Monopolistic Competition, Managerial Compensation, and the Distribution of Firms in General Equilibrium*

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Abstract

We develop a general equilibrium model to show how the distribution of firm qualities, moral hazard, and monopolistic competition in the product market interact to affect the distributions of firm size and managerial compensation. We exploit the properties of the unique, stationary general equilibrium of the model to derive a number of novel implications for the relations between the firm size and managerial compensation distributions, and the effects of firm and product market characteristics on these distributions. Our results highlight a novel general equilibrium channel through which firm and product market characteristics affect managerial compensation and incentives.

Different determinants of competition have contrasting effects on the distributions of firms and managerial compensation. An increase in the entry cost or exit probability decreases expected managerial compensation and the average size of firms, but increases the number of active firms. An increase in the elasticity of product substitution, however, decreases expected compensation if firm size is below an endogenous threshold, but increases expected compensation if firm size is above the threshold. An increase in productivity risk raises expected managerial compensation and the number of active firms. In general equilibrium, aggregate shocks to the manager-firm match quality distribution and firms' productivity levels affect compensation and incentives. Expected managerial compensation and average firm size decrease with the productivity level, while the number of active firms increases.

We use our theoretical results to develop ten robust empirically testable hypotheses that relate industry characteristics—the entry cost, the exit probability, the elasticity of product substitution, the productivity risk, and the productivity level—to managerial compensation and the number of firms in the industry. We show support for nine of the ten hypotheses in our empirical analysis.

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1 Introduction

How are the distributions of firm size, managerial compensation levels and incentives related to each other? How do characteristics of the product market in which firms are competing (such as entry costs, likelihood of failure, and the price elasticity of demand) affect those distributions? How are the relations among risk, productivity, and managerial compensation schemes affected by market structure? To address these questions, we develop a general equilibrium model in which the distribution of heterogeneous firm qualities, moral hazard, and monopolistic competition in the product market interact to affect the distributions of firm size and managerial compensation. Our framework integrates aspects of models developed in two groups of studies that have hitherto remained relatively independent of one another: the impact of asymmetric information on managerial compensation (see Bolton and Dewatripont, 2005); and the determinants of the firm size distribution (FSD) (see Cabral and Mata, 2003). With the exception of some important recent studies that we discuss in Section 2, the latter strand of the literature abstracts away from issues stemming from asymmetric information and the provision of incentives, while the former examines managerial compensation and incentives using partial equilibrium principal–agent frameworks.

We build an infinite horizon general equilibrium model that incorporates moral hazard into a framework with monopolistically competitive firms in a particular industry. As in Krugman (1979) and Melitz (2003), production is driven by labor. A group of entrepreneurs establish a firm by supplying sunk labor, and subsequently hire a manager to run the firm. Each firm has a constant exogenous probability of exiting the market in each period that could be interpreted as the failure risk of the industry. The *quality* of the firm, which affects its productivity, is realized after one period. It is drawn from a known distribution and remains constant through time.

A firm's realized productivity in each period depends on its quality, an idiosyncratic productivity shock, and the manager's costly effort. Higher effort increases the likelihood of a favorable productivity shock, and there is complementarity between effort and the firm's quality. As in Dixit and Stiglitz (1977), firms are monopolistically competitive in that they take the *aggregate price index*—the weighted average of the prices charged by each firm—as given when they make their output and pricing decisions after their productivities are realized. The firms' risk-neutral owners offer incentive contracts to their risk-averse managers at the beginning of each period to induce costly effort by the managers.

Equilibrium is characterized by a mass of firms and incentive contracts for managers in each period. The mass of firms is determined by the general equilibrium condition that the aggregate revenue in each period equals the aggregate wages paid out to managers and workers. Furthermore, with free entry of firms, the net present value of the profits of each firm must equal the entry cost. The aggregate price index, which is endogenously determined by the equilibrium conditions, influences managerial incentives because it affects the marginal product of managerial effort. Our model, therefore, highlights a novel, *general equilibrium* channel through which firm and market characteristics influence managerial incentives through their effects on the aggregate price index.

There exists a unique, stationary general equilibrium in which exiting firms are exactly replaced by new entrants. Aggregate variables—the aggregate price index, the mass of producing firms, aggregate revenue and profit—are constant through time. The expected utility of the representative consumer equals aggregate revenue divided by the aggregate price index. Because aggregate revenue is fixed by the population of consumers, it follows that consumer welfare is inversely proportional to the aggregate price index. The predictions we derive for the effects of firm and market characteristics on the aggregate price index, therefore, directly lead to implications for consumer welfare.

The size of a firm along with the compensation and incentives of its manager are endogenously determined by the firm's realized quality. The complementarity between firm quality and effort ensures that firm size (measured in terms of expected revenue, gross or net profit, or the labor force), managerial effort, and the expected compensation of managers increase with firm quality. Under an additional elasticity condition on the managers' utility function, managers with higher quality matches have *lower* pay-performance sensitivities. The additional condition ensures that the elasticity with respect to effort of the manager's compensation if the firm has a favorable productivity shock increases in effort. Under this condition, the complementarity between firm quality and effort ensures that managers with higher quality matches can be induced to exert greater effort with lower pay-performance sensitivities. The predicted positive relation between the managerial compensation level and firm size, and a negative relation between managerial pay–performance sensitivity (PPS) and firm size are consistent with considerable empirical evidence (Hall and Liebman, 1998; Schaefer, 1998; Baker and Hall, 2004). The relations are, however, not causal as firm size and managerial compensation are endogenously determined by firm quality.

Next, we explore the effects of product market characteristics on the distributions of firms, managerial compensation levels and incentives. The entry cost and exit probability influence managerial incentive compensation through the general equilibrium channel by affecting the aggregate price index. A decrease in the entry cost or exit probability of firms increases competition and correspondingly lowers the aggregate price index. The decline in the aggregate price index decreases the marginal product of managerial effort so that effort, expected managerial compensation, and the average size of firms decrease. If the additional elasticity condition on managers' utility function holds, managerial pay-performance sensitivities increase to compensate for the dampening effect of the decline in the aggregate price index. Because the average size of firms decreases, the mass of active firms increases.

In a framework with monopolistic competition, the elasticity of substitution between products equals the price elasticity of demand. We show that managerial effort and expected managerial compensation decrease with the elasticity of substitution if and only if the firm quality is below an endogenous threshold. The intuition for these results hinges on the fact that an increase in the elasticity of substitution affects managerial effort and output differentially. It causes managers of higher quality firms to benefit relatively more from exerting greater effort compared to managers of lower quality firms. As with our earlier results, the elasticity condition on the managers' utility function ensures that managerial pay-performance sensitivity moves in the opposite direction relative to effort.

Our results show that differing determinants of competition among firms—the entry cost, exit probability, and the elasticity of product substitution—have contrasting effects on the number of firms, the firm size distribution (FSD), managerial compensation levels and incentives. The differing effects arise from the fact that the entry cost and exit probability affect the distributions of firms and managerial compensation *indirectly* by affecting the aggregate price index, that is, *solely* through the general equilibrium channel. In contrast, the elasticity of product substitution affects these distributions both directly through its effects on managerial effort and output as well as indirectly by influencing the aggregate price index. The contrasting effects suggest that empirical analyses of the effects of competition on the FSD, compensation and incentives should appropriately account for different facets of competition.

Next, we examine the effects of productivity risk—the spread in the firm's productivities in the "high" and "low" states—on managerial compensation levels and incentives. In our framework, an increase in productivity risk raises the marginal product of managerial effort, thereby increasing managerial effort at the optimum that, in turn, enhances the expected compensation of managers. Because managerial effort increases, the pay-performance sensitivity (PPS) *declines* provided the additional elasticity condition on the managers' utility function holds. If the additional elasticity condition does not hold, however, the relation between PPS and risk could be positive or even vary in sign depending on the firm quality. The relation between PPS and risk, therefore, could be negative, positive, or even ambiguous depending on the manager's utility function.

These findings provide an explanation for the tenuous empirical link between firm risk and incentives. As discussed by Prendergast (2002), some empirical studies find that the link is positive (e.g., Rajgopal et al., 2006); some find that the link is insignificant (e.g., Conyon and Murphy, 2000); and others find that the link is negative (e.g., Aggarwal and Samwick, 1999a). Our analysis shows that, under more general utility functions, the relation between PPS and risk (productivity risk or firm risk) as well as the relations between PPS and other variables such as the entry cost, exit probability and the elasticity of product substitution crucially depend on the nature of the utility function. The parameters of the utility function are "deep" structural parameters that are unknown to the econometrician and for which it is difficult to find reasonable proxies. Consequently, our analysis suggests that empirical investigations of the determinants of PPS are likely to be misspecified.

The effects of productivity risk on the aggregate price index (therefore, consumer welfare) crucially hinge on the the entry cost and exit probability. If the entry cost and/or exit probability are above respective (endogenous) thresholds, an increase in productivity risk lowers the aggregate price index and raises consumer welfare. If they are below the thresholds, however, an increase in productivity risk raises the aggregate price index and lowers consumer welfare.

Our general equilibrium framework also leads to novel implications for the effects of the firm quality distribution on managerial compensation, incentives, the mass of firms, and average firm size. An increase in the firm quality distribution in the sense of first-order stochastic dominance (FOSD) *lowers* the aggregate price index (and raises consumer welfare) because firms are more productive. The decline in the aggregate price index has a dampening effect on the effort and compensation of the manager with a *given* firm quality. Under the additional elasticity condition on the managers' utility function, managers receive *stronger* incentives. Insofar as shocks to the firm quality distribution could be viewed as aggregate shocks, our results show that managerial incentives are affected by aggregate shocks through their effects on the aggregate price index, that is, via the general equilibrium channel. This prediction contrasts sharply with that of traditional partial equilibrium principal–agent models in which incentives are only affected by idiosyncratic shocks.

Finally, we investigate the effects of an increase in the productivity *level* of firms keeping productivity risk fixed. The effects of the productivity level also operate through the general equilibrium channel. An increase in the productivity level, *ceteris paribus*, makes it more attractive for firms to enter the market, which raises the extent of competition and thereby lowers the aggregate price index. Managerial effort, expected managerial compensation, the average revenue of active firms, and the average gross profit of active firms all decrease with the productivity level. An increase in the productivity level lowers managers' incentives to exert effort. Furthermore, the extent of competition increases and the aggregate price index falls, which also dampen the incentive to exert effort. To counteract these effects, managerial pay-performance sensitivities increase (under the additional elasticity condition on managers' utility function).

Our theoretical results lead to ten robust empirically testable hypotheses that relate industry characteristics—the entry cost, the exit probability, the elasticity of product substitution, productivity risk, and the productivity level—to managerial compensation levels and the number of active firms in the industry. (i) Managerial compensation increases with firm size. (ii) Managerial compensation increases with the entry cost and exit probability, while the number of active firms decreases. (iii) Managerial compensation declines with the elasticity of product substitution if firm size is below a threshold, but increases with the elasticity of product substitution if it is above the threshold. (iv) Managerial compensation and the number of firms increase with productivity risk. (v) Managerial compensation declines with the productivity level, while the number of firms increases. We use industry data from COMPUSTAT, executive compensation data from EXECUCOMP, and several alternate proxies for the key independent variables to test our ten hypotheses. With the exception of the predicted negative effect of the productivity level on managerial compensation, we show empirical support for all our hypotheses.

2 Literature Review

We contribute to the literature by developing a general equilibrium model of firms in an industry in which the distribution of heterogeneous firm qualities, moral hazard, and monopolistic competition interact to affect the distributions of firms and managerial compensation. We complement a number of previous studies that derive important insights into various facets of these relationships.

First, we develop a model with a continuous distribution of *heterogeneous* firm qualities. Because heterogeneity gives rise to a firm size distribution, we examine the endogenous relationships among firm size and managerial compensation (levels and incentives). In this respect, we complement the study of Raith (2003) who develops a model with an endogenous number of *homogenous* firms. An alternative explanation of the positive association between firm size and managerial compensation is provided by competitive assignment models in which the number of firms, managers, and the firm size distribution are exogenous (Gabaix and Landier, 2008; Tervio, 2008). We offer a complementary perspective in which firm size and managerial compensation (levels and incentives) are simultaneously and endogenously determined by firm quality that is *unknown at the outset*. Further, agency conflicts, which the above studies abstract away from, play a key role in generating the positive relation between firm size and the managerial compensation level. Edmans et al. (2009) extend the model of Gabaix and Landier (2008) and show a negative relation between pay-performance sensitivities (PPS) and firm size. The total compensation of managers is unaffected by moral hazard in their model, whereas moral hazard simultaneously affects compensation levels and incentives in our model. Edmans and Gabaix (2010) build on the model of Gabaix and Landier (2008) by incorporating risk aversion and moral hazard. They show that talent assignment is distorted by the agency problem. Baranchuk et al. (2009) develop an industry equilibrium model and show a positive relation between firm size and managerial compensation as well as a negative relation between PPS and firm size. Falato and Kadyrzhanova (2008) examine the link between industry dynamics and managerial compensation schemes. They find that industry leaders have lower pay-performance sensitivities than industry laggards.

Second, we have monopolistic competition between firms. By contrast, firms compete along a Salop circle in Raith (2003); the two firms engage in Bertrand or Cournot competition in Aggarwal and Samwick (1999b); there is no product market competition in Gabaix and Landier (2008), Tervio (2008), Edmans et al. (2009), and Edmans and Gabaix (2010); and firms are perfectly competitive in Baranchuk et al. (2009) and Falato and Kadyrzhanova (2008). As discussed by Dixit and Stiglitz (1977), monopolistic competition allows all firms to compete against each other, yet enjoy a monopoly in their specific product market. The price elasticity of demand for a firm's product is given in equilibrium by the elasticity of substitution across any pair of products purchased by the representative consumer. The incorporation of monopolistic competition, therefore, also leads to novel implications for the effects of the elasticity of product substitution on the distributions of firms and managerial compensation. We also complement the above studies by incorporating nonzero entry costs and exit probabilities for firms. Our results show that differing determinants of competition—the entry cost, exit probability, and the elasticity of product substitution—have contrasting effects on the distributions of firm, managerial compensation, and consumer welfare.

Third, we have a general equilibrium framework in which agents supply human capital to firms and use their wages to consume the products of firms, while all the studies mentioned above analyze partial equilibrium models. We show that general equilibrium effects are important and significantly change the impact of various parameters of interest in the empirical executive compensation literature on PPS, total compensation, and market structure. In particular, as discussed earlier, the contrasting effects of differing facets of competition on the distributions of firms and managerial compensation arise through the general equilibrium channel. Furthermore, our general equilibrium framework leads to relationships between aggregate shocks and incentives, and generates implications for the effects of product market characteristics on consumer welfare.

Fourth, managers in our model have general utility functions. We show that the effects of underlying variables on PPS crucially depend on an additional "elasticity" condition on the utility function. In contrast, managers are risk neutral in Edmans et al. (2009) and Falato and Kadyrzhanova (2008), have CARA preferences in Raith (2003) and Baranchuk et al. (2009), and risk aversion is irrelevant in Gabaix and Landier (2008) and Tervio (2008) since there are no agency problems.

3 The Model

We first present a brief overview of the model. We incorporate asymmetric information stemming from moral hazard in a framework with a continuum of monopolistically competitive firms in an industry (Dixit and Stiglitz, 1977). The time horizon is infinite with the set of dates $T = \{0, 1, 2, ...\}$. At any date $t \in T$, a group of entrepreneurs establish a firm. The entrepreneurs (the "principal") hire a manager (the "agent") to operate the firm. The *quality* of the firm is realized after one period and then stays constant over time. The firm quality determines the firm's productivity in each period that it is active.[1]

At the beginning of each period, the principal offers a contract to the agent. The agent then

[1] The firm quality is, in general, the result of the composition between manager-specific characteristics such as ability and firm-specific characteristics such as project quality and technical efficiency. The manner in which these characteristics interact to determine firm quality is irrelevant to our analysis.

exerts effort. An idiosyncratic productivity shock whose distribution depends on the firm quality and the agent's effort is realized. In our framework with monopolistically competitive firms, each firm manufactures one good in which it enjoys a monopoly, but takes the aggregate price index—a weighted average of prices chosen by all firms—as given when it chooses its output quantity and price. The firm exits the market in any period (for exogenous reasons) with probability δ that could be viewed as the failure risk of the industry. As in Melitz (2003) and the various models discussed in Rogerson et al (2005), the exit probability δ could also be interpreted as a *discount rate.* We choose this interpretation of the parameter δ in our empirical analysis in Section 5. In the stationary general equilibrium of the model with free entry of firms, exiting firms are exactly replaced by new entrants; the net present value of profits generated by each entering firm equals the entry cost; and the aggregate revenue of all firms equals the aggregate consumption expenditure by consumers. In the following sub-sections, we describe the various elements of the model in detail.

3.1 Preferences

In each period, the representative consumer has preferences for consumption defined over a continuum of goods (indexed by ω) that are described by

$$U = \left[\int_{\Omega} q(\omega)^{\rho} d\omega\right]^{\frac{1}{\rho}}; 0 < \rho < 1, \tag{1}$$

where Ω is the set of available goods and ω is a finite measure on the Borel σ-algebra of Ω. If $p(\omega)$ is the price of good ω then, as shown by Dixit and Stiglitz (1977), the optimal consumption and expenditure decisions for individual goods are

$$q(\omega) = U\left[\frac{p(\omega)}{P}\right]^{-\sigma}; \tag{2}$$

$$r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma}, \tag{3}$$

where $R = PU$ is the aggregate expenditure of the representative consumer and

$$P = \left[\int_{\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{4}$$

Following the terminology of Dixit and Stiglitz (1977), we refer to P as the *aggregate price index.* It determines the consumption and expenditure decisions for individual goods by (2) and (3).

In (4), the elasticity of product substitution is

$$\sigma = \frac{1}{1-\rho} > 1. \tag{5}$$

Following Dixit and Stiglitz (1977), each active firm produces a single product (that is consumed by the representative consumer) in which the firm has a monopoly. However, the firms compete monopolistically in the sense that they take the aggregate price index P as given in making the output and pricing decisions for their individual products. Each active firm faces the price elasticity of demand σ. Given that there is a continuum of firms, no single firm perceives itself as having an impact on aggregate equilibrium outcomes.

3.2 Entry and Exit of Firms

There is an unbounded pool of prospective entrants into the industry and firms are identical prior to entry. Firms are set up by entrepreneurs who supply labor $f_e > 0$ that is sunk and then hire a manager from the pool of agents in the economy. Agents are *ex ante* identical so that every agent has the same probability of becoming a manager. The quality α of the firm is realized after one period and remains constant through time. As we describe shortly, the firm quality determines the firm's productivity in each period. Firm qualities are drawn from a distribution $g(.)$ that is common knowledge and has positive support over $(0, \infty)$ and a continuous cumulative distribution $G(.)$. A firm faces a constant exogenous probability δ in each period of receiving a bad shock that forces it to exit.

3.3 Production

Production requires two factors: raw labor that is inelastically supplied by production workers at the aggregate level L and specialized human capital that is supplied by managers. As in Melitz (2003), the aggregate expenditure R of the representative consumer is determined by the aggregate labor supply and is exogenous. To simplify the analysis, we assume that the manager of a firm also supplies one unit of raw labor. *In addition*, however, the manager also supplies human capital that we label "effort".

In each period, the firm's realized productivity, which determines its marginal cost of production over the period, depends stochastically on its firm quality and the manager's costly effort. If the manager exerts effort $e \in [0,1]$ in any period, then the firm's realized productivity ϕ is a random variable that takes on two possible values: αh and αl with probabilities e and $1-e$, respectively, where $h > 1, l < 1$. Productivity shocks are independent across firms and time, that is, they are idiosyncratic.

Production decisions in each period are made *after* the productivity ϕ is realized. Each firm has a constant marginal cost (measured in units of labor). The labor used by a firm is therefore a linear function of output and is given by

$$x = \frac{q}{\phi}. \tag{6}$$

In equilibrium of our model, the respective payoffs of the firm's owners (the entrepreneurs) and the manager both increase with the firm's gross profits (inclusive of managerial compensation). Consequently, output and/or pricing decisions maximize the firm's gross profits, that is, it is irrelevant whether the owners or the manager make the output and/or pricing decisions, since either one makes the decision that maximizes gross profits. To simplify the exposition, we assume this result in the following.

In making its output and pricing decisions, the firm anticipates the demand schedule (2). Further, each firm takes the aggregate price index P and the utility U of the representative consumer as given in making its output and pricing decisions. If the price of the firm's product is p, let $q(p)$ denote the demand for the product as given by (2). If the firm's realized productivity is ϕ, the price $p(\phi)$ set by the firm maximizes its gross profit, that is, it solves

$$p(\phi) = \arg\max_{p} pq(p) - w\frac{q(p)}{\phi}, \tag{7}$$

where w is the constant labor wage rate. We hereafter normalize w to 1, that is, the labor wage rate is the numeraire with respect to which all payoffs are measured. It follows immediately from (2) and (7) that the optimal price set by the firm is (since $w = 1$)

$$p(\phi) = \frac{1}{\rho\phi}. \tag{8}$$

The firm's gross profit for the period if its realized productivity is ϕ is therefore given by

$$\begin{aligned} \pi(\phi) &= p(\phi)q(\phi) - x(\phi) \\ &= r(\phi)/\sigma, \end{aligned} \tag{9}$$

where $r(\phi)$ is the firm's revenue that is given by

$$r(\phi) = R\left(P\rho\phi\right)^{\sigma-1}. \tag{10}$$

From (9) and (10), the firm's gross profit is

$$\pi(\phi) = \frac{R\left(P\rho\phi\right)^{\sigma-1}}{\sigma}. \tag{11}$$

3.4 Managerial Preferences and Contracts

Each manager is risk-averse and is protected by limited liability. Because the ex post productivity ϕ is observable and is the only source of randomness, we can, without loss of generality, assume that each manager's contractual compensation is contingent on the realized productivity, which is denoted by $t(\phi)$. That is, $t(\phi)$ represents the compensation the manager receives in excess of the labor wage rate, which (we recall) is normalized to 1. As mentioned earlier, since firms compete monopolistically, they make output and pricing decisions taking the aggregate price index P as given. Because managers and firms do not internalize the effects of their decisions on the aggregate economy, a manager's compensation contract is only contingent on the realized productivity of her firm.

Let $\overline{u}$ denote a manager's von Neumann-Morgenstern utility function over monetary payoffs. If a manager exerts effort e, her expected period utility from a given compensation contract $t(.)$ is

$$E_{\alpha,e}\left[\overline{u}(1+t(\phi))\right] - \kappa(e) = e\overline{u}\left[1+t(\alpha h))\right] + (1-e)\overline{u}\left[1+t(\alpha l)\right] - \kappa(e) \tag{12}$$

where $\kappa(.)$ is the strictly increasing and convex disutility of effort, both of which are common across managers. Define

$$u(x) = \overline{u}(1+x) - \overline{u}(1). \tag{13}$$

Hereafter, we simply refer to u as the manager's utility function. We assume that $u(.)$ and $\kappa(.)$

are twice continuously differentiable with $\kappa(0) = \kappa'(0) = 0$. In addition, we assume that $\kappa''(e)$ is increasing. Let $v(.) \equiv u^{-1}(.)$ denote the inverse of the utility function.

As in the traditional principal-agent literature, it is convenient to augment the definition of a manager's contract to also include her effort. In this case, the contract is required to be *incentive compatible* or *implementable* with respect to the manager's effort; that is, a contract $(t(.), e)$ is incentive compatible for the manager if and only if

$$e = \arg\max_{\tilde{e}} E_{\alpha,\tilde{e}}\left[u[t(\phi)]\right] - \kappa(\tilde{e}). \tag{14}$$

A contract $(t(.), e)$ is *feasible* if and only if it is incentive compatible and satisfies the following constraints that we hereafter refer to as the "limited liability" constraints for the manager:

$$t(\alpha h) \geq 0;\ t(\alpha l) \geq 0. \tag{15}$$

The constraints (15) ensure that the manager receives at least the production wage of 1 in each state. Since any agent who is not a manager becomes a production worker who earns a wage of 1, it follows from (13) and (15) that a feasible contract guarantees the manager a reservation expected utility payoff of zero.[2]

The firm chooses the manager's contract to maximize its expected *net* profit, that is, its expected gross profit less the manager's compensation. The manager's contract $(t^*_\alpha(.), e^*_\alpha(.))$ (the subscript denotes dependence on the firm quality) therefore solves

$$\begin{aligned} (t^*_\alpha(.), e^*_\alpha(.)) &= \arg\max_{(t(.),e)} E_{\alpha,e}\left[\pi(\phi) - t(\phi)\right] \\ &= \arg\max_{(t(.),e)} E_{\alpha,e}\left[\frac{R\,(P\rho\phi)^{\sigma-1}}{\sigma} - t(\phi)\right], \end{aligned} \tag{16}$$

subject to the implementability constraint (14) and the limited liability constraints (15). The second equality in (16) follows from the expression (11) for gross profit.

[2] We can modify the model to allow for managers to have nonzero reservation utilities without altering our main implications. Managerial compensation would simply have a lower bound that depends on the reservation utility.

3.5 Aggregate Variables

An equilibrium is characterized by a mass M of firms (and hence M goods), a distribution $\mu(\alpha)$ of match qualities over $(0, \infty)$, and a contract $(t^*_\alpha(.), e^*_\alpha(.))$ for the manager with firm quality α. Since there is a continuum of firms, and the productivity shocks are independent across firms (implying that there is no aggregate uncertainty), the aggregate price defined by (4) is constant over time. By (8), it is given by

$$\begin{aligned} P &= \left[\int_0^\infty E_{\alpha, e^*_\alpha} \left(p(\phi)^{1-\sigma} \right) M\mu(\alpha) d\alpha \right]^{\frac{1}{1-\sigma}} \\ &= \left[\int_0^\infty \left(e^*_\alpha p(\alpha h)^{1-\sigma} + (1 - e^*_\alpha) p(\alpha l)^{1-\sigma} \right) M\mu(\alpha) d\alpha \right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{17}$$

In (17), the expectation appears inside the integral by the (generalized) law of large numbers for a continuum of firms, that is, the sum of the prices charged by the continuum of firms with firm quality α is replaced by the expected price charged by a firm with firm quality α multiplied by the mass of firms with firm quality α.

We note that the aggregate revenue (or expenditure) and profit are given by

$$R = \int_0^\infty E_{\alpha, e^*_\alpha} \left(r(\phi) \right) M\mu(\alpha) d\alpha; \tag{18}$$

$$\Pi = \int_0^\infty E_{\alpha, e^*_\alpha} \left(\pi(\phi) \right) M\mu(\alpha) d\alpha, \tag{19}$$

where $r(.)$ and $\pi(.)$ are given by (10) and (11), respectively.

By Section 3.1, *consumer welfare* (i.e., the utility of the representative consumer) satisfies $U = \frac{R}{P}$. Because aggregate revenue R is exogenous, consumer welfare *decreases* with the aggregate price P. Thus, all results presented henceforth about the aggregate price directly translate to consumer welfare implications.

We examine stationary equilibria in which the aggregate variables remain constant over time. Since match qualities do not change over time, the expected profit earned by a firm in each period is constant. An entering firm with firm quality α earns expected gross profit of $E_{\alpha, e^*} [\pi(\phi)]$ in each period, where e^* is the effort exerted by the manager in each period that is also constant over time.

As will be shown later, a firm's expected net profit in each period is positive so that a surviving firm produces in every period. A firm's value function is therefore

$$b(\alpha) = \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t E_{\alpha,e_\alpha^*}\left[\pi(\phi) - t_\alpha^*(\phi)\right]\right\} = \max\left\{0, \frac{1}{\delta}\xi_\alpha\right\}, \tag{20}$$

where

$$\begin{aligned} \xi(\alpha) &= E_{\alpha,e_\alpha^*}\left[\pi(\phi) - t_\alpha^*(\phi)\right] \\ &= e_\alpha^*\left[\pi(\alpha h) - t_\alpha^*(\alpha h)\right] + (1-e_\alpha^*)\left[\pi(\alpha l) - t_\alpha^*(\alpha l)\right] \end{aligned} \tag{21}$$

is the firm's *expected net profit* (net of the manager's compensation) in each period. Since a firm produces regardless of its firm quality, we have that the distribution of active firms equals the distribution of match qualities:

$$\mu(\alpha) = g(\alpha). \tag{22}$$

It follows from (20) that the parameter δ also plays the role of a time discount factor. We choose this interpretation of the parameter δ in our empirical analysis in Section 5.

3.6 Equilibrium

In equilibrium with free entry of firms, the expected net profit earned by entering firms must equal the entry cost f_e:

$$\int_0^\infty b(\alpha)g(\alpha)d\alpha = \int_0^\infty \frac{1}{\delta}\xi(\alpha)g(\alpha)d\alpha = f_e. \tag{23}$$

Define the equilibrium *expected* incentive compensation of a manager with firm quality α

$$\widehat{t}(\alpha) = e_\alpha^* t_\alpha^*(\alpha h) + (1-e_\alpha^*)t_\alpha^*(\alpha l), \tag{24}$$

and the equilibrium *expected* gross profit of the firm

$$\widehat{\pi}(\alpha) = e_\alpha^*\pi(\alpha h) + (1-e_\alpha^*)\pi(\alpha l) = \frac{e_\alpha^* r(\alpha h) + (1-e_\alpha^*)r(\alpha l)}{\sigma} = \frac{\widehat{r}(\alpha)}{\sigma}, \tag{25}$$

where the second equality follows from (9) and $\widehat{r}(\alpha)$ is the equilibrium expected revenue of the firm.

By (21) and (23), we have

$$\int_0^\infty \frac{\widehat{r}(\alpha)}{\sigma} g(\alpha)d\alpha - \int_0^\infty \widehat{t}(\alpha)g(\alpha)d\alpha = \delta f_e. \tag{26}$$

By (11), (21), and (24), the equilibrium expected net profit of a firm with firm quality α is

$$\xi(\alpha) = \frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma}\left[e^*_\alpha h^{\sigma-1} + (1-e^*_\alpha)l^{\sigma-1}\right] - \widehat{t}(\alpha). \tag{27}$$

The following lemma describes the optimal contract for a manager with firm quality α.

Lemma 1 (Managerial Contracts) *Consider a firm with quality α.*

- *The incentive compensation of the manager in the low and high states is*

$$t^*_\alpha(\alpha l) = 0; \quad t^*_\alpha(\alpha h) = v\left[\kappa'\left(e^*_\alpha\right)\right]. \tag{28}$$

- *The manager's effort is $e^*_\alpha = 1$ if*

$$\alpha^{\sigma-1}\left[\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\left(h^{\sigma-1} - l^{\sigma-1}\right)\right] \geq v\left[\kappa'\left(1\right)\right] + v'\left[\kappa'\left(1\right)\right]\kappa''\left(1\right); \tag{29}$$

otherwise, it solves

$$\alpha^{\sigma-1}\left[\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\left(h^{\sigma-1} - l^{\sigma-1}\right)\right] = v\left[\kappa'\left(e^*_\alpha\right)\right] + e^*_\alpha v'\left[\kappa'\left(e^*_\alpha\right)\right]\kappa''\left(e^*_\alpha\right). \tag{30}$$

- *The manager's pay-performance sensitivity (PPS) is*[3]

$$PPS(\alpha) = \frac{v\left[\kappa'\left(e^*_\alpha\right)\right]}{v\left[\kappa'\left(e^*_\alpha\right)\right] + e^*_\alpha v'\left[\kappa'\left(e^*_\alpha\right)\right]\kappa''\left(e^*_\alpha\right)}. \tag{31}$$

As in a standard moral hazard model (Laffont and Martimort, 2002), it follows from (28) and (30) that the manager's optimal incentive compensation and effort depend on the difference between gross profit in the high and low states. From (30), the manager's optimal effort depends on the aggregate price index P. This follows from the fact that, by (10) and (11), the revenue and profit in

[3] We define the pay-performance sensitivity (PPS) as the ratio of the (dollar) difference in the manager's compensation in the high and low states to the (dollar) difference in the firm's output.

each productivity state depends on the aggregate price index. Consequently, the marginal product of managerial effort is affected by the aggregate price index. As we discuss in more detail later, the effect of the aggregate price index on managerial effort leads to a "general equilibrium" link between firm and product market characteristics and managerial compensation.

The following lemma establishes the existence and uniqueness of a stationary equilibrium.

Lemma 2 (Existence and Uniqueness of the Equilibrium) *There exists a unique stationary equilibrium characterized by the aggregate price P satisfying the following equation:*

$$\int_0^\infty \left[\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma} \left[e_\alpha^*(P)h^{\sigma-1} + (1-e_\alpha^*(P))l^{\sigma-1}\right] - e_\alpha^*(P)v' \left[\kappa'\left(e_\alpha^*(P)\right)\right] \right] g(\alpha)d\alpha = \delta f_e, \tag{32}$$

where we explicitly indicate the dependence of the managers' effort on P.

The equilibrium condition (32) ensures that each entering firm's expected future profit, which rationally incorporates each manager's contract and effort, is equal to the entry cost.

By (18), the mass of firms M is given by

$$M = \frac{R}{\int_0^\infty E_{\alpha, e_\alpha^*}\left(r(\phi)\right) g(\alpha)d\alpha} = \frac{R}{\int_0^\infty \widehat{r}(\alpha)g(\alpha)d\alpha} = \frac{R}{\overline{r}}, \tag{33}$$

where we recall that $\widehat{r}(\alpha)$ is the expected revenue of a firm with firm quality α and $\overline{r}$ is the average expected revenue of active firms. The mass of firms therefore declines with the average expected revenue of active firms. By (26), any factor that increases the equilibrium average expected compensation of managers also increases the average expected revenue of active firms and, therefore, decreases the equilibrium mass of firms.

4 Properties of the Equilibrium

For ease of exposition, we use the following terminology. The *average* value of a firm-specific variable refers to the average across all firms computed with respect to the firm quality distribution $g(.)$. The *expected* value of a variable that is contingent on the realized value of a firm's productivity shock is its expectation computed with respect to the productivity distribution. The *average expected* *value* of a firm-specific variable that is contingent on a firm's realized productivity is computed by taking its expected value for each firm and then determining its average across all firms.

4.1 The Distributions of Firm Size and Managerial Compensation

There are two factors contributing towards firm heterogeneity, which in turn generate a firm size distribution (FSD) and a distribution of managerial compensation (levels and incentives): firm quality and the productivity shocks, which affect the distributions indirectly through their effects on managerial effort.

Proposition 1 (Match Quality, Firm Size, and Compensation)

- *Managerial effort, expected managerial compensation, expected revenue, expected gross and net profit, and the expected labor force increase monotonically with firm quality α.*
- *Pay-performance sensitivity declines with firm quality if and only if*

$$F(e) = \frac{ev'\left[\kappa'(e)\right]\kappa''(e)}{v\left[\kappa'(e)\right]} \text{ is monotonically increasing in effort } e. \tag{34}$$

By Proposition 1, managers of firms with which they enjoy higher quality matches exert greater effort, generate greater expected revenues, gross, and net profits, and receive greater expected compensation. Consequently, firm size (measured in terms of expected revenue, gross/net profits or labor force) increases with the manager-firm firm quality. The underlying intuition is that, because firm quality and effort are complements, managers with higher quality matches optimally exert greater effort.

The function $F(e)$ is the elasticity with respect to effort of the agent's compensation in the high state. Because effort increases with firm quality, a greater elasticity with respect to effort for a manager of a higher quality firm ensures that she has a lower pay-performance sensitivity. The condition (34) holds, for example, if the utility function $u(.)$ is CRRA and the effort cost function $\kappa(.)$ has a power or exponential form.

4.2 The Effects of the Firm Quality Distribution

The following proposition shows the effects of a change in the firm quality distribution g in the sense of first-order stochastic dominance (FOSD).

Proposition 2 (FOSD Change in Match Quality Distribution) *Let g_1 and g_2 be two firm quality distributions where g_1 first-order stochastically dominates g_2. (i) The aggregate price index corresponding to distribution g_1 is lower than that corresponding to g_2. (ii) For a firm with quality α, managerial effort, expected managerial compensation, expected revenue, expected gross and net profit, and the expected labor force are lower when the firm quality distribution is g_1, while the manager's pay-performance sensitivity is greater provided condition (34) holds.*

Consider the impact of an FOSD increase in the firm quality distribution. As the firm quality distribution shifts to the right, the left-hand side (LHS) of (32) increases for a given value of the aggregate price index P. Therefore, viewed as a function of P, the entire curve representing the LHS of (32) shifts upward. It immediately follows that the aggregate price index decreases. The underlying intuition is that higher quality firms are more efficient; that is, they require less labor to manufacture their product. A more efficient firm produces a cheaper product at the optimum. In equilibrium, therefore, the more efficient is the distribution of active firms, the lower is the aggregate price index. The implications from (28) and (30) are that, for a given firm quality α, the manager exerts lower effort, receives lower expected compensation, and each firm's size (measured in terms of expected revenues, gross profit, net profit, or the labor force) is lower in response to an FOSD increase in the firm quality distribution. If condition (34) holds, the manager's pay-performance sensitivity is greater.

The average (across all managers) expected compensation is affected by two conflicting forces. On the one hand, the manager of a firm with given quality exerts lower effort and therefore receives lower expected compensation. On the other hand, an FOSD increase in the firm quality distribution leads to a larger number of higher quality firms. The effect on average expected revenue and the mass of firms depends on which effect dominates. If the "FOSD" effect dominates, average firm size increases and the mass of firms decreases. If the "managerial compensation" effect dominates, average firm size decreases and the mass of firms increases.

Shocks to the firm quality distribution could be viewed as aggregate shocks because they affect all firms. In this respect, Proposition 2 implies that, in a general equilibrium setting, managerial incentives are affected by aggregate shocks through their effects on the aggregate price index. This prediction contrasts sharply with the predictions of traditional "partial equilibrium" principal–agent models in which incentives are only affected by idiosyncratic, firm–specific shocks.

While the effects of an FOSD change in the firm quality distribution on incentives and firm size can be pinned down, the effects of second-order stochastic dominance (SOSD) are, in general, ambiguous. The integrand on the LHS of (32) could be convex, concave, or neither convex nor concave in the firm quality α. Consequently, an SOSD shift in the firm quality distribution could increase or decrease the LHS of (32) for a given aggregate price index. The implication is that the equilibrium aggregate price index could increase or decrease so that the effects on managerial incentives, effort, and firm size are ambiguous.

4.3 The Effects of the Entry Cost and Exit Probability

We now describe the effects of the entry cost f_e and exit probability δ. As one would expect, the aggregate price declines and welfare is enhanced by a reduction in the entry cost or exit probability.

Proposition 3 (Entry Cost, Exit Probability, and the Aggregate Price) *The aggregate price index (consumer welfare) increases (decreases) with the entry cost f_e and exit probability δ.*

An increase in the entry cost or exit probability increases the expected net profit that an entering firm must earn by the equilibrium condition (32). Because an increase in the aggregate price has a positive effect on managerial effort and expected net profit by (30), the equilibrium aggregate price increases (and thereby consumer welfare decreases) with the entry cost and the exit probability.

The following proposition shows the effects of the entry cost and exit probability on firm size and managerial compensation.

Proposition 4 (Entry Cost, Exit Probability, Firm Size, and Compensation) *Managerial effort, expected managerial compensation, expected revenue, expected gross and net profit, and the expected labor force increase with the entry cost and exit probability, while managerial pay-performance sensitivity declines provided condition (34) holds.*

An increase in the entry cost or exit probability raises the equilibrium aggregate price index by Proposition 3. An increase in the price index enhances the return on managerial effort, such that managerial effort increases. Because managers exert greater effort, they receive greater expected compensation. The increase in managerial effort raises the expected productivity of a firm, enhancing its expected revenue and gross profit. The increase in expected gross profit is sufficiently large to offset the increase in expected managerial compensation; thus, expected net profit also increases. Since the size of the labor force is proportional to revenue, the firm's expected labor force also increases. Therefore, regardless of whether firm size is measured in terms of revenue or the size of the labor force, an increase in the entry cost leads to larger firms. Condition (34) ensures that, because effort increases with the entry cost or exit probability, managerial pay-performance sensitivity (PPS) declines.

Proposition 5 (Entry Cost, Exit Probability, and the Mass of Firms) *The equilibrium mass of firms declines with the entry cost and exit probability.*

An increase in the entry cost or exit probability dampens competition, increases the expected revenue of each active firm and, therefore, the average expected revenue of all firms. Because total revenue is fixed at R, the equilibrium mass of firms declines.

As illustrated by the intuition for the above propositions, the entry cost and exit probability affect the distributions of firms and managerial compensation *indirectly* through their effects on the equilibrium aggregate price index by the equilibrium condition (32). Consequently, their effects arise via the general equilibrium channel in the model. As we discuss in the next sub-section, this contrasts with the effects of another dimension of competition among firms—the elasticity of product substitution.

4.4 The Effects of the Elasticity of Substitution

We now investigate the effects of the elasticity of substitution σ between any pair of products, which equals the price elasticity of demand faced by each monopolistic firm in equilibrium. The greater is σ, the more substitutable (and thereby price elastic) are the products being offered by the monopolists. The following proposition describes the effects of marginal changes in the elasticity of substitution on firm size, managerial compensation, and the mass of firms.

Proposition 6 (Elasticity of Substitution, Firm Size, Compensation, and the Mass of Firms) *There exists a threshold firm quality $\alpha_T(\sigma)$ with the following properties.*

$$\begin{aligned} \frac{\partial e_\alpha^*(\sigma)}{\partial \sigma} &> 0,\ \frac{\partial \widehat{t}_\alpha(\sigma)}{\partial \sigma} > 0,\ \frac{\partial PPS_\alpha(\sigma)}{\partial \sigma} < 0 \text{ for } \alpha > \alpha_T(\sigma), \\ \frac{\partial e_\alpha^*(\sigma)}{\partial \sigma} &< 0,\ \frac{\partial \widehat{t}_\alpha(\sigma)}{\partial \sigma} < 0,\ \frac{\partial PPS_\alpha(\sigma)}{\partial \sigma} > 0 \text{ for } \alpha < \alpha_T(\sigma), \end{aligned} \tag{35}$$

where $e_\alpha^*(\sigma)$ is the equilibrium effort, $\widehat{t}_\alpha(\sigma)$ is the equilibrium expected compensation, and $PPS_\alpha(\sigma)$

is the pay-performance sensitivity of the manager with firm quality α, and condition (34) holds. The average expected revenue and average expected gross profit of all active firms, as well as the mass of firms, may increase or decrease with σ.

An increase in the elasticity of substitution affects managerial effort and output differentially. It causes managers of higher quality firms to benefit relatively more from exerting greater effort compared to managers of lower quality firms. Specifically, the left-hand side of (30) increases (decreases, respectively) with σ when the firm quality is above (below) a threshold α_T. Hence, managerial effort increases (decreases) when the firm quality is above (below, respectively) the threshold. As with our earlier results, condition (34) ensures that managerial pay-performance sensitivity moves in the opposite direction relative to effort.

Raith (2003) identified two mechanisms by which the extent of product substitutability (modeled by the transportation cost incurred by a consumer traveling to purchase from a firm) affects managerial incentives: the business-stealing and scale effects. The business-stealing effect is that, when demand is more elastic (i.e., products are more substitutable), a firm with higher productivity can more easily attract business from its rivals. The scale effect is that a firm whose rivals charge lower prices loses market share and thus has less to gain from being more productive. In Raith (2003), for a fixed number of firms, an increase in competition due to greater product substitutability has no effect on managerial incentives: the business stealing and scale effects exactly cancel out. As shown by Proposition 6, this is not the case in our model.

The following proposition examines the effects of (marginal) changes in the elasticity of substitution on the aggregate price index and, therefore, consumer welfare.

Proposition 7 (Elasticity of Substitution and the Aggregate Price) *There exist threshold levels $f'_T(\sigma)$ and $\delta'_T(\sigma)$ of the entry cost and exit probability, respectively, such that $\frac{\partial P^*(\sigma)}{\partial \sigma} > 0$ if $f_e < f'_T(\sigma)$ or $\delta < \delta'_T(\sigma)$ and $\frac{\partial P^*(\sigma)}{\partial \sigma} < 0$ if $f_e > f'_T(\sigma)$ or $\delta > \delta'_T(\sigma)$, where $P^*(\sigma)$ is the equilibrium aggregate price index when the elasticity of substitution is σ.*

For a given aggregate price, if the entry cost and/or exit probability are below their respective (endogenous) thresholds, then the threshold firm quality defined in Proposition 6 is high enough that a relatively large proportion of managers *decrease* their effort and output as the elasticity of substitution increases. The equilibrium condition (32) then implies that the *equilibrium* aggregate price increases.

On the other hand, if the entry cost and/or exit probability are above their respective thresholds, then the threshold firm quality defined in Proposition 6 is low enough that a relatively large proportion of managers increase their effort and output, so that the equilibrium aggregate price declines in response to an increase in the elasticity of substitution.

The intuition for the above propositions shows that, in contrast with the entry cost and exit probability, the elasticity of product substitution has direct and indirect effects on the distributions of firms and managerial compensation. The direct effects arise from the effects of the elasticity of product substitution on managerial effort and output. The entry cost, exit probability and the elasticity of product substitution are all determinants of competition among firms. The results of Sections 4.3 and 4.4 show that differing determinants of competition have contrasting effects on the firm size distribution (FSD), managerial compensation and incentives. Our analysis, therefore, suggests that empirical analyses of the effects of competition on the FSD and incentive compensation should appropriately account for differing dimensions of competition.

4.5 The Effects of Productivity Risk

The notion of "risk" quantifies the variability in a firm's output. By (11), the spread in the firm's gross profits in the "high" and "low" states increases with the quantity $h^{\sigma-1} - l^{\sigma-1}$. Accordingly, we define the firm's productivity risk as $x = h^{\sigma-1} - l^{\sigma-1}$. It is determined by the spread in realized productivities in the "high" and "low" states. In the following propositions, we examine the effects of changing the productivity risk keeping $h^{\sigma-1} + l^{\sigma-1}$ fixed. The following proposition derives the effects of productivity risk on firm size and managerial compensation.

Proposition 8 (Productivity Risk, Firm Size, and Compensation) *(i) Managerial effort and expected managerial compensation increase with productivity risk, while managerial pay-performance sensitivity declines if condition (34) holds. (ii) The average expected revenue and average expected gross profit of active firms increase with productivity risk.*

By (30), an increase in productivity risk raises the marginal product of managerial effort, thereby increasing managerial effort at the optimum, which in turn enhances expected compensation. Because managerial effort increases with productivity risk, if condition (34) holds, the pay-performance sensitivity (PPS) declines with productivity risk.

The relationship between PPS and productivity risk is negative provided the elasticity condition (34) on the utility function holds, but is *positive* if it does not hold. This result provides a simple explanation for the tenuous relation between risk and incentives in the empirical literature (Prendergast, 2002). Some empirical studies find that the link is positive (Rajgopal et al., 2006); insignificant (Conyon and Murphy, 2000); and negative (Aggarwal and Samwick, 1999a). The theoretical prediction of a negative relation between risk and incentives is unambiguously obtained only if the agent is assumed to have CARA preferences (see Prendergast, 2002). Our study shows that, under more general utility functions, the relation between PPS and risk as well as the relations between PPS and other variables such as the entry cost, exit probability and the elasticity of product substitution crucially depend on the additional elasticity condition (34) on the utility function. The parameters of the utility function are "deep" structural parameters that are unknown to the econometrician and for which it is difficult to find reasonable proxies. Consequently, our analysis suggests that empirical investigations of the determinants of PPS are likely to be misspecified.

Note that, while the average expected revenue and average expected gross profit of active firms increase with productivity risk, the effects of productivity risk on the expected revenue and profit of a *particular* firm are ambiguous in general. The ambiguity arises due to the fact that, for a firm with firm quality α, an increase in productivity risk has a negative effect on productivity (and, by extension, output) in the "low" state αl. Furthermore, as shown by the following proposition, productivity risk may increase or decrease the aggregate price depending on the values of the entry cost and exit probability.

Proposition 9 (Productivity Risk and the Aggregate Price) *There exist threshold levels $f_T(x)$ and $\delta_T(x)$ of the entry cost and exit probability, respectively, such that $\frac{\partial P^*(x)}{\partial x} > 0$ if $f_e < f_T(x)$ or $\delta < \delta_T(x)$ and $\frac{\partial P^*(x)}{\partial x} < 0$ if $f_e > f_T(x)$ or $\delta > \delta_T(x)$, where $P^*(x)$ is the equilibrium aggregate price index when the productivity risk is x. Note that, in computing the partial derivatives above, the quantity $h^{\sigma-1} + l^{\sigma-1}$ is kept fixed.*

The above proposition shows that an increase in productivity risk may lower the aggregate price index and, thereby, enhance consumer welfare. If the entry cost or exit probability is above a threshold, then an increase in $x = h^{\sigma-1} - l^{\sigma-1}$ raises consumer welfare, while if it is below a threshold, consumer welfare is lowered. The intuition for these results hinges on the following key observations. First, by Proposition 3, the aggregate price increases with the entry cost and the exit probability. Second, by

Proposition 8, an increase in productivity risk increases managerial effort *and* expected compensation. Third, in equilibrium, the aggregate price must satisfy (32), where the left-hand side is the average expected *net profit* of active firms. When the entry cost or exit probability is below a threshold, the aggregate price is low. In this scenario, the increase in average expected managerial compensation (due to the rise in productivity risk) more than offsets the increase in average expected gross profit, so that the average expected net profit of active firms *decreases.* Hence, by (32), the aggregate price increases (and welfare decreases). When the entry cost or exit probability is above the threshold, the aggregate price is such that the increase in the average expected gross profit (due to the rise in productivity risk) outweighs the increase in average expected managerial compensation. As a result, the average expected net profit of active firms increases so that the aggregate price decreases (and welfare increases).

By Proposition 8, an increase in productivity risk raises the average expected revenue of active firms. Because aggregate revenue R is fixed by the size of the market (e.g., the number of consumers or the sum of their disposable income), the equilibrium mass of firms declines with productivity risk.

Proposition 10 (Productivity Risk and the Mass of Firms) *The equilibrium mass of firms decreases with productivity risk.*

4.6 The Effects of the Productivity Level

Consider the effects of changes in the *productivity level* of active firms, while holding constant productivity risk $x = h^{\sigma-1} - l^{\sigma-1}$. It is natural to interpret the productivity level as the value of the quantity $l^{\sigma-1}$. Since productivity risk is kept fixed, we are essentially examining how an increase in $l^{\sigma-1}$ and $h^{\sigma-1}$ *by the same amount* affects the variables of interest. We show that an increase in the productivity level makes it more attractive for firms to enter the market, which raises the extent of competition and thereby lowers the equilibrium aggregate price (and correspondingly raises welfare, as one would expect). These responses serve to dampen the incentive to exert effort. To counteract these effects, the principal raises PPS (provided condition (34) holds). The decline in effort and rise in competition resulting from an increase in the productivity level diminishes the payoff to incumbents, such that managerial compensation and the average expected revenue and average expected gross profit of active firms decline.

Proposition 11 (Effects of the Productivity Level) *(i) Managerial effort, expected managerial compensation, the average expected revenue of active firms, and the average expected gross profit of active firms decrease with the productivity level, while managerial pay-performance sensitivities increase (provided condition (34) holds). (ii) The mass of firms increases with the productivity level. (iii) The equilibrium aggregate price index (consumer welfare) decreases (increases) with the productivity level.*

The effects of the productivity level described by Proposition 11 are the direct consequence of our general equilibrium framework in the sense that they arise because the aggregate price index and the mass of firms are endogenously determined. In particular, the effects do not arise in a partial equilibrium setting. More precisely, suppose the aggregate price index is fixed. It follows directly from Lemma 1 that an increase in the productivity level, keeping productivity risk fixed, has *no effect* on the compensation, effort, and incentives of a manager. Furthermore, by (25) and (27), each firm's expected revenue and gross and net profit increase with the productivity level. Hence, the average expected revenue and average expected gross profit of active firms *increase* with the productivity level, which is exactly the opposite behavior predicted by Proposition 11. The general equilibrium effects arise from the fact that an increase in the productivity level attracts entry, which raises the mass of firms and lowers the aggregate price.

As with the result of Proposition 2, Proposition 11 shows that general equilibrium effects lead to a relationship between aggregate shifts in productivity that affect all firms and incentives. This prediction is in sharp contrast to traditional partial equilibrium principal–agent models in which incentives are only affected by idiosyncratic, firm–specific shocks.

5 Empirical Analysis

We now test the main robust predictions of the model that relate to managerial compensation *levels* and the number of competing firms.[4]

5.1 Empirical Hypotheses

From Proposition 1, we have the following hypothesis:

Hypothesis 1 *Managerial compensation increases with firm quality.*

[4] As discussed in the previous section, the predictions relating to managerial pay-performance sensitivities are not robust because they hinge on the elasticity condition (34) that cannot be directly verified in the data.

Proposition 4 suggests the following hypothesis:

Hypothesis 2 *Managerial compensation increases with the cost of entry of the industry.*

From Proposition 5, we have the following hypothesis:

Hypothesis 3 *The number of firms decreases with the cost of entry of the industry.*

As discussed in Section 3.5, the parameter δ in the model can also be interpreted as a time discount rate for firms. Accordingly, Proposition 4 leads to the following hypothesis:[5]

Hypothesis 4 *Managerial compensation increases with the discount rate of firms in the industry.*

From Proposition 5, we have the following:

Hypothesis 5 *The number of firms decreases with the discount rate of firms in the industry.*

By Proposition 6, expected managerial compensation increases with the elasticity of product substitution if firm quality is above a threshold, but decreases with the elasticity of product substitution if firm quality is below the threshold. By Proposition 1, firm size increases with firm quality. Consequently, we have the following testable hypothesis:

Hypothesis 6 *Managerial compensation increases with the product substitutability of the industry for large firms and decreases for small firms.*

Proposition 8 suggests two hypotheses:

Hypothesis 7 *Managerial compensation increases with the productivity risk of the industry.*

Hypothesis 8 *The number of firms decreases with the productivity risk of the industry.*

From Proposition 11, we have two hypotheses:

Hypothesis 9 *Managerial compensation decreases with the productivity level of the industry.*

Hypothesis 10 *The number of firms increases with the productivity level of the industry.*

Note that, with the exception of Hypothesis 1, all our hypotheses relate *industry-level* variables—the entry cost, discount rate, product substitutability, productivity risk, and productivity level—to managerial compensation and the number of firms.

[5] There is insufficient entry and exit in our dataset to calculate a direct measure of an industry's failure risk. Consequently, we interpret the parameter δ in the model as a time discount rate rather than an exit probability in our empirical analysis.

5.2 Sample Selection and Variable Construction

We obtain CEO compensation data from ExecuComp, financial data from Compustat, and stock return data from CRSP. We delete firm-year observations in which CEOs do not work for a full fiscal year. We also eliminate firms with negative book values of total equity to ensure that we can construct meaningful market-to-book ratios. As is standard in the literature, we exclude firms in the financial services industry since the regulatory and financial environments of such firms are significantly different from other industries. Further, the firms in our model are conventional firms that produce goods for consumption rather than financial intermediaries.

To construct the industry-level variables, we define an industry at the 2-digit SIC level. Our results are robust to grouping firms by the Fama-French 48 industry classification. Given that the majority of our variables are at the industry level, we only retain observations in industries that have at least five firms each year. (Our results are robust to retaining industry observations that have at least two firms each year.) To remove the effects of outliers, we winsorize all continuous variables at the 1% and 99% levels. Our final sample includes 16,012 firm-year observations with available data over the period 1992 to 2006. All dollar items are converted into 2006 dollars using the GDP deflator index from the Bureau of Economic Analysis.

To test our hypotheses regarding managerial compensation, the dependent variable $TOTALPAY$ is the log of the total compensation of CEOs, including salary, bonus, stock options, restricted stocks, and other long-term incentives. We compute the value of stock options using the Black-Scholes formula. The distribution of the dollar amount of total compensation is skewed, so we take the log to obtain $TOTALPAY$. To test our hypotheses regarding the number of firms, the dependent variable is the number of firms in the industry ($NFIRMS$).

$TARANK$ is the scaled rank of the firm in the industry by size, wherein firm size is measured by total assets. It is our proxy for the firm quality between a firm and its CEO since firms with high firm quality have a larger size according to Proposition 1 and in agreement with Gabaix and Landier (2008) and Tervio (2008).[6] We use the firm's rank in the size distribution, rather than the size itself, because we interpet our model as one of competing firms in a particular industry so that the relevant variable for a firm is its position relative to others in the same industry. Based on Proposition 1, and

[6] It is also reasonable to expect that firms with a high match quality tend to have superior financial performance such as return on assets (ROA). However, ROA is quite volatile, whereas the premise of the model is that match quality is relatively stable, so we do not use ROA to proxy for match quality.

as stated in Hypothesis 1, we expect a positive relation between *TARANK* and *TOTALPAY*.

All remaining variables are at the industry level. Similar to Karuna (2007), we employ two measures of the entry cost that capture the level of capital investment that each firm incurs to set up its operation to enter an industry. Our first measure of the entry cost, *ENCOST_PPE*, equals the log of the weighted average of property, plant, and equipment in the industry in that year. Our second measure, *ENCOST_EMPL*, equals the log of the gross value of property, plant, and equipment in the median plant in the industry in that year. By Hypothesis 2, we expect positive signs on our entry cost measures in the *TOTALPAY* regressions. By Hypothesis 3, we expect negative signs on our entry cost measures in the *NFIRMS* regressions.

We measure the discount rate of firms in an industry by the industry beta (*BETA*), which is the coefficient from a regression of annual stock returns on the equally weighted market return for firms in the industry. By Hypothesis 4, we expect a positive sign on the discount rate in the *TOTAL-PAY* regressions. By Hypothesis 5, we expect a negative sign on the discount rate in the *NFIRMS* regressions.

Most previous estimates of product substitutability in an industry have used the negative price-cost margin (e.g., Nevo, 2001). The higher is the degree of product substitutability of the industry, the greater is the price elasticity of demand, and the less is the price-cost margin. Hence, product substitutability is positively related to the *negative* of the price-cost margin. Consistent with Nevo (2001), we measure product substitutability by the negative value of industry sales divided by industry operating costs which include costs of goods sold, selling, general, and administrative expenses, and depreciation, depletion, and amortization. The model predicts that the impact on compensation of the elasticity of substitution depends on a threshold firm quality. Therefore, we define the dummy *LOW* to equal 1 if the firm has a value of *TARANK* less than or equal to 0.33, and 0 otherwise; similarly, we define the dummy *HIGH* to equal 1 if the firm has a value of *TARANK* greater than or equal to 0.66, and 0 otherwise. By Hypothesis 6, we expect a positive sign on the interaction term *PSUB*HIGH* and a negative sign on the interaction term *PSUB*LOW* in the *TOTALPAY* regressions. Our results are robust to alternate definitions of the high and low cutoffs, that is, the dummy variables *HIGH* and *LOW*.

RISK is the time-series variance of the industry annual stock returns from 1992 to 2006, which is our proxy for the productivity risk of the industry. (Our results are robust to defining *RISK* by the time-series variance of return on assets (ROA)) By Hypothesis 7, we expect positive signs on *RISK* in the *TOTALPAY* regressions. By Hypothesis 8, we expect negative signs on *RISK* in the *NFIRMS* regressions.

We use two proxies for the productivity level of the industry. *AR* is the abnormal industry stock return, which is the intercept from the CAPM regression used to calculate *BETA*. *MB* is the weighted average of the market to book ratio for the industry. By Hypothesis 9, we expect negative signs on the two productivity level measures in the *TOTALPAY* regressions. By Hypothesis 10, we expect positive signs on the two productivity level measures in the *NFIRMS* regressions.

Finally, we include industry sales (*INDUSTRY_SALES*) as a control variable, which is calculated as the log of total sales of the industry.

5.3 Descriptive Statistics

Table 1 lists all the variables along with their definitions. Panel A of Table 2 shows descriptive statistics of *TOTALPAY*, *TARANK*, and the industry variables. The mean and median of CEOs' total compensation are \$4.33 million and \$2.26 million, respectively, suggesting the distribution of total compensation is skewed to the right. The log of total compensation has a mean of 7.780 and a median of 7.725, suggesting the distribution of the log transformation is closer to normal. The mean and median of the number of firms (*NFIRMS*) are 55.575 and 38, respectively. The log of *NFIRMS* is used in the regressions to obtain a reasonable scale for the coefficient estimates. The mean *ENCOST_PPE* is \$8.33 billion and the median is \$4.87 billion, which are similar to the sample in Karuna (2007). The log of *ENCOST_PPE* is less skewed and is used in the regression analyses. The mean and median of *PSUB* are -1.45 and -1.352, suggesting demand is inelastic (in the context of a Dixit-Stiglitz model, wherein the elasticity of substitution equals the price elasticity of demand). The mean market to book ratio of the industry (*MB*) is 4.138. The average of industry sales (*INDUSTRY_SALES*) is \$361 billion. We use the log of industry sales to avoid the skewness of the distribution. Panel B of Table 2 lists the industries and shows the distribution of firms across industries.

Table 3 presents the Pearson correlation coefficients. The correlation between *PSUB*HIGH* and *TARANK* is -0.678 due to the fact that *PSUB* is the negative value of the price-cost margin and *HIGH* is positively related to *TARANK*. We have a similar explanation for the correlation between *PSUB*LOW* and *TARANK*. The relatively high correlation of 0.620 between *INDUSTRY_SALES* and *ENCOST_PPE* indicates that larger industries usually have higher entry barriers, as one would expect. The correlation of 0.474 between *TOTALPAY* and *TARANK* suggests that executives are paid more at large firms, providing univariate supportive evidence for Hypothesis 1 and in agreement with Gabaix and Landier (2008) and Tervio (2008). The correlation between *AR* and *RISK* is 0.422, showing that the industry abnormal return is positively associated with time-series stock price volatility in the industry. The correlation between *RISK* and *ENCOST_PPE* is 0.333, showing that industries with high entry costs tend to have greater stock price volatility. The correlation between the two measures of the entry cost is 0.662.

To avoid multicollinearity problems, we are unable to include in the same regressions both the discount rate (*BETA*) and either measure of the entry cost of an industry (*ENCOST_PPE* or *ENCOST_EMPL*). With this restriction, the results show that the variance inflation factors are below the cutoff values of 10 and 30 respectively, as given in Belsley et al. (1980).

5.4 Regression Results

Table 4 presents the results of OLS regressions of CEO total compensation with year and industry fixed effects. The results for the six regressions are similar. Our empirical findings agree with five out of six hypotheses pertaining to executive compensation. Consistent with Hypothesis 1, along with Gabaix and Landier (2008) and Tervio (2008), the coefficients on *TARANK* are positive and significant, demonstrating that executives are rewarded more in firms with which they have a high firm quality (as measured by size). In agreement with Hypothesis 2, both measures of the cost of entry (*ENCOST_PPE* and *ENCOST_EMPL*) have positive and significant coefficients, indicating that CEO compensation is higher in industries with greater entry barriers. The positive and significant coefficients on *BETA* suggest that executive compensation is positively related to the discount rate of an industry and by extension the exit probability, which is consistent with Hypothesis 4. The coefficients on the two interaction terms *PSUB*HIGH* and *PSUB*LOW* are significantly positive and negative, respectively, providing supportive evidence for Hypothesis 6; that is, executive compensation increases (decreases) with the degree of product substitutability in an industry among large (small) firms. The coefficients on *RISK* are significant and positive in some regressions, providing some support for Hypothesis 7, indicating that executive compensation increases with productivity risk.

The positive and significant coefficients on the two proxies for the productivity level (*AR* and *MB*) are inconsistent with Hypothesis 9. Industries characterized by greater abnormal stock returns and market to book ratios tend to offer higher pay to their managers.

Table 5 reports the results for the number of firms in an industry. All regressions yield similar results, and all hypotheses pertaining to the number of firms are supported. As expected in Hypothesis 3, the significantly negative coefficients on $ENCOST\_PPE$ and $ENCOST\_EMPL$ show that the number of firms in an industry decreases with the entry costs of that industry. The significantly negative coefficients on $BETA$ provide supportive evidence for Hypothesis 5, which indicates that the number of firms decreases with the discount rate of firms in the industry. The significantly negative coefficients on $RISK$ show that the number of firms decreases with the productivity risk of the industry, in agreement with Hypothesis 8. The significantly positive coefficients on AR show the number of firms in an industry increases with the productivity level of the industry as proxied by its average abnormal return. The coefficients on MB are significantly positive in some regressions, but not all. Thus, taking both sets of results into account, there is support for Hypothesis 10.

5.5 Robustness Tests

We perform a number of robustness checks that do not alter our results. First, we classified firms based on the 48 Fama-French industries. Second, we tried different benchmarks for $HIGH$ and LOW including 0.1 and 0.25. Third, our results are robust to only including industries with 2 or more firms. Fourth, the results are similar for industry measures calculated by year as for those calculated for the entire sample period, which is consistent with the premise of the model that industry characteristics remained relatively unchanged throughout the years. Fifth, we obtain similar results for the $RISK$ measure defined by the time-series variance of return on assets (ROA).

6 Conclusion

Using a general equilibrium model, we show how the distribution of heterogeneous firm qualities, moral hazard, and monopolistic competition in the product market interact to affect the distributions of firm size and managerial compensation. We exploit the properties of the unique, stationary general equilibrium of the model to derive a number of novel implications for the relations between the firm size and managerial compensation distributions, and the effects of firm and product market characteristics on these distributions.

(i) Different determinants of competition have contrasting effects on firm size and managerial compensation. An increase in the intensity of competition due to a reduction in the entry cost or exit probability decreases expected managerial compensation and the average size of firms, but increases the number of active firms. An increase in the intensity of competition due to an increase in the elasticity of product substitution, however, decreases expected compensation if firm size is below an endogenous threshold, but increases expected compensation if firm size is above the threshold.

(ii) An increase in productivity risk raises expected managerial compensation and the number of active firms. Our results show that the relationship between PPS and risk crucially depends on an additional elasticity condition on the managers' utility function. Consistent with empirical findings, our study shows that, for general utility functions, the relation between PPS and risk is tenuous. We also show that productivity risk could increase or decrease consumer welfare depending on the levels of the entry cost and exit probability.

(iii) Our general equilibrium model also generates novel implications for the effects of aggregate shocks to the manager-firm firm quality distribution and firms' productivity levels. An increase in the firm quality distribution in the sense of "first order stochastic dominance" decreases expected managerial compensation. Expected managerial compensation, the average revenue of active firms, the average gross profit of active firms decrease with the productivity level, while the number of active firms increases.

We use our theoretical results to develop ten robust empirically testable hypotheses that relate industry characteristics—the entry cost, the exit probability, the elasticity of product substitution, the productivity risk, and the productivity level—to managerial compensation and the number of active firms. We show support for nine of the ten hypotheses in our empirical analysis.

Appendix

Proof of Lemma 1

We first derive the contract that implements a particular effort level e for the manager and has the least cost to the principal. If $t^e(\alpha h)$ and $t^e(\alpha l)$ denote the payoffs to the manager in the high and low states, respectively, then it follows from (12) that

$$\kappa'(e) = u\left[t^e(\alpha h)\right] - u\left[t^e(\alpha l)\right]. \tag{36}$$

Because the implementability of effort e only depends on the difference between the manager's utility payoffs in the high and low states, it easily follows from (15) that it is optimal for the principal to set $t^e(\alpha l) = 0$. Hence, by (36), we have

$$t^e(\alpha h) = u^{-1}\left[\kappa'(e)\right] = v\left[\kappa'(e)\right]. \tag{37}$$

By (16) and (37), the optimal effort e^*_α for a manager with firm quality α solves

$$e^*_\alpha = \arg\max_e e\alpha^{\sigma-1}\left[\frac{R(P\rho)^{\sigma-1}}{\sigma}\left(h^{\sigma-1} - l^{\sigma-1}\right)\right] + \alpha^{\sigma-1}\frac{R(P\rho)^{\sigma-1}}{\sigma}l^{\sigma-1} - ev\left[\kappa'(e)\right]. \tag{38}$$

By the first order condition for a maximum, the optimal effort satisfies (30) if condition (29) does not hold and is equal to 1 otherwise. By (37), the optimal compensation contract for the manager is given by (28). The manager's optimal pay-performance sensitivity (PPS) is the ratio of the difference between her payoffs in the high and low states to the difference between the firm's revenue, that is,

$$PPS(\alpha) = \frac{t^*_\alpha(\alpha h) - t^*_\alpha(\alpha l)}{\alpha^{\sigma-1}\left[\frac{R(P\rho)^{\sigma-1}}{\sigma}\left(h^{\sigma-1} - l^{\sigma-1}\right)\right]} = \frac{v\left[\kappa'(e^*_\alpha)\right]}{v\left[\kappa'(e^*_\alpha)\right] + e^*_\alpha v'\left[\kappa'(e^*_\alpha)\right]\kappa''(e^*_\alpha)},$$

where the last equality above follows from (28) and (30).

Proof of Lemma 2

Suppose that $e^*_\alpha(P) < 1$. By (30),

$$e^*_\alpha(P)\alpha^{\sigma-1}\left[\frac{R(P\rho)^{\sigma-1}}{\sigma}\left(h^{\sigma-1} - l^{\sigma-1}\right)\right] = e^*_\alpha(P)v\left[\kappa'(e^*_\alpha(P))\right] + \left(e^*_\alpha(P)\right)^2 v'\left[\kappa''(e^*_\alpha(P))\right]\kappa''(e^*_\alpha(P)). \tag{39}$$

Plugging (39) in (32), we have

$$\int_0^\infty \left[\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma} \left[e^*_\alpha(P) h^{\sigma-1} + (1 - e^*_\alpha(P)) l^{\sigma-1} \right] - e^*_\alpha(P) v \left[\kappa' \left(e^*_\alpha(P) \right) \right] \right] g(\alpha) d\alpha \tag{40}$$
$$= \int_0^\infty \left[\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma} l^{\sigma-1} + \left(e^*_\alpha(P)\right)^2 v' \left[\kappa' \left(e^*_\alpha(P) \right) \right] \kappa'' \left(e^*_\alpha(P) \right) \right] g(\alpha) d\alpha.$$

Because v, v', κ' and κ'' are increasing, it follows from (30) that $e^*_\alpha(P)$ increases with P. Therefore, the right-hand side of (40) and the left-hand side (LHS) in (32) is an increasing function of P. We can easily show that the LHS of (32) is increasing in P if $e^*_\alpha(P) = 1$. By (40), the LHS of (32) tends to infinity as $P \longrightarrow \infty$. Hence, there exists exactly one aggregate price that satisfies the equation (32).

Proof of Proposition 1

Since v, v', κ' and κ'' are increasing, it follows from (30) that the managers' effort choices $e^*_\alpha(P)$ increase with firm quality α. By (28), the expected compensation of managers also increases with their firm quality. By (10), the expected revenue of a firm with firm quality α is

$$\alpha^{\sigma-1} R \left(P\rho\right)^{\sigma-1} e^*_\alpha(P) \left[h^{\sigma-1} - l^{\sigma-1} \right] + \alpha^{\sigma-1} R \left(P\rho\right)^{\sigma-1} l^{\sigma-1}, \tag{41}$$

which increases with α because $e^*_\alpha(P)$ increases with α. By (9), firms' expected gross profit also increase with firm quality. By (40), a firm's expected net profit is

$$\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma} l^{\sigma-1} + \left(e^*_\alpha(P)\right)^2 v' \left[\kappa' \left(e^*_\alpha(P) \right) \right] \kappa'' \left(e^*_\alpha(P) \right), \tag{42}$$

which also increases with firm quality α.

By (6) and (8), the total labor employed by a firm if its realized productivity is ϕ is

$$x(\phi) = \rho r(\phi). \tag{43}$$

Since expected revenue increases with firm quality, it follows that the expected amount of labor employed also increases.

By (31),

$$PPS(\alpha) = \frac{1}{1 + \frac{e^*_\alpha(P) v'[\kappa'(e^*_\alpha(P))] \kappa''(e^*_\alpha(P))}{v[\kappa'(e^*_\alpha(P))]}}.$$

By condition (34), managerial pay-performance sensitivities increase with firm quality because managerial effort increases.

Proof of Proposition 2

By (30),

$$\left[\frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma}\left[e^*_\alpha(P)h^{\sigma-1}+(1-e^*_\alpha(P))l^{\sigma-1}\right]-e^*_\alpha(P)v'\left[\kappa'\left(e^*_\alpha(P)\right)\right]\right]$$
$$=\frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1}+\left(e^*_\alpha(P)\right)^2 v'\left[\kappa'\left(e^*_\alpha\right)\right]\kappa''\left(e^*_\alpha\right).$$

Since $e^*_\alpha(P)$ increases with by Proposition 1, and v', κ' and κ'' are increasing, the integrand on the left hand side of (32) increases with α. It follows from the properties of first order stochastic dominance that, for a given P, the left hand side of (32) is greater when the firm quality distribution is g_1. Hence, the equilibrium aggregate price index is lower when the firm quality distribution is g_1.

By (30) and (34), for a given firm quality α, the manager's effort is lower while her pay-performance sensitivity is higher. By (41), the expected revenue of a firm with firm quality α is lower when the firm quality distribution is g_1 because the aggregate price index and the manager's effort are lower. By (9) and (42), the firm's firm's expected gross and net profits are also lower.

Proof of Proposition 3

As the entry cost f_e (or the exit probability δ) increases, the right-hand side of (32) increases. In the proof of Proposition 2, we showed that the left-hand side of (32) increases with P. It follows that the equilibrium aggregate price increases with f_e and δ.

Proof of Proposition 4

Because the equilibrium aggregate price increases with the entry cost and the exit probability, it follows from (30) that each manager's effort increases with the entry cost and the exit probability. By (28), the expected compensation of managers also increases with the entry cost and the exit probability because their effort choices increase. By (10), the expected revenue of a firm with firm quality α is

$$\alpha^{\sigma-1}R\,(P\rho)^{\sigma-1}\,e^*_\alpha(P)\left[h^{\sigma-1}-l^{\sigma-1}\right]+\alpha^{\sigma-1}R\,(P\rho)^{\sigma-1}\,l^{\sigma-1},$$

which increases with the entry cost and the exit probability because $e^*_\alpha(P)$ increases. By (9), firms' expected gross profit also increase with the entry cost and the exit probability. By (40), a firm's expected net profit is

$$\frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma}l^{\sigma-1}+\left(e^*_\alpha(P)\right)^2 v'\left[\kappa'\left(e^*_\alpha(P)\right)\right]\kappa''\left(e^*_\alpha(P)\right),$$

which also increases with the entry cost and the exit probability. By (43), the expected employed labor increases with the entry cost and the exit probability because the expected revenue increases. By (31),

$$PPS(\alpha)=\frac{1}{1+\frac{e^*_\alpha(P)v'[\kappa'(e^*_\alpha(P))]\kappa''(e^*_\alpha(P))}{v[\kappa'(e^*_\alpha(P))]}}.$$

By condition (34), managerial pay-performance sensitivities decrease with the entry cost and the exit probability because managerial effort increases.

Proof of Proposition 5

The equilibrium mass of firms is equal to $\frac{R}{\overline{r}}$ where R is the aggregate revenue and $\overline{r}$ is the average revenue. Because $\overline{r}$ increases with the entry cost, the mass of firms declines.

Proof of Proposition 6

Let $f(\alpha, \sigma)$ denote the left hand side of (30) in equilibrium. We can write

$$f(\alpha, \sigma) = \alpha^{\sigma-1}\left(P^*(\sigma)^{\sigma-1}\right) g(\sigma), \tag{44}$$

where

$$g(\sigma) = \left[\frac{R\,(\rho)^{\sigma-1}}{\sigma}\left(h^{\sigma-1} - l^{\sigma-1}\right)\right] \tag{45}$$

By (44) and (45),

$$\frac{\partial}{\partial\sigma}\ln f(\alpha,\sigma) = \ln\alpha + (\sigma-1)\frac{\partial}{\partial\sigma}\ln P^*(\sigma) + \ln P^*(\sigma) + \frac{\partial}{\partial\sigma}(\sigma-1)\ln g(\sigma) \tag{46}$$

Notice that the only term that depends on α on the right hand side above is the first term. Further, $\ln\alpha \longrightarrow \pm\,\infty$ as $\alpha \longrightarrow \pm\,\infty$. It immediately follows that there exists a trigger level $\alpha_T(\sigma)$ of firm quality such that $\frac{\partial \ln f}{\partial\sigma}(\alpha,\sigma) > 0$ for $\alpha > \alpha_T(\sigma)$ and $\frac{\partial \ln f}{\partial\sigma}(\alpha,\sigma) < 0$ for $\alpha < \alpha_T(\sigma)$. The variations in managerial effort and expected managerial compensation described in (35) then follow from (30) and (28).

By (31),

$$PPS(\alpha) = \frac{1}{1 + \frac{e^*_\alpha v'[\kappa'(e^*_\alpha)]\kappa''(e^*_\alpha)}{v[\kappa'(e^*_\alpha)]}}.$$

Since managerial effort increases with σ for $\alpha > \alpha_T(\sigma)$ and decreases with σ for $\alpha < \alpha_T(\sigma)$., it follows from the above and condition (34) that managerial pay-performance sensitivities decrease with a (marginal) increase in σ for $\alpha > \alpha_T(\sigma)$ and increase for $\alpha < \alpha_T(\sigma)$.

The effects of σ on the average expected revenue and average expected gross profit of all firms depend on the relative proportions of managers with match qualities that are greater or less than $\alpha_T(\sigma)$ because their effort choices move in opposite directions. In other words, the effects of σ on the average expected revenue and average expected gross profit of all firms are affected by the firm quality distribution g. The equilibrium mass of firms is equal to $\frac{R}{\overline{r}}$ where R is the aggregate revenue and $\overline{r}$ is the average revenue. Because $\overline{r}$ could increase or decrease with σ depending on the firm quality distribution, the equilibrium mass of firms could also decrease or increase.

Proof of Proposition 7

Define

$$f(\sigma,P)=\int_0^\infty\left[\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma}\left[e_\alpha^*(P,\sigma)h^{\sigma-1}+(1-e_\alpha^*(P,\sigma))l^{\sigma-1}\right]-e_\alpha^*(P,\sigma)v\left[\kappa'\left(e_\alpha^*(P,\sigma)\right)\right]\right]g(\alpha)d\alpha, \tag{47}$$

where $e_\alpha^*(P,\sigma)$ is the optimal effort choice of the manager with firm quality α when the aggregate price is P and the elasticity of substitution is σ. If $P^*(\sigma)$ denotes the equilibrium aggregate price when the elasticity of substitution is σ, then it follows from (32) and (47) that

$$f(\sigma,P^*(\sigma))=\delta f_e. \tag{48}$$

We note that

$$\frac{\partial f}{\partial\sigma}=\int_0^\infty\left[\begin{array}{c}\frac{\partial}{\partial\sigma}\left(\frac{R(P\rho\alpha h)^{\sigma-1}}{\sigma}\right)e_\alpha^*(P,\sigma)+\frac{\partial}{\partial\sigma}\left(\frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\right)(1-e_\alpha^*(P,\sigma))\\ +\left(\frac{R(P\rho\alpha h)^{\sigma-1}}{\sigma}-\frac{R(P\rho\alpha l)^{\sigma-1}}{\sigma}\right)\frac{\partial e_\alpha^*(P,\sigma)}{\partial\sigma}\\ -v\left[\kappa'\left(e_\alpha^*(P,\sigma)\right)\right]\frac{\partial e_\alpha^*(P,\sigma)}{\partial\sigma}-e_\alpha^*(P,\sigma)v'\left[\kappa'\left(e_\alpha^*(P,\sigma)\right)\right]\kappa''\left(e_\alpha^*(P,\sigma)\right)\frac{\partial e_\alpha^*(P,\sigma)}{\partial\sigma}\end{array}\right]g(\alpha)d\alpha. \tag{49}$$

By (30),

$$\frac{R\left(P\rho\alpha h\right)^{\sigma-1}}{\sigma}-\frac{R\left(P\rho\alpha l\right)^{\sigma-1}}{\sigma}=v\left[\kappa'\left(e_\alpha^*(P,\sigma)\right)\right]+e_\alpha^*(P,\sigma)v'\left[\kappa'\left(e_\alpha^*(P,\sigma)\right)\right]\kappa''\left(e_\alpha^*(P,\sigma)\right). \tag{50}$$

Substituting (50) in (49), we have

$$\frac{\partial f}{\partial\sigma}=\int_0^\infty\left[\frac{\partial}{\partial\sigma}\left(\frac{R\left(P\rho\alpha h\right)^{\sigma-1}}{\sigma}-\frac{R\left(P\rho\alpha l\right)^{\sigma-1}}{\sigma}\right)e_\alpha^*(P,\sigma)+\frac{\partial}{\partial\sigma}\left(\frac{R\left(P\rho\alpha l\right)^{\sigma-1}}{\sigma}\right)\right]g(\alpha)d\alpha. \tag{51}$$

After calculating the derivatives in the integrand above, we can show that $\frac{\partial f}{\partial\sigma}>0$ if P exceeds a threshold $P_T(\sigma)$ and is less than zero otherwise. By arguments similar to those used in the proof of Proposition 9, $\frac{\partial f}{\partial P}>0$.

It follows from (48) that the equilibrium price $P^*(\sigma)$ increases with δ and f_e. Moreover, its support is $(0,\infty)$. Therefore, there exists a threshold level $f_T'(\sigma)$ $(\delta_T'(\sigma))$ of the entry cost (exit probability) such that $P^*(\sigma)>P_T(\sigma)$ if $f_e(\sigma)$ $(\delta_T'(\sigma))>f_T'(\sigma)$ $(\delta_T'(\sigma))$ and $P^*(\sigma)<P_T(\sigma)$ if $f_e(\sigma)$ $(\delta_T'(\sigma))<f_T'$ (δ_T'). By (48) and the implicit function theorem,

$$\frac{dP^*(\sigma)}{d\sigma}=-\frac{\partial f/\partial\sigma}{\partial f/\partial P}|_{P=P^*(\sigma)}. \tag{52}$$

By the above arguments, it follows that $\frac{dP^*(\sigma)}{d\sigma}>0$ if $f_e(\sigma)$ $(\delta_T'(\sigma))<f_T'(\sigma)$ $(\delta_T'(\sigma))$ and is greater than zero if $f_e(\sigma)$ $(\delta_T'(\sigma))>f_T'(\sigma)$ $(\delta_T'(\sigma))$.

Proof of Proposition 8

By (30), the equilibrium effort of a manager with firm quality α solves

$$\alpha^{\sigma-1}\left[\frac{R\left(P(x)\rho\right)^{\sigma-1}}{\sigma}x\right]=v\left[\kappa'\left(e^*_\alpha(x)\right)\right]+e^*_\alpha(x)v'\left[\kappa'\left(e^*_\alpha(x)\right)\right]\kappa''\left(e^*_\alpha(x)\right),\tag{53}$$

where we explicitly indicate the dependence of the effort on the productivity risk x. If $P(x)$ increases with x then it follows from the fact that κ', κ'', v, and v' are all increasing that $e^*_\alpha(x)$ increases.

Suppose that $P(x)$ decreases with x. We show that the left-hand side of (53) is still increasing with x. Suppose that the contrary holds. Because κ', κ'', v, and v' are all increasing, $e^*_\alpha(x)$ decreases. Since $P(x)$ decreases with x and $l^{\sigma-1}$ decreases with x (because $y=h^{\sigma-1}+l^{\sigma-1}$ is fixed), it follows from (59) that $f(x,P(x))$ is decreasing, which contradicts the equilibrium condition (55). Hence, the left hand side of (53) is increasing with x. Hence, managerial effort increases with the productivity risk.

By (28), expected managerial compensation, $e^*_\alpha(x)v\left[\kappa'\left(e^*_\alpha(x)\right)\right]$ also increases with the productivity risk. By (31) and condition (34), managerial pay-performance sensitivities decline with productivity risk. Because the average managerial compensation increases, it follows from the equilibrium condition (55) that the average gross profit of all firms increases. By (10) and (11), the gross profit of each firm is a constant proportion of its revenue. Hence, the average revenue of all firms also increases.

Proof of Proposition 9

Let $x=h^{\sigma-1}-l^{\sigma-1}$ and $y=h^{\sigma-1}+l^{\sigma-1}$. Define

$$f(x,P)=\int_0^\infty\left[\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma}\left[e^*_\alpha(P,x)h^{\sigma-1}+(1-e^*_\alpha(P,x))l^{\sigma-1}\right]-e^*_\alpha(P,x)v\left[\kappa'\left(e^*_\alpha(P,x)\right)\right]\right]g(\alpha)d\alpha,\tag{54}$$

where $e^*_\alpha(P,x)$ is the optimal effort choice of the manager with firm quality α when the aggregate price is P and the productivity risk is x. If $P^*(x)$ denotes the equilibrium aggregate price when the productivity risk is x, then it follows from (32) and (54) that

$$f(x,P^*(x))=\delta f_e.\tag{55}$$

We note that

$$\frac{\partial f}{\partial x}=\int_0^\infty\left[\begin{array}{c}\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}e^*_\alpha(P,x)+\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}x\frac{\partial e^*_\alpha(P,x)}{\partial x}-\frac{R(P\rho\alpha)^{\sigma-1}}{\sigma}0.5\\ -v\left[\kappa'\left(e^*_\alpha(P,x)\right)\right]\frac{\partial e^*_\alpha(P,x)}{\partial x}-e^*_\alpha(P,x)v'\left[\kappa'\left(e^*_\alpha(P,x)\right)\right]\kappa''\left(e^*_\alpha(P,x)\right)\frac{\partial e^*_\alpha(P,x)}{\partial x}\end{array}\right]g(\alpha)d\alpha.\tag{56}$$

By (30),

$$\frac{R\left(P\rho\alpha\right)^{\sigma-1}}{\sigma}x=v\left[\kappa'\left(e^*_\alpha(P,x)\right)\right]+e^*_\alpha(P,x)v'\left[\kappa'\left(e^*_\alpha(P,x)\right)\right]\kappa''\left(e^*_\alpha(P,x)\right).\tag{57}$$

Substituting (57) in (56), we see that

$$\frac{\partial f(P,x)}{\partial x} = \int_{0}^{\infty} \left[\frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma} \left(e^*_\alpha(P,x) - 0.5\right) \right] g(\alpha) d\alpha. \tag{58}$$

We note that $\frac{\partial f(P,x)}{\partial x}$ is greater than or less than zero depending on the sign of $\int_{0}^{\infty} \left[\left(e^*_\alpha(P,x) - 0.5\right) \right] g(\alpha) d\alpha$. Because $\kappa'(.), \kappa''(.)$, and $v'(.)$ are all increasing, it follows from (57) that $e^*_\alpha(P,x)$ increases with P. Therefore, there exists a threshold $P_T(x)$ such that, if $P < P_T(x)$, $\frac{\partial f(P,x)}{\partial x} < 0$, and if $P > P_T(x)$, $\frac{\partial f(P,x)}{\partial x} > 0$.

It follows from (55) that the equilibrium price $P^*(x)$ increases with δ and f_e. Moreover, its support is $(0, \infty)$. Therefore, there exists a threshold level $f_T\ (x)(\delta_T(x))$ of the entry cost (exit probability) such that $P^*(x) > P_T(x)$ if $f_e\ (x)(\delta_T(x)) > f_T(x)\ (\delta_T(x))$ and $P^*(x) < P_T(x)$ if $f_e(x)\ (\delta_T(x)) < f_T(x)$ $(\delta_T(x))$.

We can rewrite the expression (54) as follows:

$$f(x,P) = \int_{0}^{\infty} \left[\frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma} \left[e^*_\alpha(P,x)x + l^{\sigma-1}\right] - e^*_\alpha(P,x) v \left[\kappa'\left(e^*_\alpha(P,x)\right)\right] \right] g(\alpha) d\alpha.$$

By (57), the above expression can be further rewritten as

$$f(x,P) = \int_{0}^{\infty} \left[\left(e^*_\alpha(P,x)\right)^2 v' \left[\kappa'\left(e^*_\alpha(P,x)\right)\right] \kappa''\left(e^*_\alpha(P,x)\right) + \frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma} l^{\sigma-1} \right] g(\alpha) d\alpha. \tag{59}$$

It follows from the above, therefore, that $\frac{\partial f}{\partial P} > 0$. By (55) and the implicit function theorem,

$$\frac{dP^*(x)}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial P}\Big|_{P=P^*(x)}. \tag{60}$$

It follows from the above that $\frac{dP^*(x)}{dx} > 0$ if $f_e\ (x)(\delta_T(x)) < f_T(x)\ (\delta_T(x))$ and $\frac{dP^*(x)}{dx} < 0$ if $f_e(x)$ $(\delta_T(x)) > f_T(x)\ (\delta_T(x))$.

Proof of Proposition 10

The equilibrium mass of firms is equal to $\frac{R}{\bar{r}}$ where R is the aggregate revenue and $\bar{r}$ is the average revenue. Because $\bar{r}$ increases with productivity risk, the mass of firms declines.

Proof of Proposition 11

First, we note that, by (30) and the fact that the productivity risk is kept fixed, managerial effort does not depend on the productivity level for a *given* aggregate price P. We can rewrite the left-hand side (LHS) of (32) as

$$\int_0^\infty \left[\frac{R\,(P\rho\alpha)^{\sigma-1}}{\sigma} \left[e_\alpha^*(P)x + l^{\sigma-1}\right] - e_\alpha^*(P)v\left[\kappa'\left(e_\alpha^*(P)\right)\right]\right] g(\alpha)d\alpha.$$

It follows from the above that the LHS of (32) increases with $l^{\sigma-1}$ keeping x fixed. Since the right-hand side of (32) does not vary with $l^{\sigma-1}$, it follows that the equilibrium aggregate price that solves (32) must decrease with the productivity level $l^{\sigma-1}$.

Since the equilibrium aggregate price declines with the productivity level, and the productivity risk is kept fixed, it follows from (30) that each manager's effort declines with the productivity level. By (28), expected managerial compensation also declines with the productivity level because managerial effort declines. By (27), the expected gross profit of a firm is the expected net profit plus the expected managerial compensation. By the equilibrium condition (32), the average expected gross profit of all firms is equal to δf_e plus the average expected compensation of all managers. Since the average expected compensation of managers declines with the productivity level, the average expected gross profit of all firms also declines. By (10) and (9), the average expected revenue of all firms also declines with the productivity level. By (31),

$$PPS(\alpha) = \frac{1}{1 + \frac{e_\alpha^* v'[\kappa'(e_\alpha^*)]\kappa''(e_\alpha^*)}{v[\kappa'(e_\alpha^*)]}}.$$

Since managerial effort declines with the productivity level, it follows from the above and condition (34) that managerial pay-performance sensitivities increase.

The equilibrium mass of firms is equal to $\frac{R}{\overline{r}}$ where R is the aggregate revenue and $\overline{r}$ is the average revenue. Because $\overline{r}$ decreases with the productivity level, the mass of firms increases.

References

[1] Aggarwal, Rajesh K., and Andrew A. Samwick, 1999a, The other side of the trade-off: the impact of risk on executive compensation, *Journal of Political Economy* 107 (1), 65-105.

[2] Aggarwal, Rajesh K., and Andrew A. Samwick, 1999b, Executive compensation, strategic competition, and relative performance evaluation: theory and evidence, *Journal of Finance* 54 (6), 1999-2043.

[3] Antle, Rick, and Abbie Smith, 1986, An empirical investigation of the relative performance evaluation of corporate executives, *Journal of Accounting Research* 24, 1-39.

[4] Baker, George P., and Brian J. Hall, 2004, CEO incentives and firm size, *Journal of Labor Economics* 22 (4), 767-798.

[5] Baranchuk, Nina, Glenn MacDonald, and Jun Yang, 2009, The economics of super managers, unpublished working paper.

[6] Belsley, D., Kuh, E., and Welsch, R., 1980, Regression Diagnostics, Wiley, New York.

[7] Bertrand, Marianne, and Antoinette Schoar, 2003, Managing with style: the effect of managers on firm policies, *Quarterly Journal of Economics* 118 (4), 1169-1208.

[8] Bolton, Patrick and Matthias Dewatripont, 2005, *Contract Theory,* The MIT Press, Cambridge, Massachusetts and London, England.

[9] Cabral, Luis M. B., and Jose Mata, 2003, On the evolution of the firm size distribution: facts and theory, *American Economic Review* 93 (4), 1075-1090.

[10] Conyon, Martin J., and Kevin J. Murphy, 2000, The prince and the pauper? CEO pay in the United States and United Kingdom, *Economic Journal* 110 (467), F640-F671.

[11] DeFond, Mark L., and Chul W. Park, 1999, The effect of competition on CEO turnover, *Journal of Accounting and Economics* 27, 35-56.

[12] Dixit, Avinash, and Joseph Stiglitz, 1977, Monopolistic competition and optimum product diversity, *American Economic Review* 67, 297-308.

[13] Edmans, Alex and Xavier Gabaix, 2010, The effect of risk on the CEO market, *Working Paper,* Wharton School, University of Pennsylvania and Stern School of Business, New York University.

[14] Edmans, Alex, Xavier Gabaix, and Augustin Landier, 2009, A multiplicative model of optimal CEO incentives in market equilibrium, *Review of Financial Studies* 22, 4881-4917.

[15] Falato, Antonio, and Dalida Kadyrzhanova, 2008, Optimal CEO incentives and industry dynamics, unpublished working paper.

[16] Gabaix, Xavier, and Augustin Landier, 2008, Why has CEO pay increased so much? *Quarterly Journal of Economics* 123 (1), 49-100.

[17] Hall, Brian J., and Jeffrey B. Liebman, 1998, Are CEOs really paid like bureaucrats? *Quarterly Journal of Economics* 113 (3), 653-691.

[18] Janakiraman, Surya, Richard A. Lambert, and David F. Larcker, 1992, An empirical investigation of the relative performance evaluation, *Journal of Accounting Research* 30, 53-69.

[19] Karuna, Christo, 2007, Industry product market competition and managerial incentives, *Journal of Accounting and Economics* 43, 275-297.

[20] Krugman, Paul R., 1979, Increasing returns, monopolistic competition, and international trade, *Journal of International Economics* 9, 469-479.

[21] Laffont, Jean-Jacques, and Jean Tirole, 1993, *A Theory of Incentives in Procurement and Regulation* (MIT Press, Cambridge, MA).

[22] Melitz, Marc J., 2003, The impact of trade on intra-industry reallocations and aggregate industry productivity, *Econometrica* 71 (6), 1695-1725.

[23] Nevo, A., 2001, Measuring market power in the ready-to-eat cereal industry, *Econometrica* 69 (2), 307–342.

[24] Prendergast, Canice, 2002, The tenuous trade-off between risk and incentives, *Journal of Political Economy* 110 (5), 1071-1102.

[25] Raith, Michael, 2003, Competition, risk, and managerial incentives, *American Economic Review* 93 (4), 1425-1436.

[26] Rajgopal, Shivaram, Terry Shevlin, and Valentina Zamora, 2006, CEOs' outside employment opportunities and the lack of relative performance evaluation in compensation contracts, *Journal of Finance* 61 (4), 1813-1844.

[27] Rogerson, Richard, Robert Shimer and Randall Wright, 2005, Search-theoretic models of the labor market: a survey, *Journal of Economic Literature* 43, 959-988.

[28] Schaefer, Scott, 1998, The dependence of pay-performance sensitivity on the size of the firm, *Review of Economics and Statistics* 80 (3), 436-443.

[29] Tervio, Marko, 2008, The difference that CEOs make: an assignment model approach, *American Economic Review* 98 (3), 642-668.

[30] White, H. 1980, A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 48 (4), 817–838.