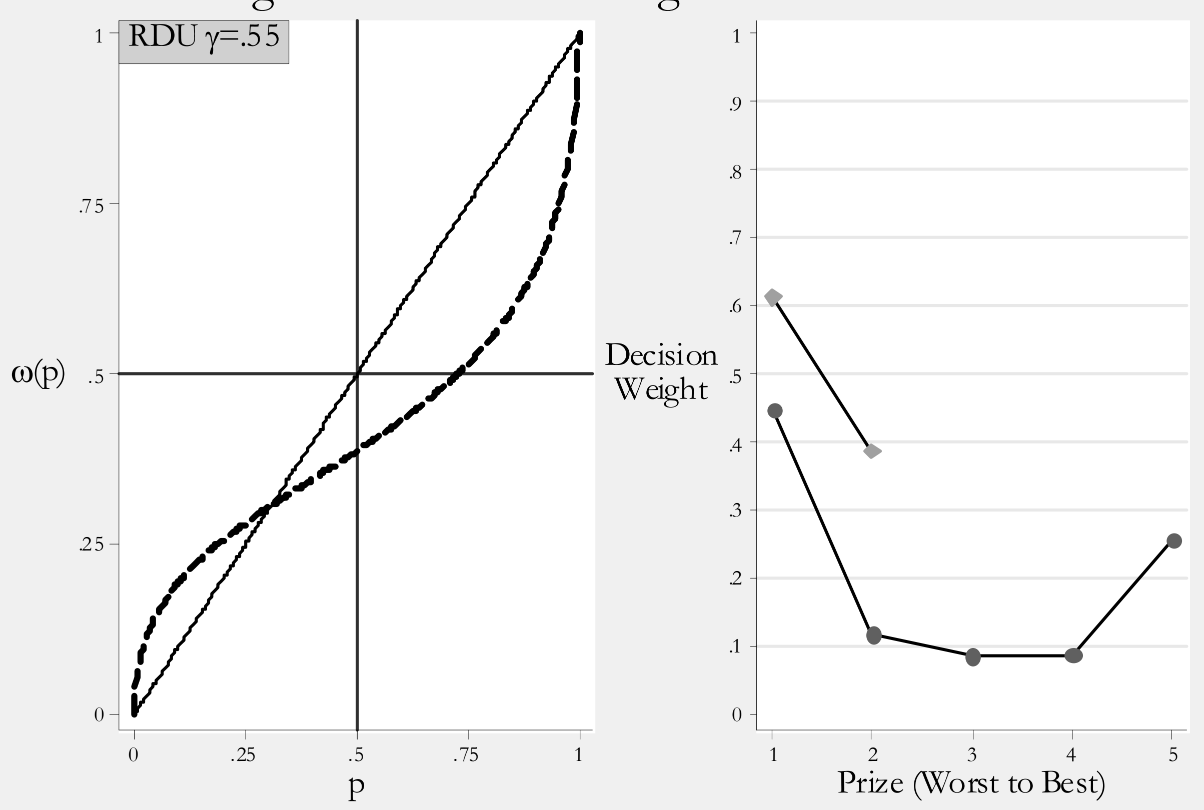
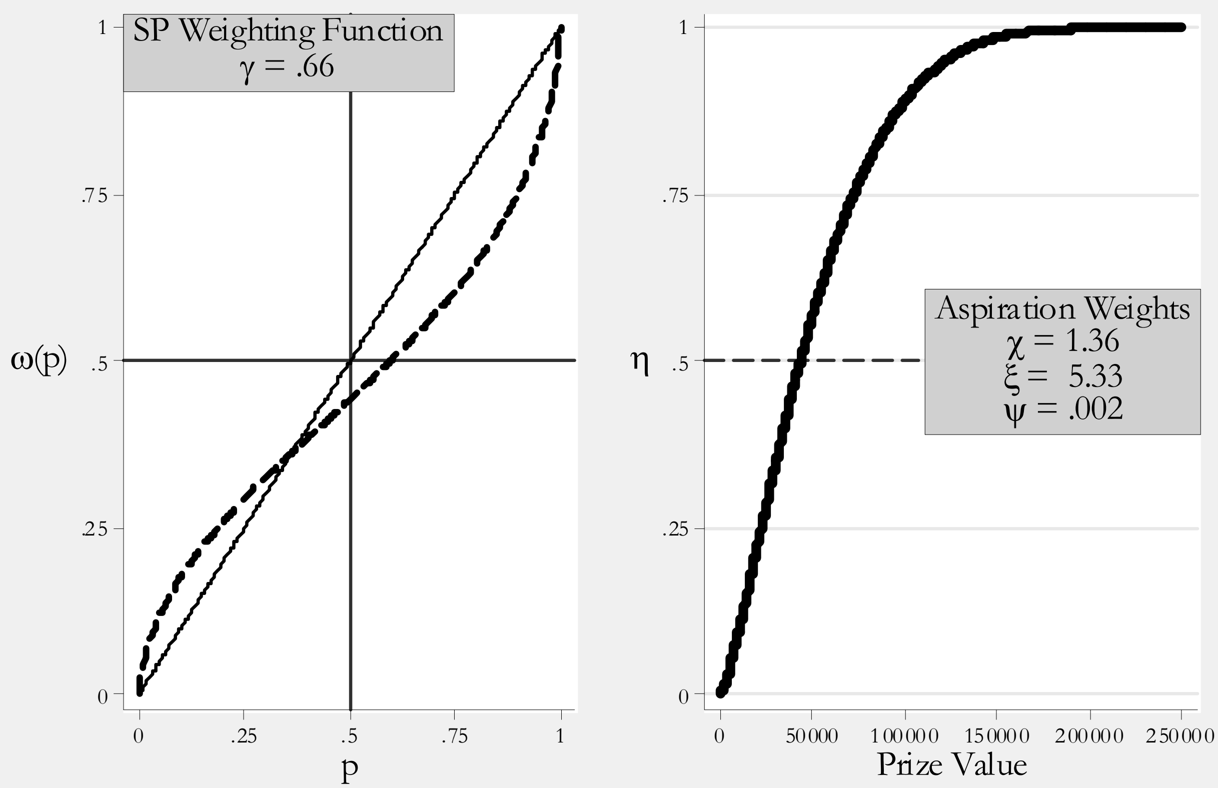


Table 2: Estimates for *Deal or No Deal* Game Show Assuming SP/A

Parameter	Estimate	Standard Error	Lower 95% Confidence Interval	Upper 95% Confidence Interval
ϱ	0.654	0.027	0.601	0.707
γ	0.664	0.052	0.562	0.767
χ	1.355	0.146	1.068	1.642
ξ	5.330	1.510	2.371	8.289
ψ	0.002	0.001	-0.001	0.005
μ	0.183	0.029	0.125	0.241
π^{SP}	0.353	0.025	0.304	0.401
$\pi^{A}=1-\pi^{SP}$	0.647	0.025	0.599	0.696

Figure 2: SP/A Weighting and Aspiration Functions



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