It’s all about speed and costs: 

The impact of digital technology on the insurance market structure

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Abstract  Digital technology is costly, but it allows insurers to more quickly learn about the risk type of their policyholders. We study the implications of this speed-versus-cost tradeoff for equilibrium pricing and coverage decisions in an insurance market featuring adverse selection. In particular, we develop a theoretical model of dynamic competitive equilibrium featuring individuals who differ in their privately known risk types, and a large number of two types of insurers: conventional insurers and “tech” insurers who employ digital technologies. We consider three distinct dynamic equilibrium concepts: a finite horizon structure with foresight, an infinite horizon “overlapping generations” structure, and an infinite horizon myopic structure. Equilibrium in each setting features a sorting of low-risk types into tech firms and high-risk types into conventional firms. Depending on the setting, however, the equilibrium tech-firm market share may negatively, positively, or mixed correlate with the learning speed of conventional insurers.

Keywords  Digitalization, Speed of learning, Asymmetric learning, Dynamic equilibrium, Adverse selection, Market structure

JEL Classification  G22, D40, O33

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1 Introduction

Digital technology generates big data, enabling insurance firms to more effectively and efficiently understand the risk they underwrite. A telematics device installed in a car, for example, provides an insurer with much more risk-relevant information than the conventional underwriting information. Moreover, it opens up opportunities for real-time monitoring and risk assessment. These digital technologies can therefore—at least in some contexts—be expected to mitigate information asymmetries, both of the adverse selection and moral hazard varieties. Qualitatively speaking, then, digital technologies such as telematics devices will operate in much the same way as “old fashioned” insurer learning based on past claims experience and policyholder disclosure (Nilssen, 2000; de Garidel-Thoron, 2005; Cohen, 2012; Kofman and Nini, 2013)—learning processes which have been shown to be effective tools to mitigate adverse selection (Eling, Jia, and Yao, 2017) and moral hazard (Dionne, Michaud, and Dahchour, 2013). Quantitatively speaking, however, there are two important differences: speed and cost. As shown empirically in Eling, Jia, and Yao (2017), learning by conventional technologies is slow—and learning via digital technologies is fast or even instantaneous (Gemmo, Browne, and Gründl, 2017). Since telematics and other digital technologies involve installation and potentially data processing costs, they are—at least for the moment—more costly than conventional learning techniques.

This paper studies the implications of this speed versus cost tradeoff for equilibrium pricing and coverage decisions in an insurance market featuring adverse selection. In particular, we develop a theoretical model of dynamic competitive equilibrium featuring a continuum of individuals who differ in their risk types and featuring a large number of two types of insurers: those using conventional underwriting techniques only (or “conv” insurers) and those “tech” insurers who employ digital learning technologies. Tech insurers have an advantage because they can rapidly learn the risk type of any given individual and thus can effectively price based on each risk; but installing (and/or maintaining) their technology is costly. Conventional insurers cannot immediately observe risk types, and hence cannot employ risk-based pricing and face adverse selection, but we do allow them to learn over time about the risk type of their customers.

Within this broad modeling framework, we consider three distinct dynamic equilibrium concepts. One is a “myopic” (behavioral) model where customers choose firms based on current prices. In this setting, conventional insurers understand their risk pool based on backward-looking experience and update prices with a lag as they “bleed” customers to tech insurers. This is best seen as a heuristic, but realistic model to describe the learning process of conventional insurers and the market evolution in response to the introduction of new technologies. The other two models consider firms and individuals as fully rational and forward-looking; the essential difference is in their assumptions about whether conventional firms can distinguish potential buyers who have newly entered the market from buyers who have been
revealed as high-risks to their incumbent insurer and are returning to the market for a fresh start.² We refer to the model where firms can distinguish these two types of buyer as the “finite horizon” (FH) and the model where firms cannot as the “overlapping generations” (OLG) model. (The reason for this will be clearer later.)

We show that, in all three settings, equilibria will feature a sorting of low-risk types into tech firms and high-risk types into conventional firms. We then consider comparative statics of equilibrium with respect to two key underlying parameters: the cost of digital technology, and the speed at which conventional insurers learn the risk types of their buyers. Across all three settings, we establish the intuitive result that lowering the cost of the digital technology will raise the equilibrium market share of tech firms. Interestingly, however, the effect of the speed of learning on the tech market share depends on the dynamic model. In the FH model, faster learning leads to a higher market share for conventional firms. Intuitively, this is because incumbent firms can earn information rents once they learn about their clients, so the faster they learn the more they will “lowball” prices for new customers (Kunreuther and Pauly, 1985; Nilssen, 2000). In the OLG model, the same lowballing incentive is present, but it is counteracted by the fact that individuals who reveal themselves to be high risk will be dumped (via high prices) by their insurers. They will then return to the conventional market, “polluting” the risk pool for conventional insurers writing new business and hence raising the equilibrium price of conventional insurance. This “polluting” effect always dominates, so in the OLG model, faster learning by conventional firms actually raises the equilibrium share of tech insurers. In the myopic model, we show that the effects of faster learning of conventional insurers on the market share of tech insurers are ambiguous.

Our paper fits in a thick literature on asymmetric learning and dynamic pricing. For example, our model of conventional firm learning and consequent “lowballing” is consistent with the asymmetric learning model used in Kunreuther and Pauly (1985), Nilssen (2000) and de Garidel-Thoron (2005) in which the incumbent insurer knows the risk type of the policyholder, but competing insurers do not (and in contrast to the symmetric learning model used e.g. in Watt and Vazquez (1997) and Hendel (2016)) wherein all insurers have the same information in all periods). The key difference in our paper is the simultaneous presence of tech insurers who effectively know the risk type in real time and have no incumbency advantage.

Our work also contributes to the ongoing discussion on the impact of digitalization on the equilibrium of insurance market (Filipova-Neumann and Welzel, 2010; Gemmo et al., 2017). Gemmo et al. (2017) use a one-period menu contract framework to analyze the trade-off between the reduction of information asymmetry and the willingness to share the private information (transparency aversion). While Gemmo et al. (2017) assume perfect observation of risk type at contract inception. Filipova-Neumann & Welzel

² Both cases can be motivated empirically, given that in many countries for certain types of insurance products information exchange platforms exists, while for other products such platforms do not exists.
(2010) analyze a similar setting where risk type is revealed only after an accident. Both studies find welfare increases greater or equal to zero in a separating equilibrium setting. We assume risks do not have the problem of transparency aversion and instead focus on the trade-off between the speed of learning and the digital technology cost. To the best of our knowledge, we are the first to analyze the impact of learning speed on the market structure at the competitive dynamic equilibrium.\(^3\)

The rest of the paper is structured as follows. In Section 2, we present our theoretical framework by summarizing the common features and qualitative differences of the three models. In Section 3, we describe the three models, their (quasi-)equilibriums, and the corresponding propositions at the (quasi-)equilibriums. We conclude in Section 4.

2 Theoretical Framework

2.1 Common features of all three models

We consider three distinct models of competitive market structure. In each model, there are a large numbers of conventional and tech insurers that compete on price to sell an insurance product with homogeneous coverage to a large number of buyers who are differentiated by their loss probability. All models feature a set of periods \(t = 0,1,2,...,T\), where \(T \geq 1\) and \(T \leq \infty\). They all feature a set of risk types, indexed by the per-period risk of loss \(p\). Losses are assumed to be independent across periods, and of constant size \(L\) out of a per-period income \(W\). We abstract from saving, so that, absent insurance, individuals have a net consumption of \(y_t^L = W - L\) or \(y_t^N = W\) in the event of a loss or no loss, respectively.

Firms sell full insurance as one period contracts at a premium \(q\), which, depending on that firm’s ability to observe information, may depend on risk types of the buyer. Tech firms observe risk types and hence offer premiums \(q^\tau(p)\) depending directly on the risk type. Conventional firms can distinguish between their incumbent insureds and other potential insureds, and hence can offer them different premiums; conventional firms may learn information about their incumbent insureds over time, in which case they can potentially offer different incumbent insureds different premiums.

An individual who buys an insurance contract at a price \(q\) in period \(t\) will have loss-independent consumption \(y_t^L = y_t^N = W - q\). A conventional firm who sells such a contract to a risk type \(p\) will earn period profits \(\pi^c = q - pL\). A tech firm who sells such a contract will earn \(\pi^\tau = q - pL - C\), where \(C \geq 0\) are monitoring costs, which we potentially allow to be history-dependent in order to allow for one-time installation costs as well as ongoing maintenance and monitoring costs.

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\(^3\) Our paper is also related to the debate on the usage of genetic information for risk calculation. A genetic test is also costly, but if shared with the insurance company decreases information asymmetry. Hoy & Ruse (2005) argue that the reduction of adverse selection and therefore the increasing efficiency is accompanied by the effect that people who are in poor health are punished twice (higher premium and health problems); some people will refuse to take a genetic test, because they are afraid that it will increase their insurance premium. Doherty & Posey (1998) find that for uninformed individuals a genetic test has a positive private value if prevention is sufficiently effective in lowering the premium, even though the information must be shared with the insurer.
Individuals are assumed to have standard risk averse von Neumann-Morgenstern preferences over consumptions within periods; we assume the utility of individuals is additively separable and time-discounted across periods, i.e., given by:

$$\sum_t \beta^t [pU(y_t^L) + (1-p)U(y_t^N)].$$

We assume that $U$ is twice differentiable, increasing, and strictly concave, and $\beta \leq 1$ is the time discount factor of individuals. Firms are risk neutral and they all discount their per-period profits over time with the same market discount rate $\beta$.

Finally, we assume that there is a large number of firms, so that competition drives expected profits down to zero on a contract-by-contract basis. As we describe below, the precise meaning of this will vary depending on whether firm beliefs are myopic or forward looking. In all cases, however, competition will drive tech firms to price at $q = pL + C$, so that they break even given the realized (and known) risk $p$, and this tech market will provide a static “outside option” to insurance buyers who are considering buying from a conventional firm. We assume, purely for expositional simplicity, that

$$pU(W - L) + (1-p)U(W) < U(W - pL - C)$$

for all $p \in [p, \bar{p}]$. This (which is always true for sufficiently low $C$) ensures that all individuals prefer insuring with a (break-even pricing) tech firm to going without insurance.

2.2 Qualitative differences across models

We consider three families of models differ in their assumptions about dynamics and rationality. We describe these differences qualitatively here and then formalize them in the subsequent section.

The first “finite horizon” (FH) model is a two-period rational expectations model in the spirit of Nilssen (2000) and De Garidel-Thoron (2005). In this model, conventional firms compete in period 0, taking into account two things that firms in the myopic model ignore. First, they take into account the endogenous distribution of risk types who actually buy from them. In other words, they correctly anticipate that some buyers (in practice, the low risks) will buy from tech firms or opt out of the market in the current period, and adjust their prices accordingly. Second, they take into account the future consequences of their current actions. Specifically, conventional insurers may asymmetrically learn between periods 0 and 1 about the risk type of the individual they insured. If so, they will earn some information quasi-rents in period 1—though these rents are limited by the competitive tech market that coexists with it. A central dynamic in this model is “lowballing” of conventional prices in period 1, in anticipation of these period 2 quasi-rents.

The second “overlapping generations” (OLG) model is an infinite horizon rational expectations model. As in the FH model, firms (and buyers) are forward-looking and correctly anticipate the equilibrium distribution of risk types who actually buy from them at each moment in time. As such, the same
“lowballing” dynamic applies, wherein conventional firms make losses on initials sales in anticipation of the information quasi-rents they will earn when they learn the risk types of their incumbent insureds. A key difference from the FH model is that these quasi-rents are (also) limited by the ability of individuals to return to the conventional insurance market, where they will be indistinguishable from new entrants into the market. Indeed, a central dynamic in this model is that high risk types “cycle through” different conventional firms, sticking with their new firm until that firm learns their type and then returning to the broad pool for a new, uninformed conventional insurer. The third “myopic” model makes stylized—but plausible—behavioral assumptions about pricing and purchasing behavior, in the spirit of Kunreuther and Pauly (1985). Individuals in this model are assumed to choose by (myopically) maximizing their current-period payoff—i.e., they choose the cheapest contract available to them today. Similarly, competition is assumed to drive current period profits down to zero, however, some conventional insurers may suffer from negative profits since they do not know the risk type of new customers and thus underprice them, given that the beliefs of firms are also myopic. Furthermore, the beliefs of conventional firms are assumed to be formed in a backward-looking way, based initially on the population distribution of risk types, and then, later, from the risk distribution realized at conventional firms in the preceding period. We consider a family of these myopic models, where conventional firms learn over time about the risk types of their incumbent insureds. A central dynamic in these models is an unraveling over time a la Akerlof (1970), as conventional firms realize worse and worse risks over time, raising prices and driving more individuals towards tech firms.

3. Models

3.1 Finite Horizon Rational Expectations Model

In this model, there are two periods: \( t = 0, 1 \). A continuum of types with (cumulative) distribution \( F(p) \) and continuous pdf \( f(p) \) with support on \( [p, \bar{p}] \subset (0, 1) \) contemplate buying insurance in each period.

We make the technical assumption that \( \frac{1 - F(p)}{f(p)} \) is decreasing in \( p \) (which is obviously true, e.g., for a uniform distribution).

The cost \( C \) for tech firms to provide insurance is (potentially) time dependent, with cost \( C_0 \) for a first-time tech buyer, and \( C_1 \leq C_0 \) for an individual who was insured with a tech firm in period 0 and buys insurance from a tech firm again in period 1. Note that we assume that any tech firm will have a cost of \( C_1 \) in period 1 for such a buyer; one interpretation is that there is a fixed cost of putting in equipment and then a variable cost of using it in each period—but all firms can use the same equipment once it is installed and only bear the monitoring cost. This assumption ensures that tech firms are effectively competing statically in each of the two periods over the premium \( q_t^1(p) \) for each potential buyer, so that, in equilibrium, \( q_0^1(p) = q_1^{n_1}(p) = pL + C_0 \) and \( q_1^{1_1}(p) = pL + C_1 \), where \( q_1^{n_1}(p) \) denotes prices offered in period 1 to individuals who were not insured with tech firms in period 0 and \( q_1^{1_1}(p) \) denotes
prices for those who were. We also assume that there is no time discounting between periods, i.e., $\beta = 1$.

Conventional firms cannot observe buyer type when setting prices in period 0 and thus compete over a single premium $q_0^c$. If they insure an individual in period 0, then, with probability $\alpha$, they will learn the buyer’s type before setting prices again in period 1, and will choose a price $q_1^{cL}(p)$. With probability $1 - \alpha$, they do not observe the type, and can offer a single premium $q_1^{cU}$ to these unlearned types. We also assume, again purely for analytical simplicity, that in period 1 conventional firms cannot (or do not) offer contracts to non-incumbent insureds—so individuals who do not wish to stay with their original conventional insurer can either forgo insurance or switch to tech firms.\(^4\)

With the goal of defining a competitive, rational expectations equilibrium for this economy, consider first the sequentially rational decisions of individuals and firms in period 1. Individuals who bought from a tech firm in period 0 choose rationally in period 1 between forgoing insurance and buying from a tech firm at their competitive price $q_1^{tL}(p) = pL + C_1$. Because of our assumption on $[p, \bar{p}]$ and $C_1$, they will all choose to purchase insurance.

Similarly, individuals who purchased from conventional firm in period 0 in principle face a choice among three options: foregoing insurance, switching to buy from a tech firm at the premium $q_1^{cL}(p) = pL + C_0$, or remaining with their conventional firm at the offered price of $q_1^{cL}(p)$ or $q_1^{cU}$, depending on whether their type was learned or not. However, as we will show below they will never choose to forego insurance in equilibrium, and they effectively choose between switching to a tech firm and remaining with their incumbent firm. We assume that they remain with their conventional firm if they are indifferent. In other words, they choose to remain precisely when $q_1^{cL}(p) \leq q_1^{tL}(p)$ if their type was learned and when $q_1^{cU} \leq q_1^{tL}(p)$ if it was not.

In light of the optimizing behavior by individuals, an incumbent firm in period 1 who has learned the type $p$ of a given insured clearly maximizes profits by choosing $q_1^{cL}(p) = pL + C_0$, i.e., the highest price at which they will retain their customer—which is a profitable price since $C_0 > 0$. The profit-maximizing price for their unlearned customers, namely $q_1^{cU}$, is less obvious. At any price $q_1^{cU}$, they will sell to those types for whom $q_1^{cU} \leq q_1^{cL}(p) = pL + C_0$, i.e., to any customers with $p \in \left[\frac{(q_1^{cU} - C_0)}{L}, \bar{p}\right]$. Any incumbent unlearned types with $p < \frac{(q_1^{cU} - C_0)}{L}$ will depart for tech firms. Lowering the premium thus involves a tradeoff: on the one hand, the firm retains additional, profitable lower risks,\(^4\) This assumption is intuitively plausible, as incumbent conventional firms have an informational advantage over other conventional firms for these individuals: even though they do observe their type, they know that they have not been revealed to be a high risk types. Non-incumbent firms would attract risks of both the unrevealed types and the high-risk revealed types who the incumbent firms would be happy to offload. For this reason, it is not hard to show that non-incumbent conventional firms will never make positive sales in equilibrium in which such sales are allowed. But the presence of these potential competitive firms makes the analysis of the equilibrium less transparent. Hence our technical assumption.
but, on the other hand they lower their per-unit profits on the higher-risk customers who were already planning to purchase at the higher premium. The assumption that \( \frac{1-F(p)}{f(p)} \) is decreasing in \( p \) will ensure that there is a unique profit maximizing price \( q^*_1(p) \) whenever the set of period-0 conventional firm purchasers has the interval form \([\check{p}, \bar{p}]\) (as it will in equilibrium), and that there will also be a cutoff individual \( p^*(\hat{p}) \in [\check{p}, \bar{p}] \) for whom \( q^*_{c,U}(\hat{p}) \equiv p^*(\hat{p})L + C_0 \) who is indifferent between staying with the conventional insurer and switching to a tech firm.

In period 0, we aim to describe a competitive equilibrium with perfect foresight. Intuitively, individuals will choose between tech and conventional firms in period 0 fully anticipating what pricing will be in period 1 (and hence consistent with the preceding). Conventional firms will compete over premiums and drive that premium to the level at which such firms earn zero lifetime profits. Since conventional firms earn positive profits in period 1, the will imply “lowballing”.

Towards formalizing this basic intuition, note first that individuals who choose a tech firm pay \( q^*_1(p) = pL + C_0 \) in period 0 and rationally anticipate paying a premium \( q^*_1(p) = pL + C_1 \) in period 1. Individuals who choose a conventional firm pay some premium \( q^*_0 \) and then rationally anticipate their conventional firm’s period 1 pricing. They thus anticipate paying \( q^*_1(p) = pL + C_0 \) when, with probability \( \alpha \), their type is learned. With probability \( 1 - \alpha \), their type is not learned. If \( p < \frac{(q^*_1(p) - C_0)}{L} \), they will switch to a tech firm in period 1 and again pay \( q^*_1(p) = pL + C_0 \). Higher \( p \) types will instead anticipate staying with their conventional firm and paying some rationally anticipated \( q^*_1(p) \).

It is intuitive that in period 0 purchase decisions will take a cutoff form: higher \( p \) individuals will buy from conventional firms and lower \( p \) individuals from tech firms.\(^5\) The cutoff individual \( \hat{p} \) who buys from a conventional firm will always buy insurance at a price \( \hat{p}L + C_0 \) in period 1. (This follows from period 1 profit maximization if their type is learned. If it is not learned, it follows from the fact that \( q^*_1(p) \geq \hat{p}L + C_0 \) as, at \( q^*_1(p) = \hat{p}L + C_0 \), the firm will retain all of their incumbent customers, and there is no “additional sales” benefit to lower prices.) If interior, the cutoff type \( \hat{p} \) is thus indifferent between buying from a conventional firm and paying \( q^*_0 \) then \( \hat{p}L + C_0 \) or buying from a tech firm and paying \( \hat{p}L + C_1 \) and then \( \hat{p}L + C_1 \). Since there is no time discounting, this implies \( \hat{p}(q^*_0) = \hat{p}L + C_1 \), or \( \hat{p}(q^*_0) = (q^*_0 - C_1)/L \).

If the market price is \( q^*_0 \), then the lifetime profits will be:

\[
\pi^c(q^*_0) = \int_{\hat{p}(q^*_0)}^{\bar{p}} (q^*_0 - Lp) f(p)dp + \alpha C_0 \left(1 - F(\hat{p}(q^*_0))\right) + (1 - \alpha) \int_{\hat{p}(q^*_0)}^{\bar{p}} (q^*_U(\hat{p}(q^*_0)) - Lp) f(p)dp.
\]

\(^5\) It follows formally from two simple observations given any fixed prices. First, the (non-stochastic) utility difference between buying from a tech firm in both periods and buying from a conventional firm in period 1 and then a tech firm in period 2 is \( U(W - pL - C_1) - U(W - q^*_0) \), which is decreasing in \( p \). Second, the period 1 option value of staying at a conventional firm is increasing in \( p \).
The first term is the period-0 profits. The second term is the profits in period 1 for individuals whose types have been learned (i.e., $C_0$, since the profit-maximizing price is $q^{CL}(p) = pL + C_0 = q^{TR}(p)$). The final term is the profit from unlearned types. The equilibrium price $q^*_0$ is determined by a zero lifetime profit condition, $\pi^{C}(q^*_0)$. We say that a zero-profit equilibrium is \textit{locally stable} if $\pi^{C}(q^*_0)$ is increasing in $q^*_0$ (otherwise, a small decrease in premium by a single firm will be profitable).

We gather the preceding reasoning into the following definition of an FH-equilibrium.

**Definition: Equilibrium in the FH Market**

An equilibrium is a set of prices $q^*_0, q^{CL}_1, q^{CL}_1(p), q^*_1(p), q^{L}_1(p), q^{C}(p)$, and a pair of cutoffs $p_0$ and $p_1 \geq p_0$ such that:

(i) Tech firm competition: $q^*_0(p) = q^{TR}_1(p) = pL + C_0$. $q^{CL}_1(p) = pL + C_1$.

(ii) Conventional firm period 1 profit maximization for learned types: $q^{CL}_1(p) = pL + C_0$.

(iii) Conventional firm period 1 profit maximization for unlearned types:

\[
q^{CL}_1 = \arg \max \int_{\hat{p}(q)} (q - pL)f(p)dp, \text{ where } \hat{p}(q) = \max \left\{ p_0, \frac{q - C_0}{L} \right\}.
\]

(iv) Period-0 conventional firm competition and stability: $\pi^{C}(q^*_0) = 0$ and $\frac{d\pi^{C}(q^*_0)}{dq^*_0} > 0$.

(v) Individual optimization: $p_1 = \frac{(q^{CL}_1 - C_0)}{L}$ and $p_0 = \max \left\{ p_0, \frac{q^*_0 - C_1}{L} \right\}$, and individuals with $p < p_t$ purchase from tech firms while those with $p > p_t$ purchase from conventional firms.

Equilibrium depends on the exogenous cost parameters $C_0, C_1$ and the speed of learning $\alpha$. The following proposition is our main result for the FH model. It establishes formally that the equilibrium cutoffs are decreasing in $\alpha, C_0$, and $C_1$. In other words, when the speed of learning increases or the cost of digital technology increases, the market shares of conventional firms in both periods increases. The formal proof is in the appendix.

**Proposition FH:** In any locally smooth family of equilibria:

1. The equilibrium cutoff $p_0(\alpha, C_0, C_1)$ is (weakly) decreasing in $\alpha, C_0$, and $C_1$, strictly so if $p_0 \neq p_t$.

2. The equilibrium cutoff $p_1(\alpha, C_0, C_1)$ is (weakly) decreasing in $\alpha, C_0$ and $C_1$. It is strictly decreasing in $C_0$ if $p_0 \neq p_1$.

### 3.2 Infinite Horizon OLG Model

In this model, there are an infinite number of periods. There is a constant mass of potential insurance buyers in each period, which we normalize to 1. These potential buyers (individuals) make insurance decisions over one-period contracts at the beginning of the period, then losses are realized and payouts are made for covered losses. Before the next period starts, each individual has an independent probability
\((1 - \eta) \in [0,1]\) of “exiting” the market (e.g., dying), and a mass \((1 - \eta)\) of new individuals enter the market (are “born”).

Otherwise, the structure is broadly similar to the FH market. We assume that new types have the distribution \(F(p)\) (with continuous pdf \(f(p)\) with support on \([\underline{p}, \bar{p}] \subset (0,1)\)). The distribution of types in the market is thus constant and described by \(F(p)\), as in the FH market, but here there is a constant inflow and outflow of new and old agents.

We again assume that the cost \(C\) for tech firms to provide insurance is again (potentially) time dependent, with cost \(C_0\) for a first-time tech buyer, and \(C_1 \leq C_0\) for an individual who was previously insured with any tech insurer. We allow discounting \(\beta \in (0,1]\), but impose that \(\beta \eta < 1\).

The key difference in modeling assumptions are closely related to the finite vs infinite time horizon. With a finite horizon, a conventional firms knows in period 1 whether they insured a given individual in the preceding period, and thus would know that any new potential customer must have previously insured with another provider—and thus is likely to be high risk. It is thus natural to assume that individuals in period 1 will find it impossible to purchase insurance at competing, less informed, conventional firms in period 1. In contrast, conventional firms in the infinite horizon model know that there are newly born potential customers in each period, so they have an incentive to make new sales. We assume—critically—that these firms are unable to distinguish these new customers from customers who were previously insured at another firm in the Infinite Horizon model.

The tech firms still compete statically each period, setting \(q_t^{RN}(p) = pL + C_0\) for customers who have new tech customers and \(q_t^{OL}(p) = pL + C_1\) for old ones, and we drop the \(t\) subscript henceforth since the environment is, by construction, static over time. The ability for individuals to “return to the market” and find a new conventional firm significantly changes—and in some ways simplifies—conventional firm pricing. First, conventional firms who have incumbent types whose type they have learned set some price \(q_c^{IL}(p)\) so as to maximize profits, but the outside option for these types is no longer the same: they can go to a tech firm, as in the FH model, but now they additionally have the option of leaving for a new conventional firm. Second, conventional firms who do not learn the type of their incumbent buyer have no incentive to retain those buyers. Intuitively: they have learned nothing about them (other than the fact that they are high enough risk to prefer conventional to tech insurers), and because these buyers can always go to a new firm, they cannot charge a higher price or extract any rents. In effect, we can treat unlearned types as “returning” to the conventional pool and buying from a random new firm. Together, this means that the only conventional firm premiums needed to describe equilibrium in the OLG model are \(q_c^{OL}(p)\) and \(q_c\), with the latter denoting the price in the competitive market for “new” insureds. We will be looking for a steady-state equilibrium in which these premiums are stable over time.

Towards describing that steady-state equilibrium, note that an individual can be in one of only four (4) mutually exclusive and exhaustive states: they can be an incumbent conventional firm who knows their
type (L), or not (U), and they can have purchased from a tech firm in the past (i), or not (n). Describe the four states in the obvious way by Li, Ln, Ui, and Un.

Consider first an individual in state Ui. She has the option of choosing a tech firm at price \( q^{i,i}(p) \) or from a conventional firm at price \( q^c \). Contrast this with an individual in state Un. She has the option of choosing a tech firm at price \( q^{i,n}(p) \) or from a conventional firm at price \( q^c \). Because \( q^{i,n}(p) \geq q^{i,i}(p) \) (with strict inequality if \( C_1 < C_0 \)), it is obvious that if the Un individual finds it optimal to choose the tech firm, so will the Nτ individual. But this means that an individual who purchases from a tech firm in the period in which she is born (into the Un state) will always purchase from a tech firm—that is, there will never be a type in the Li state. Conversely, an individual who purchases from a conventional firm when they are born will never purchase from a tech firm: she has revealed that she prefers \( q^c \) at a conventional firm over buying from a tech firm, and she will always have the option of doing so. Together, this means that individuals sort permanently into tech and conventional firms in the year they are born.

Since tech firms can observe risk type better than conventional firms, standard adverse selection intuition suggests that this initial sorting will take a “cutoff” form. To show this formally, consider again the pricing problem for a conventional firm who has learned a buyer’s type and thus can earn some information quasi-rents from her. Because that buyer “sorted” into conventional firms, we know that the relevant “outside option” for this buyer is a return to the conventional market where she will pay \( q^c \). The conventional firm can and therefore will charge up to \( q^c \) to incumbent buyers they wish to retain. (They will charge more to those buyers with \( q^c < p_L \), because there is no price they can profitable charge while retaining them, but without loss of generality we can imagine them charging a price of \( q^c \) to all and only retaining types with \( q^c \geq p_L \).) This means that conventional buyers face a time independent insurance premium, independent of their type—and this is true regardless of whether their type is learned or not. In light of this the payoff to “sorting” into conventional firms when born is independent of type, while the payoff to “sorting” into tech firms is decreasing in \( p \). There is therefore some cutoff type \( \hat{p} \) who is indifferent between the tech and conventional firms, with lower types sorting into tech and higher types sorting into conventional firms. In fact, we can easily characterize the cutoff \( \hat{p}(q^c) \) as a function of the price \( q^c \) via the indifference condition:

\[
\frac{1}{1 - \beta \eta} U(W - q^c) = U \left( W - q^{i,n}(\hat{p}(q^c)) \right) + \frac{\beta \eta}{1 - \beta \eta} U \left( W - q^{i,i}(\hat{p}(q^c)) \right).
\]

The final thing needed to characterize equilibrium is the price \( q^c \) offered to conventional buyers in the “non-incumbent” pool. Towards describing the distribution of types in this pool, let \( \bar{p}(q^c) = \frac{q^c}{L} \), which is the maximum risk type that incumbent firms will retain after learning their types. The pool of non-incumbent buyers consists of all living individuals with \( p > \bar{p}(q^c) \) (a mass \( \left( 1 - F(\bar{p}(q^c)) \right) \) of them)
and a fraction \( \frac{1-\eta}{1-\eta(1-\alpha)} \) of individuals with \( p \in [\hat{p}(q^c), \bar{p}(q^c)] \), a mass \( F(\bar{p}(q^c)) - F(\hat{p}(q^c)) \). A randomly selected individual in the pool will thus be drawn from \( p \in [\hat{p}(q^c), \bar{p}(q^c)] \) with probability

\[
Q_M(q^c) \equiv \frac{1 - \eta}{1 - \eta(1 - \alpha)} \frac{1}{1 - \eta(1 - \alpha)} \left( F(\hat{p}(q^c)) - F(\bar{p}(q^c)) \right) + 1 - F(\bar{p}(q^c)).
\]

Lifetime expected profits from such a sale are thus:

\[
\pi^c(q^c) = \left( Q_M(q^c) \mathbb{E}[q^c - Lp | p \in [\hat{p}(q^c), \bar{p}(q^c)] \right) + \left( 1 - Q_M(q^c) \mathbb{E}[q^c - Lp | p \in [\hat{p}(q^c), \bar{p}(q^c)] \right) + \alpha \eta \beta Q_M(q^c) \frac{1}{1 - \eta \beta} \left( \mathbb{E}[q^c - Lp | p \in [\hat{p}(q^c), \bar{p}(q^c)] \right).
\]

The first line is just the premium minus the expected losses in the current period. The second term is the future profits if the type is learned (probability \( \alpha \)), does not “die” (\( \eta \)) and is worth retaining \( (Q_M(q^c)) \). In this case, the firm earns positive information rents \( q^c - L \mathbb{E}[p | p \in [\hat{p}(q^c), \bar{p}(q^c)] \) for all future periods in which the individual remains alive (the present discounted number of which is \( \frac{1}{1 - \eta \beta} \).

**Definition: Equilibrium in the OLG Market**

An equilibrium is a set of prices \( q^{L,n}(p), q^{L,i}(p), q^{C,L}(p) \), and \( q^c \) and a cutoffs \( \bar{p}^* \) such that:

(i) Tech firm competition: \( q^{L,n}(p) = pL + C_0, q^{L,i}(p) = pL + C_1 \).
(ii) Incumbent conventional firm profit maximization: \( q^{C,L}(p) = q^c \)
(iii) Conventional firm competition: \( \pi^c(q^c) = 0, \) and \( \frac{d \pi^c(q^c)}{dq^c} > 0 \).
(iv) Individual optimization \( \frac{1}{1 - \beta \eta} U(W - q^c) = U(W - q^{L,n}(\bar{p}^*)) + \frac{\beta \eta}{1 - \beta \eta} U \left( W - q^{L,i}(\bar{p}^*) \right) \).

Equilibrium will again depend on the exogenous cost parameters \( C_0, C_1 \) and the speed of learning \( \alpha \). The following proposition is our main result for the OLG model. It establishes formally that the equilibrium cutoffs are again decreasing \( C_0 \) and \( C_1 \). In contrast to the FH model, however, faster learning by conventional firms actually lowers the conventional firm market share (raises the cutoff). The formal proof is in the appendix.

**Proposition OLG:** In any locally smooth family of equilibria:

1. The equilibrium cutoff \( \hat{p}^* \) is (weakly) decreasing \( C_0 \) and \( C_1 \), and strictly so if \( \hat{p}^* \neq p_- \).
2. The equilibrium cutoff is increasing in \( \alpha \), strictly so if \( \hat{p}^* \in (0,1) \) and \( \beta < 1 \).

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6 These individuals remain in the pool as long as they are (a) alive and (b) have not yet had their type learned. The number of such individuals is \( (1 - \eta) \left( F(\hat{p}) - F(\bar{p}) \right) (1 + \eta(1 - \alpha) + \cdots + \eta^k(1 - \alpha)^k + \cdots) \). The total number of such individuals in total is \( F(\bar{p}) - F(\hat{p}) \).
3.3 Infinite Horizon Myopic Model [Construction Zone]

We again assume an infinite number of periods and a continuum of types with (cumulative) distribution $F(p)$ and continuous pdf $f(p) > 0$ on $[\underline{p}, \overline{p}] \subset (0,1)$, buying insurance in each period. The cost $C$ for tech firms to provide insurance is (potentially) time dependent, with cost $C_0$ for a first-time tech buyer, and $C_k \leq C_0$ for an individual who was previously insured with any tech insurer for $k$ periods. (As we shall see, the $C_k$ will not end up being relevant for $k > 0$.) In contrast to the FH and OLG model, customers choose firms based on the current-period prices without considering the consequences of current-period choices for prices in future periods. Also, conventional insurers use backward-looking expectations about their risk pool and update prices with a lag of one period. The tech firms compete in each period, offering $q_t^{\tau,n}(p) = pL + C_0$ for new customers and $q_t^{\tau,t}(p) = pL + C_k$ for old ones.

In period 0, conventional firms cannot observe buyers’ types when determining prices, and offer average premium $q_0^{CU} \equiv \overline{p}_0 L$ equal to the expected loss of all risks in the market to each customer (where $\overline{p}_0 \equiv \int_{\underline{p}}^{\overline{p}} p f(p) dp$.)

In the subsequent periods ($t = 1, 2, \ldots$), if a conventional firm insured an individual in the period $t - 1$, it will learn that policyholder’s risk type with probability $\alpha$. We assume that firms will offer these learned types a fair premium $q_t^{CL}(p) = pL$. We assume that firms will offer unknown types (unlearned types and new customers) a premium $q_t^{CU} = E[pL|conv_{t-1}^{U}] \equiv \hat{p}_t L$, namely the premium that would be fair given loss experience they experienced within their unknown insured population in the previous period—whose mean risk is defined to be $\overline{p}_t$. We assume symmetry across all conventional firms, so the firm-specific distribution $conv_{t-1}^{U}$ coincides with the market-wide distribution of customers who buy in the “unknown” market. As such, we can without loss assume that all customers, whose type was unlearned in period $t - 1$ and who choose to buy from a conventional firm, re-enter a common pool and choose randomly among the conventional insurers.

Individuals who purchased from a conventional firm in period $t - 1$ whose type was learned by their incumbent firm thus choose in period $t$ among three options: (1) switching to a tech firm offering a premium of $q_t^{\tau,n}(p) = pL + C_0$, (2) stay with the incumbent conventional firm offering a premium of $q_t^{CL}(p) = pL$, and (3) entering the “unknown” market paying a premium of $q_t^{CU}$. Individuals who purchased from a conventional firm in period $t - 1$ and did not have their type learned have only choices (1) and (3). Since customers are myopic, they choose the lowest-premium option. It follows that, among individuals who purchased in the “unknown” conventional market in period $t - 1$: 

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- The fraction \( (1 - \alpha) \) whose insurers did not learn their type will leave for a tech firm if \( q_{t,n}^\alpha(p) < q_{t,U}^\alpha \), i.e., \( p < \tilde{p}_t - \frac{c_0}{L} \). Otherwise, they will continue to participate in the “unknown” conventional market.\(^7\)
- The fraction \( \alpha \) whose insurers learned their type will leave the “unknown” market and stay with their incumbent firm, paying the premium \( pL \), if \( p < \tilde{p}_t \) and otherwise will continue to participate in the “unknown” conventional market (by randomly choosing a new firm that does not know their type).\(^8\)

There are thus two key cutoffs in each period: \( \tilde{p}_t \) and \( p^*_t \equiv \max\left\{ \tilde{p}_t - \frac{c_0}{L}, p \right\} \). All types below \( p^*_t \) exit the unknown conventional market, as does the fraction \( \alpha \) of all learned types below \( \tilde{p}_t \).

It is straightforward to see that these cutoffs are non-decreasing in \( t \). Inductively: \( \tilde{p}_0 \leq \tilde{p}_1 \) (with equality if and only if \( p^*_0 > p \)) since the lowest risk types (potentially) leave the unknown conventional market for tech firms; and if \( \tilde{p}_{t-1} \leq \tilde{p}_t \), then no individuals will re-enter the unknown conventional market (but will instead stay either in the tech market or with their incumbent conventional firm who knows their type). Per the preceding bullets, all exit from the unknown conventional market will occur from individuals with \( p < \tilde{p}_t \) i.e., from individuals with below-average risk. Hence \( \tilde{p}_{t+1} \geq \tilde{p}_t \).

In fact, for any \( \alpha > 0 \), \( \lim_{t \to \infty} \tilde{p}_t = \tilde{p} \) it must converge, and, since at least a fraction \( \alpha \) of the below-average risks leave the unconventional market each period, the only values to which it can converge must be those for which there is a zero measure of types below the average (which is only true for \( \tilde{p} \)).

**Definition: Quasi-equilibrium in the Infinite Horizon Myopic Market**

In each period \( t \) (\( t=1,2,\ldots\)), the quasi-equilibrium is a set of prices \( q_{t,n}^\alpha(p), q_{t,i}^\alpha(p), q_{t,L}^\alpha(p), q_{t,U}^\alpha \), and a set of cutoffs \( \tilde{p}_t \) and \( p^*_t = \max\left\{ \tilde{p}_t - \frac{c_0}{L}, p \right\} \) such that:

(i) Tech firm competition: \( q_{t,n}^\alpha(p) = pL + C_0 \), \( q_{t,i}^\alpha(p) = pL + C_1 \).

(ii) Conventional firm quasi-competition: \( q_{t,L}^\alpha(p) = pL \) and \( q_{t,U}^\alpha = L\tilde{p}_t \), with \( \tilde{p}_t = E[p | \text{conv}_{t-1}^\alpha] \).

(iii) Individual quasi-optimization: in period 0, individuals with \( p < \tilde{p}_0^* \) purchase from tech firms \( p \geq \tilde{p}_0^* \) purchase from conventional firms. In period \( t \): individuals who purchased from a tech firm in \( t-1 \) continue to purchase from tech firms; individuals who purchased from a conventional firm who knew their type continue to purchase from that firm; for individuals who purchased in the “unknown type” market, a fraction \( \alpha \) individuals who had

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\(^7\) We assume that the indifferent types with \( p = \tilde{p}_t + \frac{c_0}{L} \) remain in the unknown conventional market. Because there is a measure zero of such types, this assumption is unimportant.

\(^8\) Again, the measure zero of indifferent types are (safely) assumed to remain in the unknown conventional market.
their type learned continue to purchase from their current firm if \( p < \bar{p}_t \) and otherwise buy in the “unknown type” market (from another random conventional firm), and the fraction \( 1 - \alpha \) of such individuals who did not have their type learned purchase from a tech firm if \( p < p^*_t \) and otherwise stay in the “unknown type” market.

In contrast to the FH and OLG models, the comparative statics of the share of individuals insuring with a tech firm may not be monotone with respect to \( \alpha \). Consider the effect of an increase in \( \alpha \). On the one hand, a higher \( \alpha \) increases learning and means that more low risk types have their types learned early and thus stay with conventional firms forever. Thus, larger \( \alpha \) would seem to imply lower tech shares. On the other hand, a higher \( \alpha \) means that some individuals who would have stayed in the “unlearned type” market exit for the learned type conventional market. Since those types are lower-than-average risk, this exit raises the average riskiness of the “unlearned type” pool, implying a higher \( p^*_t \) and hence more types who leave for tech firms in the subsequent period. To put it another way: all else equal, for a fixed rate of “unlearned type” market unraveling, a higher \( \alpha \) implies more “unraveling” towards the learned conventional market instead of the tech market; but it also implies faster unraveling.

Figure 1: Temporal evolution of the tech firm market share. Assumes uniform distribution of \( p \) on \([0.3, 0.9]\), \( C_0 = 0.27L \).

Figure 1 illustrates the potential for non-monotone comparative statics with respect to \( \alpha \). The tech shares for four different levels of \( \alpha \) “cross” each other—and hence switch order—over time. In period \( t = 8 \),
for example, the tech share increases from \( \alpha = 0.05 \) to \( \alpha = 0.15 \) but then decreases when \( \alpha \) is further increased to 0.25 and 0.35.

A higher \( C_0 \) implies, for any given distribution of types in the “unlearned type” market, a smaller share of individuals leaving for tech firms in the current period. On the other hand, it also implies slower unraveling of the market. Intuitively, this slower unraveling should give individuals in the “unlearned types” market more opportunities to be learned before the price is driven up to the point where they will exit to a tech firm—and hence should also lead to a smaller tech share. All of our simulations to date have confirmed this intuition: a higher cost leads to a lower tech share. A formal proof remains elusive, however, so we cannot, at this stage, rule out the theoretical possibility of non-monotone comparative statics with respect to \( C_0 \).

“Proposition” Myopic:

The period \( t \) tech market share may be increasing or decreasing in \( \alpha \). The comparative statics of the tech share with respect to \( C_0 \) are unknown. The tech market share is independent of \( C_k, k > 0 \).
4. Conclusions

Table 1 summarizes our main results, i.e. the (quasi)-equilibrium predictions of the three models.

<table>
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<th>Questions</th>
<th>Model structure</th>
<th>Notes: For details, see Propositions FH, OLG, and Myopic</th>
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<tr>
<td>1 How do risk types distribute between tech and conventional firms?</td>
<td>Infinite horizon myopic model&lt;br&gt;In each period, there is a cutoff $p^<em>_t$ for current buyers in the conventional market with unlearned types. Types with $p &lt; p^</em>_t$ buy from tech firms unless their type is learned by their insurer.</td>
<td>All three models feature a “cutoff” structure, where low risk types buy from tech firms while higher risks buy from conventional firm. This cutoff structure reflects a basic tradeoff at the heart of all models: on the one hand, tech firms have an informational advantage; on the other hand, they bear a higher cost of operation, which they pass on to customers. In general, this tradeoff allows both types of firm to coexist. As expected (although we have not yet formally shown it in the myopic case), higher cost disadvantages lead to lower tech market shares. In the finite horizon model, the decreasing of the informational advantage of tech firms—by increasing the learning rate of conventional firms—also lowers the tech market shares. Interestingly, however, the effect of shrinking the informational disadvantage between tech and conventional firms may not decrease the share of tech firms in other market structures. In the myopic model, the effect is ambiguous (and examples can be shown where the effects go in either direction). And in the OLG model, increasing the speed of learning of conventional firms necessarily increases the tech market share. Intuitively, this “reversal” stems from the fact that increasing the speed of learning of conventional firms can actually exacerbate adverse selection within the conventional market, as high risk types leave their insurer to find a new one who does not know their type, thereby polluting the risk pool. Worse adverse selection in the conventional market redounds to the benefit of tech firms, who do not suffer from it.</td>
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<tr>
<td>2 How does the market share of tech insurers correlate with the learning speed of conventional insurers?</td>
<td>Finite horizon model with foresight&lt;br&gt;There is a period 1 cutoff type $p^<em>_1$. Types $p &lt; p^</em>_1$ buy from tech insurers. There is a a period 2 cutoff type $p^<em>_1 \geq p^</em>_0$. Individuals with $p &lt; p^*_1$ switch to tech insurers.</td>
<td>$\hat{\beta}^*$ is increasing in the speed of learning.</td>
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<td>3 How does the market share of tech insurers correlate with the additional cost of digital technology</td>
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<td>$\hat{\beta}^*$ is decreasing in the cost.</td>
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$p^*_1$ is decreasing and $p^*_2$ is non-increasing with the speed of learning. $p^*_1$ and $p^*_2$ are decreasing in the cost.
References


Appendix

Proof of proposition FH

We consider two distinct cases: case 1: $p_0 = \frac{q_0 c_1}{l}$ and case 2: $p_0 = \frac{q_0 c_1}{l}$. 

Case 2: in this case, conventional firms sell to everybody in period 0, $p_0 = p^*$, and, along a smooth family of equilibrium, this cutoff will be locally independent of small changes in $\alpha$, $C_0$, and $C_1$. For local changes in $\alpha$, $C_0$ and $C_1$, the period 1 cutoff $p_1$ solves:

$$\max_{p^t \in [p^*, \bar{p}]} \int_{p^t}^{\bar{p}} (p^t L - C_0 - p L) f(p)dp \equiv \max_{p^t \in [p^*, \bar{p}]} \pi^1(C_0, p^t).$$

Since $\pi^1$ is independent of $\alpha$ and $C_1$, so is $p_1$. We compute $\frac{\partial^2 \pi^1}{\partial C_0 \partial p^t} = f(p^t)$, which is strictly positive (as, otherwise, $\frac{\partial^2 \pi^1}{\partial p^t} > 0$ and $p^t$ would not be optimal). By Topkis’s Theorem, the cutoff $p_1$ is increasing in $C_0$, strictly so if $p_1 > p^*$.

Case 1: in this case, we can parameterize the zero-profit problem in terms of the first period cutoff $p_0$ instead of $q_0^*$:

$$\pi^c(p_0, \alpha, C_0, C_1) = (p_0 L + C_1) - \int_{p_0}^{\bar{p}} pf(p)dp + \alpha C_0 (1 - F(p_0)) + (1 - \alpha) \max_{p^t \in [p^*, \bar{p}]} \int_{p^t}^{\bar{p}} (p^t L - C_0 - p L) f(p)dp.$$

This is obviously strictly increasing in $\alpha$, $C_0$, and $C_1$. By item (iv) in the definition of an FH equilibrium and $p_0 = \frac{q_0 c_1}{l}$, it is increasing in $p_0$ and satisfies $\pi^c(p_0, \alpha, C_0, C_1) = 0$ for all equilibrium. It follows that, along a smooth equilibrium path, $p_0$ must be strictly decreasing in $\alpha$, $C_0$, and $C_1$.

The period 1 cutoff satisfies:

$$\max_{p^t \in [p_0(\alpha, C_0, C_1), p^*]} \int_{p^t}^{\bar{p}} (p^t L - C_0 - p L) f(p)dp \equiv \max_{p^t \in [p_0(\alpha, C_0, C_1), p^*]} \pi^1(C_0, p^t).$$

An increase in $\alpha$ or $C_1$ only affects this problem by lowering the lower bound $p_0(\alpha, C_0, C_1)$. Hence, $p_1$ is weakly decreasing in $\alpha$ and $C_1$. By the same argument as in case 2, the $p^t$ solving $\max_{p^t \in [p^*, \bar{p}]} \pi^1(C_0, p^t)$ for any fixed $p^*$ is decreasing in $C_0$, and strictly so unless the optimum is at $p^*$. Together with the fact that the lower bound $p_0(\alpha, C_0, C_1)$ is strictly decreasing in $C_0$ implies that $p_1$ is decreasing in $C_0$, and strictly so unless $p_1 = p_0$. 

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**Proof of proposition OLG**

First, observe that

\[ V(q^c, \bar{p}^*; C_0, C_1) = \frac{1}{1 - \beta \eta} U(W - q^c) - \left[ U(W - q^{c,n}(\bar{\rho}^*)) + \frac{\beta \eta}{1 - \beta \eta} U(W - q^{c,l}(\bar{\rho}^*)) \right] \]

\[ = \frac{1}{1 - \beta \eta} U(W - q^c) - [U(W - \bar{p}^* L - C_0) + \frac{\beta \eta}{1 - \beta \eta} U(W - \bar{p}^* L - C_1) \]

is strictly increasing in \( C_0 \) and \( C_1 \) and strictly decreasing in \( \bar{p} \). Thus, for any given \( q^c \), the \( \bar{p}^*(q^c; C_0, C_1) \) satisfying \( V(q^c, \bar{p}^*; C_0, C_1) = 0 \) is strictly decreasing in \( C_0 \) and \( C_1 \) unless \( \bar{p}^*(q^c; C_0, C_1) = p \). Given any \( q^c, \bar{p}^*(q^c; C_0, C_1) \) is obviously independent of \( \alpha \).

Denoting by \( \mathbb{E}\pi_M \equiv \mathbb{E}[q^c - L p | p \in [\bar{p}(q^c), \bar{\rho}(q^c)]] \) and \( \mathbb{E}\pi_H \equiv \mathbb{E}[q^c - L p | p \in [\bar{p}(q^c), \bar{\sigma}]] \), lifetime per-sale firm profits in the “unlearned pool” can be written, using the definition of \( Q_M(q^c) \) as:

\[ \pi^c(q^c) = Q_M(q^c) \left( 1 + \frac{\alpha \eta \beta}{1 - \eta \beta} \right) \mathbb{E}\pi_M + (1 - Q_M(q^c)) \mathbb{E}\pi_H \]

\[ = Q_M(q^c) \left( \frac{1 - \eta + \alpha \eta}{1 - \eta} \right) \left( \frac{1 - \eta \beta + \alpha \eta \beta}{1 - \eta \beta} \right) \mathbb{E}\pi_M + (1 - Q_M(q^c)) \mathbb{E}\pi_H \]

\[ = (1 - Q_M(q^c)) \frac{1 - F(\tilde{\rho}(q^c))}{1 - F(\hat{\rho}(q^c))} \mathbb{E}\pi_M + \frac{1 - F(\tilde{\rho}(q^c))}{1 - F(\hat{\rho}(q^c))} \mathbb{E}\pi_H \]

Hence, \( \pi^c(q^c) \) is independent of \( C_0 \) and \( C_1 \) for any given \( q^c \) except through the effect on \( \bar{p}^*(q^c; C_0, C_1) \), and it is easy to see that \( \pi^c(q^c) \) is strictly decreasing in \( \bar{p}^*(q^c; C_0, C_1) \). Together with the preceding paragraph, this implies

**Observation 1:** \( \pi^c(q^c) \) is increasing in \( C_0 \) and \( C_1 \), strictly so unless \( \bar{p}^*(q^c; C_0, C_1) = p \).

Finally, observe that the expression:

\[ \frac{1 - \eta \beta + \alpha \eta \beta}{1 - \eta + \alpha \eta} \]

is strictly decreasing in \( \alpha \) unless \( \beta = 1 \), in which case it is independent of \( \beta \). It follows that (starting from an equilibrium where profits are zero), \( \pi^c(q^c) \) is strictly decreasing in \( \alpha \) as long as \( \bar{p}(q^c) > \hat{\rho}(q^c) \). And \( \bar{p}(q^c) > \hat{\rho}(q^c) \) obviously holds in equilibrium if \( \alpha > 0 \): if \( \bar{p}(q^c) = \hat{\rho}(q^c) \), conventional firms never retain any customers, which means that they must break even on the average risk type above \( \hat{\rho}(q^c) \). But then would be strictly profitable to sell to learned types with type \( \hat{\rho}(q^c) \). This yields:

**Observation 2:** For any \( q^c \), \( \pi^c(q^c) \) is decreasing in \( \alpha \), strictly so if \( \beta < 1 \).
Observations 1 and 2, together with the fact that $\pi^c(q^c) = 0$ and $\frac{d\pi^c(q^c)}{dq^c}$ is increasing in any equilibrium, implies that the equilibrium $q^c$ is increasing in $C_0$ and $C_1$ (strictly so if $p \neq \hat{p}^*(q^c)$) and decreasing in $\alpha$ (strictly so if $\beta < 1$).

Observing that $\hat{p}^*(q^c)$ is decreasing in $q^c$ (strictly so if $p \neq \hat{p}^*(q^c)$) completes the proof.

It is worth commenting briefly on the intuition behind the $\alpha$ result. The expression for $\pi^c$ above implies that $\pi^c = 0$ if and only if

$$\left[ \frac{F(\hat{p}(q^c)) - F(\hat{p}(q^c))}{1 - F(\hat{p}(q^c))} \frac{1 - \eta + \alpha \eta}{1 - \eta} \right] \mathbb{E}_{\pi_M} + \frac{1 - F(\hat{p}(q^c))}{1 - F(\hat{p}(q^c))} \mathbb{E}_{\pi_H} = 0.$$

When $\beta = 1$, this is a weighted average of the profits from “retained” types and “non-retained” types, where the weights are the population shares of these types. To see why, note that all types above $\hat{p}(q^c)$ are buying from conventional firms at all times. When there is no time discounting, the cross-sectional profits per period must be zero. Time discounting does not (directly) the distribution of types over time—it still matches the cross sectional distribution. But from an individual firm’s point of view a sale loses money immediately, and will make that profit up later if they learn that they had sold to a lower risk type. With time discounting, though, being later in time is less valuable. So if the cross sectional profits were zero, then the per-firm discounted profits from a sale would be negative. Prices must therefore be higher to break even when there is discounting. Moreover, the faster is the learning, the more back-loaded is the profit stream, and the higher the prices have to be. This is manifesting itself in the preceding formula in the term

$$\frac{1 - \eta \beta + \alpha \eta \beta}{1 - \eta \beta} \frac{1 - \eta + \alpha \eta}{1 - \eta},$$

The numerator of which is, effectively, the present discounted “number of periods” of sales to profitable learned types. The denominator is the undiscounted “number of periods”. The gap between these two grows with $\alpha$. 