Capital Structure under Heterogeneous Beliefs*

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Abstract

We develop a dynamic structural model to examine the effects of differing beliefs of the manager and outsider investors regarding the profitability of a firm’s projects and manager-shareholder agency conflicts on its capital structure. The manager receives dynamic incentives through explicit contracts with shareholders whose implementation through financial securities leads to a dynamic capital structure consisting of inside equity, outside equity, long-term debt and short-term debt. The analysis of the model generates novel testable implications for the effects of project characteristics on different components of capital structure: (i) Long-term debt declines with managerial optimism, while the manager’s inside equity stake and short-term debt increase. (ii) Long-term debt and the manager’s inside equity stake decline with the firm’s transient risk; the component of the firm’s risk that is resolved over time due to Bayesian learning. Short-term debt, however, increases with the transient risk. (iii) Long-term debt increases with the firm’s intrinsic risk—the component of the firm’s risk that is invariant through time—while short-term debt and the manager’s inside equity stake decline. (iv) Long-term and short-term debt increase with the firm’s expected future profitability. We calibrate our structural model to a sample of general firms as well as a sample of IPO firms, and show the quantitative impact of asymmetric beliefs on capital structure. Without making any assumptions about their relative magnitudes at the outset, the calibrated parameter values corresponding to the two sets of firms suggest that managers/entrepreneurs of IPO firms are, indeed, significantly more optimistic than general managers, and the degree of uncertainty about the profitability of IPO firms is substantially higher. Overall, our findings show that imperfect information and asymmetric beliefs are important determinants of firms’ financial policies.

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Key Words: Capital Structure, Imperfect Information, Heterogeneous Beliefs, Agency Conflicts

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1 Introduction

There is growing evidence to suggest that managers and outside investors often have differing beliefs about the profitability of projects in addition to asymmetric attitudes towards their risks (Malmendier and Tate, 2005 and 2008, Baker et al, 2007, Ben-David et al, 2007, Malmendier et al, 2009). While a substantial literature in dynamic corporate finance provides quantitative guidance into the effects of bondholder–shareholder and manager–shareholder conflicts on capital structure, the quantitative effects of asymmetric beliefs are relatively unexplored in structural frameworks.

We develop a dynamic, structural model to examine how asymmetric beliefs and agency conflicts interact to affect capital structure. Our model incorporates internal imperfections arising from asymmetric beliefs and moral hazard as well as external imperfections arising from taxes and bankruptcy costs. We calibrate the model to aggregate data and derive a number of novel testable implications that link characteristics of a firm’s projects—the degree of asymmetry in beliefs, permanent and transient components of risk, and expected future profitability—to different components of its capital structure: long-term debt, short-term debt, inside equity and outside equity. (i) Long-term debt declines with the degree of managerial optimism, while the manager’s inside equity stake and short-term debt increase. (ii) Long-term debt and the manager’s inside equity stake decline with the transient risk of a firm’s earnings—the degree of uncertainty in agents’ priors about the profitability of a firm’s projects—while short-term debt increases. (iii) Long-term debt increases with the intrinsic risk of a firm’s earnings—the component of risk that is invariant through time—while short-term debt and the manager’s inside equity stake decline. (iv) Long-term and short-term debt increase with the expected future profitability of the firm’s projects.

In our discrete-time, finite horizon framework, the manager of an all-equity firm raises financing for a positive NPV project through outside equity and long-term debt that is non-callable and completely amortized as in Leland (1998). The manager has an initial ownership stake and receives a proportion of the net payoff from external financing (the total proceeds from financing net of the required initial investment outlay). In addition to equity and long-term debt, the firm’s
capital structure also consists of non-discretionary short-term (single-period) debt financing of the firm’s working capital requirements. The firm’s earnings in each period are distributed among its stakeholders—the manager, shareholders, bondholders, and the government (through corporate taxes). We abstract away from personal taxes for simplicity.

The key state variable in the model is the firm’s total earnings gross of long-term debt interest payments, short-term debt payments associated with working capital, corporate taxes, and the manager’s compensation. As in Giat, Hackman and Subramanian (2010), the firm’s total earnings in each period consist of two components; the normally distributed base earnings, and the discretionary earnings that are generated by the shareholders’ incremental capital investments and the manager’s effort. The mean of the project’s base earnings is its intrinsic quality or profitability. Outside investors and the manager have imperfect information and possibly differing priors about the project’s profitability. They “agree to disagree” about their respective mean assessments. The difference of their mean assessments is the degree of managerial optimism. The variance of the project’s earnings in each period is its intrinsic risk, which is invariant through time. The project’s discretionary earnings are deterministic and determined by a Cobb-Douglas production function. The common variance of outside investors’ and the manager’s assessments of the project’s base earnings is its transient risk. In contrast with the intrinsic risk, the transient risk is resolved over time as intermediate observations of the project’s earnings enable the players to update their assessments of the project’s profitability.

Outside investors are risk-neutral and capital markets are competitive (see Chapters 3 and 4 of Tirole (2006)). The undiversified manager has multiplicatively separable CARA preferences. We consider an incomplete contracting environment in which the manager receives dynamic incentives through a sequence of explicit contracts contingent on the firm’s earnings. The contracts must be incentive compatible for the manager. The contracts must also guarantee that shareholder value under the contracts is at least as great as the shareholder value in the hypothetical absence of the manager’s human capital inputs, that is, if the firm receives only the project’s base earnings in each period. As the manager has multiplicatively separable CARA preferences, and the firm’s earnings
are normally distributed, we follow Gibbons and Murphy (1992) in restricting consideration to contracts that are affine in the firm’s earnings in each period.

As in Leland (1994), debt is serviced entirely in financial distress by the additional issuance of equity, and bankruptcy occurs endogenously when the equity value falls to zero. For simplicity, we assume that the firm is liquidated at bankruptcy and the absolute priority of debt is enforced. The “bankruptcy payoff” to debtholders is equal to a portion of the firm’s asset value at bankruptcy, which is the present value of the firm’s base earnings flow less bankruptcy costs.

We characterize the equilibrium in which the firm’s capital structure and the manager’s contracts are endogenously determined. Because the manager has an equity stake in the initial all-equity firm, she receives a portion of the net proceeds from external financing. The manager chooses the firm’s capital structure to maximize the total expected utility she derives from her initial payoff from external financing and her stream of future contractual compensation payments.

We implement the risky component of the manager’s compensation in each period through an inside equity stake in the firm, and the performance-invariant or “cash" component through a cash reserve. Since cash is effectively negative short-term debt (see DeMarzo and Fishman, 2007), the cash reserve offsets the firm’s short-term debt associated with the financing of its working capital requirements. In our implementation of the manager’s contracts, therefore, the firm’s capital structure consists of inside and outside equity, long-term debt, and the short-term debt financing of working capital as well as the manager’s cash compensation.\footnote{The explicit modeling of short-term debt associated with the firm’s working capital does not affect the testable implications of the model, but facilitates its calibration because the financing of working capital requirements is a significant component of a firm’s short-term debt in the data.}

As in Giat et al. (2010), the manager’s equilibrium inside equity stake is a deterministic function of time, but her cash compensation depends on the project’s earnings history through its effect on the players’ posterior assessments of the project’s profitability. The manager’s inside equity stake in each period increases with the initial degree of managerial optimism. When the manager is optimistic, she overvalues the project’s future earnings relative to outside investors. The optimal contract exploits the manager’s optimism by providing the manager with more powerful incentives,
that is, by increasing her equity compensation relative to her cash compensation.

The manager’s choice of long-term debt financing at date zero reflects its effects on her initial payoff from external financing, and the expected utility from her future contractual compensation payments—hereafter, her continuation utility. The manager’s initial payoff is proportional to the total proceeds from external financing net of the initial required investment. Under rational expectations, the proceeds from external financing—the market value of debt plus the market value of outside equity—equal the market value of the firm’s total after-tax earnings net of the manager’s stake. Because capital markets are competitive, the manager captures the surplus she generates from her human capital. Consequently, the manager’s beliefs and actions affect her continuation utility, but do not affect the proceeds from external financing at date zero.

Long-term debt has conflicting effects on the manager’s total expected utility. On the positive side, because debt interest payments are shielded from corporate taxes, the manager can potentially increase the proceeds from external financing at date zero (therefore, her initial payoff) by choosing greater long-term debt. Choosing greater long-term debt, however, increases the expected bankruptcy costs for the firm and personal bankruptcy costs for the manager, which negatively affect her continuation utility.

We show that long-term debt declines with the degree of managerial optimism. As discussed earlier, an increase in the manager’s optimism increases the manager’s inside equity stake in each period and, therefore, the output she generates. Consequently, the manager’s expected contractual compensation in each period and continuation utility increase with her degree of optimism, while her initial payoff is unaffected as discussed earlier. At the margin, therefore, the manager gives relatively more weight to her continuation utility than her initial payoff in choosing the firm’s long-term debt. Because long-term debt lowers the manager’s continuation utility through the likelihood of bankruptcy, the manager chooses lower long-term debt to lower the probability of bankruptcy.

We numerically explore additional implications of the model. We calibrate the model to aggregate data to obtain a reasonable set of baseline parameter values for our analysis. In particular, we indirectly infer the “deep” structural parameters of the model—outside investors’ and the man-
ager’s mean assessments of the firm’s profitability, the transient risk, the manager’s risk aversion, and her disutility of effort—so that key statistics predicted by the model match their average values in the data. In our first calibration exercise, we use a sample of general firms (excluding financial firms and regulated utilities) from Compustat. Our calibration shows that managers are, indeed, significantly optimistic relative to outside investors. Relative to outside investors, the average manager in the sample of general firms overestimates the market to book ratio by 182%. Further, the ratio of the project’s transient risk to its intrinsic risk is 2.4, which suggests that the degree of uncertainty about profitability—the transient risk—is significant relative to the intrinsic risk.

We derive additional testable implications of the model by varying parameters about their baseline values. Long-term debt declines with the project’s transient risk, but increases with its intrinsic risk. An increase in the transient risk increases the project’s “signal to noise ratio,” that is, intermediate observations of the project’s earnings are more informative about its intrinsic quality. Consequently, the standard deviation of the evolution of posterior assessments of the project’s quality increases. As a result, the “option value” of continuing to service long-term debt interest payments increases, which delays bankruptcy. Consequently, the marginal impact of the manager’s continuation utility on her long-term debt choice increases relative to her initial payoff. The manager, therefore, chooses lower long-term debt to increase her continuation utility. An increase in the intrinsic risk, however, decreases the signal to noise ratio and the option value of delaying bankruptcy. Further, an increase in intrinsic risk increases the costs of risk-sharing between the manager and outside investors, which weakens the manager’s incentives and the output she generates in each period. As a result, an increase in the intrinsic risk lowers the marginal impact of the manager’s continuation utility on her long-term debt choice. The manager, therefore, chooses greater long-term debt to increase her initial payoff.

Next, we explore the effects of project characteristics on short-term debt. As the manager’s degree of optimism increases, she receives more powerful incentives so that her cash compensation declines relative to her equity compensation. Because cash is effectively negative short-term debt, the firm’s short-term debt increases with managerial optimism. Short-term debt increases with the
project’s transient risk because an increase in the transient risk delays bankruptcy and, therefore, increases the value of the firm’s short-term debt. An increase in the intrinsic risk, however, hastens bankruptcy, increases the costs of risk-sharing and, therefore, the manager’s cash compensation relative to her equity compensation. Consequently, short-term debt declines with the intrinsic risk.

We extend the model to allow for differing allocations of bargaining power between the manager and outside investors. We show that our main predictions continue to hold as long as shareholders’ bargaining power vis-a-vis the manager is below a threshold (in our simulations, the threshold is close to 50%). Our examination of the effects of bargaining power also generates additional testable predictions. We find that long-term debt and short-term debt increases with shareholders’ bargaining power. Given that shareholders’ bargaining power is likely to be greater in physical capital intensive industries relative to human capital intensive ones, our results imply that long-term debt and short-term debt ratios are greater in physical capital intensive industries. Our results are also robust to an extension of the model in which the manager continues to service long-term debt interest payments even after the outside equity value falls to zero, that is, the firm effectively becomes privately held. The manager optimally declares bankruptcy when it is no longer optimal for her to service debt interest payments.

Finally, to provide further validation for the model, we calibrate its parameters to a sample of IPO firms for which uncertainty about future profitability and asymmetric beliefs are likely to be more important. Consistent with this intuition, we find that the degree of managerial optimism and the project’s transient risk are, indeed, much more quantitatively significant relative to their calibrated values for the sample of general firms. Relative to outside investors, the average manager in the sample of IPO firms overestimates the market to book ratio by 258%. The ratio of the project’s transient risk to its intrinsic risk for the sample of IPO firms is 4.4. In other words, without any a priori assumptions on their relative magnitudes, the indirectly inferred values of managerial optimism and transient risk are much higher (in relative terms) for the sample of IPO firms compared with the sample of general firms. It is worth emphasizing that the comparisons between the calibrated parameters corresponding to the two samples are not a priori obvious be-
cause we match a range of statistics in the two samples, and the calibrated parameter set also includes parameters that are unrelated to those pertaining to the beliefs of managers and investors. Consistent with the notion that entrepreneurs are likely to be less diversified than general firm managers, the risk aversion of managers of IPO/entrepreneurial firms is, indeed, higher. Moreover, in conformity with the fact that young firms are likely to have greater growth opportunities on average, the indirectly inferred value of the total factor productivity is significantly higher for the IPO sample. Overall, our findings provide additional support for the model as a viable (albeit stylized) descriptor of firms’ financing decisions. In particular, our results highlight the importance of imperfect information and asymmetric beliefs as determinants of capital structure, especially for entrepreneurial firms.

2 Related Literature

We contribute to the literature by developing a dynamic structural model to examine the quantitative effects of asymmetric beliefs and agency conflicts on different components of capital structure. Landier and Thesmar (2009) present a two-period framework in which contracts are exogenously restricted to be debt contracts. They show that optimistic managers choose short-term debt, whereas realists opt for long-term debt. We complement their analysis by examining the effects of asymmetric beliefs on long-term debt, short-term debt, inside equity, and outside equity in a dynamic principal-agent framework with general contractual structures. Dittmar and Thakor (2007) develop a three-period model and show that a manager chooses equity financing if there is greater agreement between the manager and investors. Chemmanur, Nandy and Yan (2007) show that, if outside investors are more optimistic or more heterogenous in their beliefs, the firm is more likely to issue equity rather than debt. Hackbarth (2008) incorporates managerial traits, such as growth and risk perception biases, into a dynamic tradeoff model of capital structure and shows that a biased manager chooses a higher debt level.

Our study complements these analyses in several aspects. First, we quantitatively examine
the effects of asymmetric beliefs on capital structure in a structural model that allows for the
manager to be optimistic or pessimistic relative to outside investors. The calibration of the model
leads to an indirect inference of the level of managerial optimism (or pessimism) as implied by the
data. We find that managers are, indeed, significantly more optimistic than outside investors, and
managerial optimism is particularly pronounced in entrepreneurial firms. Second, we consider a
dynamic principal-agent framework in which the manager’s optimal dynamic contract is derived
and implemented through financial securities, which leads to a dynamic capital structure. Third,
our unified framework generates novel implications for the effects of optimism, permanent and
transitory components of risk, and profitability on various components of capital structure.

Adrian and Westerfield (2009) and Giat et al. (2010) develop general dynamic, principal-agent
models to study the impact of asymmetric beliefs and agency conflicts on optimal dynamic contracts.
They abstract away from capital structure choices. Gervais, Heaton and Odean (2007) study the
effects of manager overconfidence on investment, but also abstract away from capital structure. We
build on the model in Giat et al. (2010) by layering an additional capital structure choice problem.
We show how the dynamic tradeoff between debt tax shields and bankruptcy costs, asymmetric
beliefs, Bayesian learning, and agency conflicts interact to affect capital structure.

Our study is also related to the literature on dynamic tradeoff models of capital structure
(e.g. Fischer et al (1989), Leland and Toft (1996), Goldstein et al (2001), Hennessy and Whited
(2005), Strebulaev (2007)). In these studies, the manager is assumed to behave in the interests of
shareholders, and agents have perfect information about project characteristics.

Another group of studies analyzes financial contracting in dynamic principal-agent frameworks
in which all agents are risk-neutral and beliefs are symmetric (e.g. DeMarzo and Sannikov (2006),
DeMarzo and Fishman (2007)). Bhagat et al. (2010) examine the effects of manager-specific char-
acteristics such as ability and risk aversion on capital structure in a dynamic principal-agent model
with perfect information about project characteristics. We differ from their study by investigat-
ing the effects of imperfect information and asymmetric beliefs about project quality on capital
structure.
3 The Model

We consider a finite horizon framework with equally spaced dates, 0, 1, . . . , T. At date 0, the manager of an all-equity firm, who has an initial ownership stake, $g_{initial} \in (0, 1)$, considers a positive net present value project that requires an initial investment outlay, $I$. She raises financing for the project from public debt and equity markets that are competitive. The total earnings from the project are distributed among all the firm’s claimants - the manager, shareholders, debt-holders, and the government (through taxes). We ignore personal taxes for simplicity and assume that the corporate tax rate is a constant, $\tau \in (0, 1)$. Security issuance costs are negligible, and the risk-free interest rate, $r$, is a constant and the same for all market participants. All agents are fully rational, but (as we describe shortly) could have differing beliefs about the project’s payoff distribution.

3.1 The Total Earnings Flow

We adopt the formulation in Giat et al. (2010) to model the firm’s total earnings in each period. In any period $[i, i + 1]$, the earnings from the project are affected by physical capital investments by shareholders and human capital investments (effort) by the manager. The total earnings flow in any period has two components: the base earnings—a stochastic component that is unaffected by the physical and human capital investments, and the discretionary earnings—a deterministic component that depends on the incremental capital investment by shareholders and the manager’s effort. Specifically, if the capital investment and effort over the period $[i, i + 1]$ are $k_i$ and $\eta_i$, respectively, the total earnings flow is

$$E_{i+1} = \Theta + N_{i+1} + A k_i^\alpha \eta_i^\beta$$  (1)

The first component of the base earnings, $\Theta$, represents the project’s intrinsic quality or core profitability. The manager and outside investors—shareholders and debtholders—have imperfect information and possibly asymmetric beliefs about $\Theta$. Their respective beliefs are, however, com-
mon knowledge, that is, they agree to disagree (see Morris, 1995, Allen and Gale, 1999). Their respective priors on $\Theta$ at date zero are normally distributed as follows:

$$\Theta \sim N(\mu_0^S, \sigma_0^2); \text{ Shareholders’ Prior}$$

$$\Theta \sim N(\mu_0^M, \sigma_0^2); \text{ Manager’s Prior}$$

The equilibrium only depends on how the manager’s and outside investors’ assessments of project quality relate to each other, and not on the true project quality distribution. We, therefore, make no assumptions on the true project quality distribution in our theoretical analysis. Although we could consider the general scenario in which the manager could be optimistic or pessimistic relative to outside investors, we simplify the exposition by considering the (empirically relevant) scenario in which the manager is optimistic. It is important to mention here that, when we calibrate the model to data in Section 6, we do not assume a priori that the manager is optimistic, and indirectly infer the average level of managerial optimism (or pessimism) implied by the data. Define

$$\Delta_0 = \mu_0^M - \mu_0^S,$$

which is the degree of managerial optimism at date zero.

The second component of the base earnings, $N_{i+1}$, is a normal random variable with mean 0 and variance $s^2$. The random variables $\{N_{i+1}, i \geq 0\}$ are independent of each other, and are also independent of $\Theta$. The parameter $s^2$ is the intrinsic risk of the firm’s earnings because it is present even when there is perfect information about $\Theta$, and is invariant through time.

The discretionary earnings are described by a Cobb-Douglas production function. The total factor productivity, $A$, is a known constant and the parameters, $\alpha, \beta \in (0,1)$. The discretionary earnings are observable but non-verifiable, and, therefore, non-contractible.

All agents—the manager and outside investors—update their prior beliefs of the project’s core profitability (hereafter, profitability), $\Theta$, over time based on intermediate observations of the
Define the random variable

\[ \xi_{i+1} := E_{i+1} - A k_i^o \eta_i^o = \Theta + N_{i+1}, \quad i = 0, 1, \ldots, T - 1. \]  

(4)

The posterior distribution on \( \Theta \) for each date \( i; \ i \geq 1 \) is normally distributed under the beliefs of the manager (denoted by \( N(\mu_i^M, \sigma_i^2) \)) and outside investors (denoted by \( N(\mu_i^S, \sigma_i^2) \)), where

\[ \sigma_i^2 = \frac{s^2 \sigma_{i-1}^2}{s^2 + \sigma_{i-1}^2} = \frac{s^2 \sigma_0^2}{s^2 + i \sigma_0^2}, \]  

(5)

\[ \mu_i^\ell = \frac{s^2 \mu_{i-1}^\ell + \sigma_{i-1}^2 \xi_i}{s^2 + \sigma_{i-1}^2} = \frac{s^2 \mu_0^\ell + \sigma_0^2 \sum_{j=1}^i \xi_j}{s^2 + i \sigma_0^2}, \quad \ell = M, S. \]  

(6)

Note that \( \sigma_i \) tends to zero as \( i \to \infty \). The parameter, \( \sigma_i^2 \), is the project’s transient risk because it represents the degree of uncertainty about the project’s quality that is resolved through time due to Bayesian learning.

Define

\[ \Delta_i := \mu_i^M - \mu_i^S = \frac{s^2 \Delta_0}{s^2 + i \sigma_0^2} = \frac{\sigma_i^2}{\sigma_0^2} \Delta_0, \quad i = 0, 1, 2, \ldots \]  

(7)

We refer to \( \Delta_i \) as the degree of managerial optimism at date \( i \). By (7), the degree of managerial optimism declines deterministically over time as the manager and outside investors update their priors on \( \Theta \) in a Bayesian manner based on observations of the project’s earnings.

### 3.2 The Debt Structure

All long-term debt issued at date zero matures at date \( T \) and is non-callable. Further, if the time horizon \( T \) is sufficiently long (we set it to 30 years in the calibration of the model), we can follow Leland (1998) by assuming that long-term debt is completely amortized so that long-term debtholders (hereafter, bondholders) receive a constant coupon payment, \( d \), in each period as long as the firm remains solvent. The debt coupon payment \( d \), which determines the firm’s long-term debt structure, is later determined endogenously. In addition to long-term debt, the firm also has non-discretionary working capital requirements such as inventories, accounts payable and receivable,
employee wages, etc that are financed through short-term debt. Consistent with empirical evidence (e.g., Fazzari and Petersen, 1993), working capital requirements increase with the firm’s total earnings. For simplicity, we assume that working capital requirements are a constant proportion $\nu$ of the firm’s total earnings flow, where $\nu$ is observable. Hence the firm’s total earnings in period $[i, i + 1]$ net of working capital requirements is

$$Q_{i+1} = (1 - \nu)E_{i+1}.$$  

We hereafter refer to the process $Q_{i+1}$ as the EBITM (earnings before interest, taxes, and the manager’s compensation) because it will be distributed among bondholders, the government (through taxes), the manager, and shareholders.

For now, the firm’s capital structure consists of equity, long-term debt, and short-term debt financing of the firm’s working capital requirements. In Section 4.4, we implement the manager’s optimal contract through an inside equity stake and a cash reserve that offsets the firm’s short-term debt. The explicit incorporation of working capital requirements is necessary for the subsequent calibration of the model because the financing of working capital is an important component of a firm’s total short-term debt in the data. In the calibrated model, therefore, the firm’s capital structure consists of inside equity, outside equity, long-term debt, and short-term debt that reflects the financing of working capital requirements and the manager’s cash compensation.

### 3.3 Contracting and Bankruptcy

Outside investors are risk-neutral, while the manager is risk-averse with inter-temporal CARA preferences. If the manager’s payoff in period $[j, j + 1]$ is $c_j^m$, her effort is $\eta_j$, and bankruptcy does not occur prior to date $T - 1$, her total expected utility at date $i$ is

$$U(c, \eta) = E^M_i \left[ - \exp \left( - \lambda \sum_{j=i}^{T-1} e^{-r(j-i)}(c_j^m - \kappa\eta_j^\gamma) \right) \right].$$  

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In the above, $r$ is the risk-free interest rate, $\lambda$ is the manager’s absolute risk aversion, and $\kappa \eta_j^\gamma$ ($\kappa > 0$ is a constant) is the manager’s disutility of effort in period $[j, j+1]$.

The manager receives dynamic incentives through contracts that could be explicitly contingent on the EBITM process, $Q^i$ defined in (8). We consider an “incomplete contracting” environment in which only single-period contracts are enforceable. As in Chapter 3 of Tirole (2006), capital markets are competitive. The manager offers a contract to the firm’s competitive shareholders in each period, which specifies the division of the firm’s earnings (net of corporate taxes and interest payments) between the manager and shareholders. In our subsequent implementation of the manager’s contracts in Section 4.4, this is equivalent to the manager dynamically issuing (or buying back) financial securities in competitive capital markets.

Following Leland (1994, 1998), debt payments are serviced entirely as long as the firm is solvent. In financial distress when earnings are insufficient to meet debt interest payments, they are serviced through an additional issuance of outside equity. Bankruptcy, therefore, occurs endogenously when the equity value falls to zero. Note that, because the manager’s contracts determine the payout flows to outside equity, they also effectively determine the bankruptcy time. We can extend the model to allow for the manager to continue servicing debt after the equity value falls to zero, which is effectively equivalent to the scenario in which the firm becomes privately held. The manager declares bankruptcy when it is no longer optimal for her to service debt. The implications of the extended model do not differ from those of the simpler model presented here in which bankruptcy occurs when the equity value falls to zero.

For simplicity and concreteness, we assume that the manager leaves the firm upon bankruptcy and the firm is liquidated. The absolute priority of debt is enforced at bankruptcy. The total payoff to bondholders upon bankruptcy is

\[
\text{Bondholders’ Bankruptcy Payoff} = LD(T_b)
\]

\[
= E^S_{T_b} \sum_{i=T_b}^{T-1} e^{-r(i-T_b)} (1 - \tau)(1 - \nu)(1 - \rho)(\Theta + N_{i+1} - \zeta)
\]

\]

13
where the expectation is taken with respect to outside investors’ beliefs at the bankruptcy time, $T_b$. In (10), we see that the payoff to bondholders upon bankruptcy is the net present value of the base earnings flow, $\Theta + N_{i+1} - \zeta$, net of corporate taxes and working capital requirements. The parameter $\rho \in (0, 1)$ represents the firm’s (proportional) bankruptcy costs.

As in traditional principal-agent models with moral hazard (see Laffont and Martimort, 2002), it is convenient to augment the definition of the manager’s contract to also include the manager’s effort choices and the shareholders’ capital investments. We then require that the manager’s contract be incentive compatible or implementable with respect to her effort. Further, without loss of generality, we can view the sequence of single-period contracts for the manager as a single long-term contract that is implemented by the sequence.

Formally, a contract $\Gamma = [c^m(\cdot), \eta, k]$ is a stochastic process describing the manager’s compensation payments $c^m_i(\cdot)$, her effort choices $\eta_i$, and the capital investments $k_i$ in each period $[i, i+1]$. If $\mathcal{F}_i$ denotes the information filtration generated by the history of the firm’s earnings and the discretionary earnings, the process $\Gamma$ is $\mathcal{F}_i$-adapted. The bankruptcy time $T_b$ is an $\mathcal{F}_i$-stopping time (recall that the bankruptcy time is determined by the contract).

Consistent with Holmström and Milgrom (1987) and Gibbons and Murphy (1992), the manager has multiplicatively separable CARA preferences, as specified in (9), and the firm’s earnings flow evolves as a Gaussian process. Following these studies, therefore, we restrict consideration to contracts in which the manager’s compensation in each period has the affine form

$$c^m_i = a_i c + b_i (1 - \tau)(Q_{i+1} - \zeta), \quad i < T_b. \quad (11)$$

The component $a_i$ of the manager’s compensation is determined at the beginning of period $[i, i+1]$ and could, therefore, be interpreted as her cash compensation for the period. The second component $b_i (1 - \tau)(Q_{i+1} - \zeta)$ of the manager’s compensation is contingent on the firm’s earnings $Q_{i+1}$ that are realized at the end of the period and could, therefore, be interpreted as her equity compensation. We express the manager’s compensation in terms of the earnings net of interest payments and taxes.
because it clarifies our subsequent implementation of the manager’s contract through financial securities.

By (11), a feasible contract for the manager can be described by the quadruple \((a, b, \eta, k)\) where \(a\) is the manager’s cash compensation process, \(b\) determines her equity compensation over time, \(\eta\) is her effort process, and \(k\) is the capital investment process.

### 3.4 Feasible Contracts

In any period \([i, i+1); i < T_b\) the total after-tax earnings flow is

\[
c^a_i = (1 - \tau)Q_{i+1} + \tau d, \quad i < T_b.
\]

(12)

The above reflects the fact that corporate taxes are incurred on earnings net of debt interest payments. Next, the payoff to bondholders during the period is the long-term debt coupon payment

\[
c^d_i = d, \quad i < T_b.
\]

(13)

Finally, the payoff to outside shareholders is the after-tax earnings net of payments to the manager as well as bondholders.

\[
c^s_i = c^a_i - c^m_i - c^d_i, \quad i < T_b.
\]

(14)

We now describe the constraints that must be satisfied by feasible contracts. At any date \(i\), the manager’s conditional expected utility from a contract \(\Gamma = (a, b, k, \eta)\) is

\[
M(i) := E_i^M \left[ - \exp \left( - \lambda \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)}(c^m_j - k\eta^j_j) \right) \right) \right],
\]

(15)

where \(E_i^M\) denotes the expectation with respect to the manager’s beliefs at date \(i\). A feasible contract must be incentive compatible for the manager, that is, it must be optimal for the manager.
to exert the effort $\eta$ specified by the contract given her compensation structure. Therefore,

$$
\eta = \arg \max_{\eta'} E_i^M \left[ -\exp\left(-\lambda \left( \sum_{j=i}^{T_h-1} e^{-r(j-i)}(c_j^m - \kappa \eta_j^r)\right) \right) \right]. 
$$

(16)

By (1), the firm’s total earnings flow in the absence of any actions by the manager is simply the base earnings. By (8), therefore, the after-tax payout flow to the firm without any actions by the manager is $(1 - \tau)(1 - \nu)(\Theta + N_{i+1}) + \tau d$. Consequently, the after-tax payout flow to shareholders without any actions by the manager is $(1 - \tau)\left((1 - \nu)(\Theta + N_{j+1}) - d\right)$. Because shareholders competitively allocate capital, they accept a contract if and only if the net present value of their expected payout flows under the contract at each date is at least as great as the net present values expected payout flow without any actions by the manager, that is, the following dynamic constraints must be satisfied for a contract to be feasible:

$$
E_i^S \left[ \sum_{j=i}^{T_h-1} e^{-r(j-i)}(c_j^b - k_j) \right] \geq E_i^S \left[ \sum_{j=i}^{T_h-1} e^{-r(j-i)}(1 - \tau)\left((1 - \nu)(\Theta + N_{j+1}) - d\right) \right], \text{ for all } i < T_h.
$$

(17)

In Section 6.3, we show that our main testable implications are robust to an extended model in which shareholders have nonzero bargaining power vis-a-vis the manager. In this extended model, the manager only appropriates a portion of the surplus she generates from her human capital, that is, the constraints (17) are modified so that the right hand side includes an additional term that represents a portion of discretionary earnings in each period.

### 3.5 The Manager’s Financing and Contract Choices

The manager chooses the firm’s long-term debt at date zero and her subsequent contract to maximize the total expected utility she derives due to her payoff at date zero from financing the firm’s initial investment outlay and her future payoffs from operating the firm. In a rational expectations equilibrium, the proceeds from debt and equity issuance at date zero to finance the project are equal to their respective market values. For a given long-term debt coupon $d$ and contract $\Gamma \equiv (a, b, \eta, k)$, $c_j^d$ and $c_j^s$ are the corresponding payout flows to bondholders and shareholders in period $[i, i+1]$ as...
described in (13) and (14). The market values of long-term debt $LD(0)$ and equity $S(0)$ are given by

$$LD(0) = E^S_0 \left[ \sum_{i=0}^{T_b-1} e^{-r_i c_i^d} + e^{-r T_b} LD(T_b) \right],$$  \hfill (18)

$$S(0) = E^S_0 \left[ \sum_{i=0}^{T_b-1} e^{-r_i (c^m_i - k_i)} \right].$$  \hfill (19)

Note that the long-term debt and equity values depend on the long-term debt structure and the manager’s contract; we avoid explicitly indicating this dependence for simplicity. The net payoff generated from external financing at date zero is $[LD(0) + S(0) - I]$. Because the manager holds a stake $g_{\text{initial}}$ in the initial all-equity firm, her initial payoff is a proportion $g_{\text{initial}}$ of the net payoff from external financing.\(^2\) The manager’s valuation of her future contractual compensation payments or *continuation expected utility* is

$$M(0) = E^M_0 \left[ - \exp \left( - \lambda \left( \sum_{i=0}^{T_b-1} e^{-r_i (c^m_i - k_i)} \right) \right) \right].$$  \hfill (20)

The optimal long-term debt coupon $d^*$ and the manager’s optimal contract $\Gamma^*$, therefore, solve the following optimization problem:

$$(d^*, \Gamma^*) = \arg \max_{(d, \Gamma)} \exp \left( - \lambda \left( g_{\text{initial}}(LD(0) + S(0) - I) \right) \right) \left[ \text{Continuation Expected Utility} \right].$$  \hfill (21)

Note that the two components of the manager’s total expected utility are multiplied because the manager has multiplicatively separable CARA preferences.

\(^2\)We can show that it is optimal for the manager to sell her initial equity stake $g_{\text{initial}}$ at date zero. To avoid complicating the analysis, we assume this result in the subsequent discussion (the proof is available upon request). The intuition for the result hinges on the fact that the only potential benefit from retaining an equity stake is the provision of appropriate effort incentives for the manager. However, these incentives are already provided by her ex post contract with shareholders. More precisely, if the manager were to retain any equity stake after date zero, her ex post contract with shareholders would rationally “adjust” for her existing exposure to firm-specific risk through her equity stake so that her “total incentives,” which determine her effort in each period, would be unaltered. Further, as we show in Section 4.4, the manager’s compensation contract can be implemented by requiring the manager to hold an inside equity stake that provides her with the appropriate incentives.
4 The Equilibrium

We derive the equilibrium in two steps. In step one, for a given long-term debt structure \( d \), we derive the manager’s optimal contract, which specifies the manager’s effort, the incremental capital investments, payoffs to both parties, and the bankruptcy time. In step two, we derive the manager’s optimal choice of long-term debt, \( d^* \).

4.1 The Optimal Contract for a Given Long-Term Debt Structure

The following theorem characterizes the manager’s optimal contract for a given long-term debt structure \( d \).

**Theorem 1 (The Manager’s Optimal Contract)**

In any period \([i, i+1]\) for \( i < T_b \), the manager’s optimal contract for a given long-term debt structure \( d \) is characterized as follows:

(a) The manager’s pay-performance sensitivity \( b_i^* \) solves

\[
\max_{b_i \geq 0} F_i(b_i),
\]

where

\[
F_i(b_i) = (1 - \tau)(1 - \nu)\Delta_i b_i + Bk(b_i) - 0.5\lambda(1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s^2)b_i^2; \\
B = \left( (1 - \alpha)\gamma - \beta \right)/\alpha\gamma
\]

(b) The incremental capital investment is

\[
k_i^* = k(b_i) = \left[ \frac{\alpha\gamma}{\gamma - \beta} \psi(b_i^*) \right]^\frac{\gamma - \beta}{(1 - \alpha)\gamma - \beta}
\]

(c) The manager’s effort is

\[
\eta_i^* = \eta(b_i^*, k_i^*) = \left[ \frac{A(1 - \tau)(1 - \nu)\beta k_i^*\alpha b_i^*}{\gamma\kappa} \right]^\frac{1}{\gamma - \beta}
\]
(d) The manager’s cash compensation is

\[ a_i^* = a(b_i^*, k_i^*) = (1 - \tau)(d - (1 - \nu)\mu_i^S)b_i^* + (1 - \tau)(1 - \nu)Ak_i^*\eta_i^\beta(1 - b_i^*) - k_i^* \]  

(26)

(e) The endogenous bankruptcy time solves the following optimal stopping problem:

\[ T_b = \arg \max_{t \leq T} E_i^S \left[ \sum_{j=i}^{t-1} e^{-\tau(j-i)}(1 - \tau)(1 - \nu)(\Theta + N_{j+1}) - d \right], \]  

(27)

where the maximization is over all \( \{F_t\} \)-stopping times \( t \leq T \).

Proof. See Appendix A

Given the manager’s and outside investors’ beliefs about the core output, \( N(\mu_i^M, \sigma_i^M) \) and \( N(\mu_i^S, \sigma_i^S) \), respectively, and conditional upon the firm’s solvency at the beginning of period \([i, i+1]\), the optimal contractual parameters for this period, \((a_i^*, b_i^*, k_i^*, \eta_i^*)\), are described in Theorem 1. The equilibrium values for the pay-performance sensitivity, investment, and effort at each point in time are deterministic. The cash component of the manager’s compensation \( a_i^* \) is, however, stochastic and depends on the firm’s earnings history through its effect on shareholders’ posterior mean assessment, \( \mu_i^S \), of the project’s quality or core profitability.

4.2 Bankruptcy Time

The following theorem shows that bankruptcy occurs in any period if and only if shareholders’ mean posterior assessment of the project’s intrinsic quality falls below an endogenous trigger.

**Theorem 2** (Bankruptcy Time)

There exists a trigger \( \mu_i^* \) for \( i = 0, ..., T - 1 \) such that bankruptcy occurs at date \( i \) if and only if \( \mu_i^S \leq \mu_i^* \).

Proof. See Appendix A.

As in Leland (1994, 1998), bankruptcy occurs endogenously when the firm’s equity value falls to zero. As one would intuitively expect, the equity value increases shareholders’ mean posterior assessment of the project’s quality. Consequently, bankruptcy occurs if and only if shareholders’
mean posterior assessment is sufficiently low.

The following theorem describes the effects of project characteristics—shareholders’ initial mean assessment of project quality or the project’s expected “profitability,” the intrinsic risk, and the transient risk—on the timing of bankruptcy.

**Theorem 3 (Project Characteristics and Bankruptcy)**

The bankruptcy time $T_b$ increases with shareholders’ initial mean assessment of the project’s intrinsic quality or its expected future profitability, $\mu^S_0$, increases with the initial transient risk, $\sigma^2_0$, and decreases with the intrinsic risk, $s^2$.

**Proof.** See Appendix A

The firm’s equity value at each date, defined as the right hand side of (27), is the expected value with respect to shareholders’ beliefs. An increase in the shareholders’ initial mean assessment of the project’s future profitability also increases their posterior assessments in each subsequent period and, therefore, the equity value at each date. Accordingly, an increase in the expected future profitability of the project reduces the likelihood of bankruptcy.

The two types of risk have opposite effects on the evolution of the shareholders’ mean assessments of the project’s profitability, and, therefore, the bankruptcy time. The intuition for these differing effects hinges on the differing effects of the intrinsic and transient risk on the evolution of shareholders’ mean posterior assessments. More precisely, by (5) and (6), the variance of the evolution of shareholders’ mean assessment over period of $[i, i+1]$, is

$$(\sigma^\mu_i)^2 := Var_i[\mu_i^{S+1} - \mu_i^{S}] = \frac{s^2}{[(s/\sigma_0)^2 + i + 1][(s/\sigma_0)^2 + i]}.$$  

By (28), we see that an increase in the initial transient risk increases the variance of the evolution of the mean assessments of the project’s profitability. Roughly, an increase in the initial transient risk increases the “signal to noise ratio” so that intermediate signals are more informative about the project’s profitability. Since the equity value is convex in the shareholders’ mean assessment of the project’s profitability, an increase in the initial transient risk increases the “option value” of continuing to service debt payments and delaying bankruptcy.
On the other hand, we also see by (28) that the intrinsic risk decreases the variance of the evolution of mean assessments of project quality because it decreases the signal to noise ratio so that intermediate signals are less informative. Hence an increase in the intrinsic risk decreases the “option value” of delaying the firm’s bankruptcy.

4.3 Optimal Long-Term Debt Structure

The optimal long-term debt structure $d^*$ solves (21), where the manager’s contractual parameters for a given debt structure are described by Theorem 1. Because an analytical characterization of the optimal long-term debt structure is not available, we numerically derive it in Section 6.

As shown by (21), the optimal long-term debt structure maximizes the manager’s expected utility derived from her initial and continuation utilities. For a given long-term debt coupon payment $d$, her initial payoff is proportional to the surplus generated from external financing. As we show in the proof of Theorem 1, the dynamic participation constraints (17) are satisfied with equality at each date. The expectation on the right hand side of (17) is under the beliefs of outside investors and is also not affected by the manager’s actions. It follows that the market value of equity at date zero does not depend on the manager’s beliefs or her future actions. In other words, the competitive market for capital provision by outside shareholders ensures that the manager captures the surplus she generates from her human capital. Consequently, the surplus from external financing at date zero is not affected by the manager’s beliefs or actions. The manager’s beliefs and actions only affect her continuation expected utility $M(0)$.

In Section 6.3, we extend the model to allow for differing allocations of bargaining power between the manager and outside shareholders. In the extended model, shareholders extract a portion of the surplus generated by the manager’s human capital in each period. Consequently, the equity value at date zero is affected by the manager’s future actions. We show that our main testable implications are robust to the incorporation of moderate levels of bargaining power of shareholders.
4.4 Implementation of Manager’s Contract and Dynamic Capital Structure

We now implement the manager’s contract through financial securities. By (8), (13), and Theorem 1, we can rewrite the manager’s compensation payment in any pre-bankruptcy period \([i, i+1]\), (11), as

\[
c_i^m = b_i^*[c_i^{tot} - c_i^d - c_i^{sd}],
\]

where

\[
\begin{align*}
c_i^{tot} &= (1 - \tau)E_{i+1} + \tau d^x, \\
c_i^d &= d^x, \\
c_i^{sd} &= (1 - \nu)E_{i+1} - \bar{a}_i, \quad \text{and} \quad \bar{a}_i = a_i^x/b_i^x,
\end{align*}
\]

where \(E_{i+1}\) is the firm’s total earnings flow over the period described by (1). In (29), \(c_i^{tot}\) is the firm’s total after-tax earnings from the project, \(c_i^d\) is the long-term debt coupon payment, and \(c_i^{sd}\) represents the firm’s total short-term debt payments over the period. Note that, by (30), the total payout flow to short-term debt, \(c_i^{sd}\), reflects the financing of the firm’s working capital requirements and the manager’s cash compensation. In other words, consistent with what we observe in reality, short-term debt reflects the financing of working capital requirements (inventories, accounts receivable and payable, employee wages, etc.) and the cash compensation of the manager (more generally, insiders).

By the above discussion, the expression, \(c_i^{tot} - c_i^d - c_i^{sd}\), in (29) represents the total payout flow to all equity holders—inside and outside—over the period. Since the manager’s compensation is a proportion \(b_i^*\) of the total payout flow to equity in (29), her compensation contract is implemented through an inside equity stake, \(b_i^*\) (that evolves over time), and dynamic short-term lending or borrowing. In this implementation, the market values of long-term debt, short-term debt, and
outside equity at any date \( i < T_b \) are as follows:

Long-Term Debt Value (LD\((i)\)) = \( E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} c_j^d + e^{-rT_b} LD(T_b) \right] \),

Short-Term Debt Value (SD\((i)\)) = \( E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} c_j^{sd} \right] \),

Outside Equity Value (S\((i)\)) = \( E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} (1 - b_j^*) (c_j^{tot} - c_j^d - c_j^{sd}) \right] \). (31)

The above-described implementation of the manager’s contract through financial securities leads to a dynamic capital structure for the firm comprising of inside equity, outside equity, long-term debt, and dynamic short-term (risk-free) debt that reflects the financing of the firm’s working capital requirements and the manager’s cash compensation.

5 Asymmetric Beliefs, Risk-Sharing, and Capital Structure

In this section, we analytically explore some properties of the firm’s capital structure that is described in Section 4.4. The manager’s optimal contract in Theorem 1 has a similar structure to the optimal contract in the general framework of Giat et al. (2010). Given that the focus of this paper is on the effects of asymmetric beliefs on capital structure, the effects of key underlying parameters—the degree of managerial optimism, intrinsic risk, transient risk, and managerial risk aversion—on the manager’s cash and equity compensation are specifically pertinent to our analysis.

The following proposition, which is adapted from Giat et al. (2010), describes the effects of optimism and risk on the manager’s inside equity stake.

**Proposition 1 (Optimism, Risk, and Inside Equity)**

(a) The manager’s inside equity stake \( b_i^* \) at each date \( i < T_b \) increases with the initial degree of managerial optimism \( \Delta_0 \).

(b) The manager’s inside equity stake \( b_i^* \) at each date \( i < T_b \) declines with her risk aversion \( \lambda \) and the initial transient risk \( \sigma_0^2 \).

(c) The manager’s inside equity stake also declines with the intrinsic risk \( s^2 \), provided that \( \Delta_0 \leq 2p_0 \). If \( \Delta_0 > 2p_0 \), the manager’s inside equity stake could vary non-monotonically with the
intrinsic risk.

Proof. See Theorems 4.2 and 4.3 in Giat et al. (2010).

As discussed in Giat et al. (2010), because the manager is optimistic, she overvalues the firm’s future earnings relative to shareholders. Consequently, as the degree of managerial optimism increases, the extent to which she overvalues the performance-sensitive component of her compensation increases. The optimal contract exploits this by increasing the performance-sensitive component of the manager’s compensation, that is, her inside equity stake. The manager’s inside equity stake declines with her risk aversion, the initial transient risk, and the intrinsic risk as an increase in any of these parameters increases the costs of risk-sharing between the principal and the agent.

The above propositions describe the effects of optimism and risk on the equity component of the manager’s compensation. The manager’s cash compensation (and, therefore, the firm’s short-term debt) described in (26) cannot be unambiguously characterized analytically, because conflicting effects could cause it to vary non-monotonically (in general) with the underlying parameters. Consequently, we calibrate the model to data in Section 6 to obtain quantitative assessments of these potentially conflicting forces. Our numerical analysis of the calibrated model yields clear predictions for the variations of the manager’s cash compensation with the underlying parameters.

The following theorem analytically describes the effects of the degree of asymmetry in beliefs, the manager’s risk aversion, and her disutility of effort on long-term debt.

Theorem 4 (Optimism, Risk, and Long-Term Debt)

Long-term debt declines with the degree of managerial optimism, but increases with the manager’s risk aversion and disutility of effort.

Proof. See Appendix A

As discussed in Section 4.3, the net proceeds from external financing do not depend on the manager’s beliefs or future actions, which only affect her expected continuation utility. An increase in the initial degree of asymmetry in beliefs increases her expectation of the firm’s future earnings. Further, as we discussed after Theorem 1, the manager can be provided with more powerful incen-
tives so that she exerts greater effort and generates greater output. Consequently, the manager’s continuation expected utility increases, but her initial payoff from external financing is unaffected. As a result, at the margin, the manager cares more about her continuation expected utility relative to her initial payoff. Hence she chooses lower long-term debt, which lowers the likelihood of bankruptcy and increases her continuation payoff.

An increase in the manager’s risk aversion and/or disutility of effort increases the costs of risk sharing between shareholders and the risk-averse manager. Consequently, the manager exerts lower effort in each period, which lowers the output she generates in each period. As a result, her continuation expected utility decreases. At the margin, therefore, the manager cares more about her initial payoff from external financing. She, therefore, chooses greater long-term debt, which increases her initial payoff from leveraging the firm through the exploitation of ex post debt tax shields.

The effects of intrinsic and transient risks on long-term debt are ambiguous for general parameter values. As shown in Theorem 3, both of these risks have opposing effects on the option value of continuing to service long-term debt coupon payments and, therefore, the bankruptcy time. Moreover, these risks also affect the manager’s compensation in each period. Because the interactions between these forces are complex, an analytical characterization of their net effects for general parameter values cannot be obtained. We numerically explore the effects of the intrinsic and transient risks after calibrating the model to data in the next section.

6 Numerical Analysis

We first calibrate the basic structural parameters of the model using a sample of general firms (excluding financial and regulated firms) from Compustat. We then perform a sensitivity analysis using the baseline parameters to investigate how the firm’s capital structure variables vary with the key parameters of the model. We then calibrate the model to a sub-sample of IPO firms where asymmetric beliefs are likely to play a more important role in influencing financing decisions. A
comparison of the calibrated models corresponding to the two samples allows us to quantitatively investigate the impact of asymmetric beliefs on capital structure.

6.1 Model Calibration

**Basic Economic Parameter Values:** We set the risk-free rate \( r \) to 4.65\%, which is equal to the median rate of 3 month Treasury bills over our sample period, 1993 – 2003, as reported in FRED (Federal Reserve Economic Data). We set the corporate tax rate \( \tau \) to 0.15 that is consistent with the estimates of Graham (2000).\(^3\) The time horizon \( T \) is set to 30 years and the length of each period is one year. The exponent \( \alpha \) in the Cobb-Douglas production function is the capital share of the discretionary earnings in each period (see (1)). We set it to 0.3, which is consistent with U.S. macroeconomic data on the capital share of output. (Our implications are robust to varying choices of \( \alpha \).) We set the bankruptcy cost parameter \( \rho \) to 0.15, which is the midpoint of the estimates reported in Andrade and Kaplan (1998).

**Definitions of the Statistics:** We calibrate the underlying parameters of the model by matching key relevant moments in the data. Our proxy for the asset value is the value of the hypothetically un-levered (all-equity) firm in the absence of the manager’s human capital inputs. It follows that the asset value at any date \( i \), \( AV(i) \), is the present value (with respect to shareholders’ beliefs) of the stream of the firm’s base earnings net of corporate taxes, that is,

\[
AV(i) = E_i^S \left[ \sum_{j=i}^{T-1} e^{-r(j-i)}(1 - \tau)(\Theta + N_{j+1}) \right]. \tag{32}
\]

We set the initial investment outlay \( I \) to the asset value at date zero. Similarly, the firm value or project value is measured as the sum of the present value of the stream of the total after-tax earnings from the project \( c_{i}^{\text{tot}} \) for \( i < T_b \), which is specified in (30), and the present value of the

\(^3\)An “effective” corporate tax rate of 0.15 also incorporates the effects of personal taxes that are not explicitly modeled in our framework.
bankruptcy payoff, that is,

\[
FV(i) = E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} c_j^{tot} + e^{-rT_b} LD(T_b) \right].
\]

(33)

For simplicity, we assume that the total payoff to bondholders upon bankruptcy \( LD(T_b) \) is equal to a fraction \((1 - \rho) \) of the asset value at that date, \( AV(T_b) \).

In the calibration of the model, we match the following moments in the data: (i) the ratio of long-term debt value to asset value, \( LD(0)/AV(0) \) (ii) the ratio of short-term debt to asset value, \( SD(0)/AV(0) \) (iii) the ratio of firm value to asset value, \( FV(0)/AV(0) \) (iv) the ratio of incremental capital investment to asset value, \( k_0^i/AV(0) \) (v) the ratio of total earnings to asset value, \( E_s^S[E_1]/AV(0) \) (vi) the ratio of net income to asset value, \( E_s^S[c_0^i - k_0^i]/AV(0) \) and, finally, (vii) the initial inside equity stake, \( b_0^i \).

**Data Description**: To obtain the observed values of the statistics, we use a set of firm-year observations over the period of 1993-2003, which are reported in Standard and Poor’s Compustat industrial files. We first delete observations with missing variables or with total assets (Compustat item #6) or investment (item #30) that are either zero or negative. We also omit regulated and financial firms.

The long-term debt is obtained from item #9, while short-term debt is measured as debt in current liabilities (item #34) minus debt due in one year (item #44) minus cash (item #1). The firm value is measured by the sum of equity value, which is the closing stock price (item #199) multiplied by the number of common shares outstanding (item #25), and the total value of debt, which is long-term debt (item #9) plus short-term debt (item #34) minus cash (item #1). The total project earnings of the model corresponds to operating income (item #13), and we use investment (item #30) and net income (item #172) to proxy the ratio of net income and investment ratio, respectively. These variables are all divided by asset value (item #6), and we use the median values of the ratios in our calibration. Finally, the median value of inside equity ownership cannot be directly computed because the ownership information corresponding to our sample is not available.
Thus we use the median percentage of ownership of insiders (officers and directors), 14.4%, which Holderness, Kroszner and Sheehan (1999) estimate for exchange-listed firms in 1995 from Compact Disclosure. We set the manager’s initial inside equity stake, $g_{initial}$ to 14.4%.

The set of the remaining parameters to be calibrated is

$$\pi = (A, s, \nu, \mu_0^S, \Delta_0, \sigma, \lambda, \kappa, \gamma/\beta).$$

As described earlier, we only have seven model-predicted statistics to be matched with the corresponding observed statistics in the data. Since at least the same number of statistics as the number of unknown parameter values are required for model identification, we directly estimate the intrinsic earnings risk $s$ and the sensitivity of working capital requirements $\nu$ from the data. First, in the sense that the intrinsic risk is the pure stochastic part in the firm’s total earnings, we proxy this risk by the median value of three-year standard deviations of operating income per share in the data. Next, we set $\nu$ to the slope coefficient from a simple regression of working capital on operating income. Following Rayburn (1986), working capital is computed by current liabilities other than current maturities of long-term liabilities (item #5 - item #44) minus current assets other than cash and short-term investments (item #4 - item #1). In addition, note that the parameters $\beta$ and $\gamma$ cannot be separately identified because all economic variables only depend on the ratio $\gamma/\beta$.

We determine the baseline values of the remaining seven parameters by matching the model-predicted statistics with their observed values. More precisely, their baseline values minimize the (mean squared) difference between the predicted and observed values of the statistics. We describe the numerical implementation of the model in Appendix B.

**Baseline Parameter Values:** Table 1 compares the observed values with the predicted values of the statistics. The model reasonably matches all of the statistics except investment and net income ratios, which are relatively difficult to match because the observed ratios depend on several other factors that are not captured in our stylized model. The baseline values of the parameters are reported in Table 2, where the parameters of the model are grouped into “technology,” “belief,”
and “preference” categories.

The results show that managers are significantly optimistic relative to outside investors. Note that we do not assume that managers are optimistic at the outset; we indirectly infer this from the calibration of the model. Based on the calibrated parameter values, the average manager in the sample overestimates the market to book ratio by 182% relative to outside investors. The ratio of the baseline values of the initial transient risk to the intrinsic risk is 2.4, which suggests that there is significant uncertainty about project quality.

6.2 Sensitivity Analysis

We now explore the effects of managerial optimism $\Delta_0$, the transient risk $\sigma_\theta^2$, the intrinsic risk $s^2$, and outside investors’ initial mean assessment, $\mu_0$, (or the expected profitability of the project) on capital structure by varying these parameters about their baseline values.

6.2.1 The Effects of Managerial Optimism

Figure 1 displays the variations of various components of capital structure—the long-term debt ratio, the short-term debt ratio, and the inside equity stake—with the initial degree of managerial optimism $\Delta_0$. Consistent with Theorem 4, long-term debt declines with the degree of optimism. By contrast, short-term debt ratio increases with managerial optimism. Furthermore, the figure shows that managerial optimism has a much more significant quantitative impact on short-term debt than long-term debt.

To understand the effects of optimism on short-term debt, consider the definition of short-term debt in (30) and (31). By Proposition 1, an increase in managerial optimism increases the manager’s inside equity stake in each period that, in turn, lowers her cash compensation relative to her equity compensation. Second, because the manager receives more powerful incentives and thus exerts greater effort, the firm’s discretionary earnings increase so that the short-term debt payments associated with its working capital requirements increase. Consequently, short-term debt increases with managerial optimism.
Our results are consistent with empirical evidence. Landier and Thesmar (2009) empirically show that optimists prefer short-term debt over long-term debt financing. Using various measures of agreement about project payoffs between managers and investors, Dittmar and Thakor (2007) look at security-issuance decisions and find that managers tend to use debt in the presence of disagreement, which is similar to the result of Malmendier et al (2009) that optimistic CEOs prefer internal to external financing and debt to equity if external financing is necessary. In unreported results, we also find that outside equity declines with the degree of managerial optimism, while the leverage ratio increases. Our results, however, suggest finer predictions about the relative effects of asymmetric beliefs on different components of capital structure—inside equity, outside equity, long-term debt, and short-term debt.

6.2.2 The Effects of Transient Risk and Intrinsic Risk

Figures 2 and 3 show the effects of the initial transient risk and the intrinsic risk on capital structure. The figures show that the two risks have opposing effects on the long-term debt and short-term debt ratios. On the one hand, an increase in the transient risk and/or the intrinsic risk increases the costs of providing incentives (or the costs of risk-sharing) to the manager, which lowers her inside equity stake. The decline in the power of incentives to the manager lowers her effort and the output she generates in each period. This has the effect of lowering the manager’s continuation utility relative to her initial payoﬀ. On the other hand, for a given long-term debt structure, the two types of risk have opposite effects on the bankruptcy time as described in Theorem 3.

Long-term debt declines with the initial transient risk because the “bankruptcy effect” dominates the “risk-sharing” effect in the calibrated model. In other words, an increase in the initial transient risk increases the manager’s continuation utility because bankruptcy is delayed by Theorem 3. As discussed in the intuition for Theorem 4, the marginal impact of the manager’s continuation utility on her long-term debt choice increases. She, therefore, chooses lower long-term debt to increase her continuation utility relative to her initial payoﬀ.

An increase in the intrinsic risk, however, increases the costs of risk-sharing and also hastens
bankruptcy by Theorem 3. Both these effects work in the same direction to lower the manager’s continuation utility relative to her initial payoff. Consequently, the marginal impact of the manager’s continuation utility on her long-term choice is lowered. She, therefore, chooses greater long-term debt to increase her initial payoff from external financing through the exploitation of ex post debt tax shields.

The figures show that the short-term debt ratio declines with the intrinsic risk, but increases with the initial transient risk. An increase in the intrinsic and/or the initial transient risks increases the costs of risk-sharing and, therefore, increases the manager’s cash compensation relative to her equity compensation. By (30), this has the effect of lowering the firm’s short-term debt. An increase in the intrinsic risk lowers the bankruptcy time, which also has a negative effect on the firm’s short-term debt. Consequently, the short-term debt ratio declines with the intrinsic risk. An increase in the initial transient risk, however, increases the bankruptcy time. As discussed above, in the calibrated model, the “bankruptcy effect” of the initial transient risk dominates the “risk-sharing” effect. Consequently, the short-term debt ratio increases with the initial transient risk.

6.2.3 The Effects of Expected Future Profitability

Figure 4 shows the effects of outside investors’ initial mean assessment of the project’s profitability $\mu_0^S$ on the firm’s capital structure. An increase in $\mu_0^S$ increases the expected earnings in each period (see (1)), that is, the firm’s expected future profitability increases with $\mu_0^S$. Figure 4 shows that long-term debt and short-term debt both increase with the expected future profitability.

The effects of the expected future profitability on the manager’s long-term debt choice are subtle because they affect both components of the manager’s objective function in (21). An increase in the expected future profitability increases the market value of outside equity so that the manager’s initial payoff from external financing increases. However, an increase in the expected future profitability also delays bankruptcy by Theorem 3 and, therefore, increases the manager’s continuation utility. For the calibrated model, the former effect dominates the latter. The manager, therefore, chooses greater long-term debt to increase her initial payoff.
By (26), an increase in the expected future profitability lowers the manager’s cash compensation. The intuition is that an increase in \( \mu_0^S \) tightens the shareholders’ dynamic participation constraints (17) and, therefore, lowers the manager’s “guaranteed” cash compensation in each period. By (30) and (31), the firm’s short-term debt value increases.

6.3 Bargaining Power

In the basic model, outside investors are competitive so that the manager captures the surplus she generates from her human capital. We now examine the robustness of the implications of our theory by extending the model to allow for differing allocations of bargaining power between the manager and shareholders.

In the basic model, the discretionary earnings accrue to the manager because she has all the bargaining power. If shareholders have some bargaining power, however, they are able to extract a portion of the discretionary earnings. Let \( \phi \in (0, 1) \) denote the bargaining power of shareholders. The manager’s contract must now satisfy the following dynamic participation constraints for shareholders for all \( i < T_b(\phi) \):

\[
E_i^S \left[ \sum_{j=i}^{T_b(\phi)-1} e^{-r(j-i)} (c_j^s - k_j) \right] \geq E_i^S \left[ \sum_{j=i}^{T_b(\phi)-1} e^{-r(j-i)} (1 - \tau) \left( (1 - \nu)(\Theta + N_{j+1} + \phi \tilde{A}k_j^{s_\alpha} \tilde{\eta}_j^\beta) - d \right) \right].
\]

(34)

In the above, \( \tilde{A}k_i^{s_\alpha} \tilde{\eta}_i^\beta \) is the equilibrium discretionary surplus the manager generates that is rationally anticipated by all agents. \( T_b(\phi) \) is the endogenous bankruptcy time where the argument indicates its dependence on shareholders’ bargaining power. Comparing (17) with (34), we see that the right hand side includes the additional term \( \phi \tilde{A}k_i^{s_\alpha} \tilde{\eta}_i^\beta \). The manager’s contract must, therefore, guarantee shareholders the net present value of the base earnings flow plus a proportion \( \phi \) of the discretionary earnings flow generated by the manager (less taxes and debt payments). The following proposition shows the effects of shareholders’ bargaining power on the manager’s optimal contract.

**Proposition 2 (Bargaining Power of Shareholders)**

*Given the bargaining power \( \phi \) of shareholders, the manager’s cash compensation for period \( [i, i+1] \)
and the market value of equity at date \( i \) are given by

\[
\tilde{a}_i(\phi) = (1 - \tau) \left( d - (1 - \nu)\mu_i^S \right) \tilde{b}_i + (1 - \tau)(1 - \nu)\tilde{A}\tilde{k}_i^\alpha \eta_i^\beta (1 - \tilde{b}_i - \phi) - \tilde{k}_i,
\]

(35)

\[
S(i) = E_i^S \left[ \sum_{j=i}^{T_h(\phi)-1} e^{-r(j-i)} (1 - \tau) \left( (1 - \nu) (\Theta + N_{j+1} + \phi \tilde{A}\tilde{k}_i^\alpha \eta_i^\beta) - d \right) \right].
\]

(36)

In the above, \( (\tilde{b}_i, \tilde{k}_i, \tilde{\eta}_i) \equiv (b_i^*, k_i^*, \eta_i^*) \) where \( b_i^* \) solves (22) and (23), and \( k_i^* \), and \( \eta_i^* \) are given by (24) and (25), respectively.

**Proof.** See Appendix A

Proposition 2 shows that, while the manager’s cash compensation is affected by shareholders’ bargaining power, the manager’s pay-performance sensitivity, the investment and the manager’s effort are not. This is actually a direct consequence of the fact that the allocation of bargaining power does not affect the incentive efficiency of the contract in each period and, therefore, the total surplus that the manager generates. Bargaining power only affects the allocation of the surplus between the manager and the shareholders by changing the manager’s cash compensation.

The optimal long-term debt structure \( d^*(\phi) \) solves the problem in (21). In contrast with the basic model, however, (36) shows that the initial equity value depends on the manager’s human capital inputs because the manager cedes a proportion of the discretionary earnings she generates in each period. We numerically analyze the effects of shareholders’ bargaining power \( \phi \) using the baseline parameter values in Table 2.

Figures 5, 6, 7, and 8 display the variations of long-term debt and short-term debt with key underlying parameters for varying values of \( \phi \). First, notice that long-term debt and short-term debt increase with the bargaining power of shareholders. As shareholders’ bargaining power increases, the manager’s initial payoff increases relative to her continuation utility. Consequently, the importance of the manager’s initial payoff in driving her long-term debt choice increases relative to her continuation utility. Therefore, the manager chooses greater long-term debt. Short-term debt also increases with shareholders’ bargaining power because the manager’s cash compensation
declines as shown in (35).

Second, the figures show that the main testable implications of the theory are robust to differing levels of shareholders’ bargaining power as long as the level is less than a threshold that is approximately 50%. Our empirical implications, therefore, hold for differing allocations of bargaining power between the manager and shareholders if shareholders’ bargaining power is moderate. We believe that the assumption of moderate levels of bargaining power is reasonable in well-developed capital markets (such as the U.S. market) in which firms typically have widely dispersed shareholders.

6.4 Analysis of IPO Firms

It is likely that “new” or “young” firms are more likely to be characterized by asymmetric beliefs about their prospects than older, more mature firms. To examine this hypothesis, and to further explore the key underlying mechanisms that drive our results, we calibrate the model to a sample of firms that go public. We select a sub-sample of 3,308 firms that undertake initial public offerings over the period of 1993-2003. We trace the firms’ variables that are necessary for our calibration immediately after their offerings. We use CEO ownership estimates that Mikkelson, Partch and Shah (1997) report for a sample of 283 initial public offerings by U.S. companies in the years of 1980-1983. To reflect the fact that an initial public offering leads to significant changes in the ownership of a company’s stock, as observed in their study, we set $g_{\text{initial}}$ to 24.8%, which is the median stake of the CEO prior to the offering, and the observed statistic for the inside equity stake in the model to 15.9%, which is the median value of the CEO stake that is followed immediately after the offering.

Table 3 displays the firms’ observed median values and the model-predicted values of the statistics. Compared to the observed values for general firms in Table 1, IPO firms have lower debt ratios and higher market to book ratios. IPO firms also have higher inside equity stakes. Finally, IPO firms have slightly higher investment ratio and lower earnings ratios, which is consistent with the fact that IPO firms tend to be newly formed innovative firms that require more investments and do not yet have stable earnings flows.
Table 4 shows the calibrated parameter values. We observe that managerial optimism in IPO firms is considerably higher than in general firms. To obtain some guidance on the relative effects of managerial optimism in the two samples, we compute the “market to book ratio” (the market value at date zero divided by the initial investment outlay) where the “market value” is computed under the manager’s beliefs and under outside investors’ beliefs, respectively. Relative to outside investors, the average manager in the sample of general firms overestimates the market to book ratio by 182%, while the average manager in the sample of IPO firms overestimates it by 258%.

A comparison of the calibrated values of the intrinsic risk and the transient risk in the two samples shows that transient risk is much higher in relative terms for the IPO sample. The ratio of the project’s transient risk to its intrinsic risk for the sample of general firms is 2.4, while the ratio is 4.4 for the sample of IPO firms.

In other words, without any a priori assumptions on their relative magnitudes, the indirectly inferred values of managerial optimism and transient risk are much higher (in relative terms) for the sample of IPO firms compared with the sample of general firms. We also notice significant differences in the values of the technology and preference parameters. The higher total factor productivity is consistent with high growth opportunities that are typical of entrepreneurial firms. Further, the lower disutility of effort and higher risk aversion parameters also explain typical characteristics of entrepreneurs who are willing to take on highly innovative projects, but are under-diversified relative to managers of general firms.

It is worth emphasizing here that the implications of the calibrations of the model to the two different samples are not a priori obvious. We calibrate the model to a range of statistics and the calibrated parameter set includes a number of parameters other than those that directly pertain to the beliefs of managers and outside investors. Overall, our findings provide further support for the quantitative importance of asymmetric beliefs as a determinant of capital structure (especially for nascent firms). They also suggest that our model is a viable descriptor of the effects of asymmetric beliefs on financial decisions.
7 Conclusions

We develop a dynamic structural model to examine the effects of heterogeneous beliefs and agency conflicts on capital structure. In our dynamic agency model, the manager of a firm has discretion in financing and effort and receives dynamic incentives through explicit contracts with shareholders. Managers and outside investors have imperfect information and differing beliefs about the project’s intrinsic quality.

We implement the manager’s optimal contract through financial securities, which leads to a dynamic capital structure that reflects the effects of external imperfections arising from taxes and bankruptcy costs as well as internal imperfections arising from asymmetric beliefs and agency conflicts. The theoretical and numerical analyses of the model generate several novel testable implications that link project characteristics—the degree of managerial optimism, intrinsic risk, transient risk, and growth opportunities—to different components of capital structure; long-term debt, short-term debt, inside equity, and outside equity.

We calibrate the model parameters to a sample of general firms as well as a sample of IPO firms where uncertainty and asymmetric beliefs are likely to be more important. Consistent with this intuition, we find that managerial optimism and uncertainty about project quality are, indeed, more significant (in relative terms) for the IPO sample. As further support for the viability of the model as a descriptor of firms’ capital structure decisions, we find that the calibrated parameters corresponding to the two samples suggest that IPO firms have higher growth opportunities and the managers/entrepreneurs of IPO firms are more less diversified than general firm managers. Broadly, our study shows that asymmetric beliefs and agency conflicts are important determinants of firms’ financial policies.
Appendix A: Proofs

Proof of Theorem 1

We use backward induction to solve the manager’s optimal contracting problem. We normalize the interest rate to zero to simplify notation in this proof as well as subsequent proofs.

The Optimal Contract in Period \([T-1, T]\)

Suppose that the firm is solvent as of date \(i = T - 1\). The manager’s conditional expected utility derived from her compensation including her disutility of effort over this period is

\[
CEU_i := E_i^M \left[ -\exp \left( -\lambda \left( c_i^m - \kappa \eta_i^2 \right) \right) \right] = -\exp \left( -\lambda \Lambda(a_i, b_i, k_i, \eta_i) \right),
\]

where

\[
\Lambda(a_i, b_i, k_i, \eta_i) = a_i + (1-\tau)(1-\nu) \left( \mu_i^M + Ak_i^{\alpha} \eta_i^{\beta} \right) b_i - \kappa \eta_i^2 - (1-\tau)db_i - 0.5\lambda(1-\tau)^2(1-\nu)^2(\sigma_i^2 + s^2)b_i^2.
\]

Equation (37) is followed from (1), (8), (11), and the fact that \(\Theta + N_{i+1} \sim N(\mu_i^M, \sigma_i^2 + s^2)\) under the manager’s beliefs. Note that maximizing the manager’s conditional expected utility is equivalent to maximizing \(\Lambda(a_i, b_i, k_i, \eta_i)\).

A feasible contract must be incentive compatible, that is, it is optimal for the manager to exert the specified effort in the contract given the other contractual variables \((a_i, b_i, k_i)\). For the same reason as in Giat et al (2010), we assume that \((1-\alpha)\gamma/\beta > 2\), which guarantees a unique optimal effort choice. The manager’s optimal effort \(\eta_i\) that maximizes \(\Lambda(a_i, b_i, k_i, \eta_i)\) is

\[
\eta_i = \eta(b_i, k_i) = \left[ \frac{A(1-\tau)(1-\nu)\beta k_i}{\gamma K} \right] \frac{1}{\gamma-\beta}.
\]

In addition, the shareholders’ participation constraint (17) must be satisfied by the remaining contractual variables \((a_i, b_i, k_i)\):

\[
CV_i := E_i^S \left[ c_i^s - k_i \right] \geq E_i^S \left[ (1-\tau) \left( (1-\nu)(\Theta + N_{i+1}) - d \right) \right].
\]

It is evident that this participation constraint must be satisfied with equality to maximize the manager’s objective function. Consequently, the contractual parameter \(a_i\) as a function of the other two variables \(b_i, k_i\) is

\[
a_i = a(b_i, k_i) = (1-\tau) \left( d - (1-\nu)\mu_i^S \right) b_i + (1-\tau)(1-\nu)Ak_i^{\alpha}\eta(b_i, k_i)^{\beta}(1-b_i) - k_i.
\]
(41) into $\Lambda(a_i, b_i, k_i, \eta_i)$, the objective function to be maximized simplifies to

$$
\Lambda(b_i, k_i) = (1 - \tau)(1 - \nu)\Delta_i b_i + \psi(b_i)k_i^{\alpha_i - \beta} - k_i - 0.5\lambda(1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s^2)b_i^2, \quad (42)
$$

where

$$
\psi(b_i) := (A(1 - \tau)(1 - \nu))^{\gamma_i - \beta}(1 - \nu)(\beta b_i)\frac{\beta}{\gamma_i}(1 - \beta b_i) \quad (43)
$$

It now remains to determine the optimal incremental capital investment and the manager’s optimal pay-performance sensitivity that maximize the manager’s objective function (42). For $b_i \geq \gamma/\beta$, it follows from (42) that the optimal investment is zero. For $b_i \in (0, \gamma/\beta)$, the optimal capital investment as a function of the manager’s pay-performance sensitivity is

$$
k(b_i) = \left[\frac{\alpha \gamma}{\gamma - \beta} \psi(b_i)\right]^{\frac{\gamma - \beta}{\gamma - \beta - \beta}}. \quad (44)
$$

By (44), the optimal contract choice problem reduces to the optimal choice of the manager’s pay-performance sensitivity

$$
b_i^* = \arg\max_{b_i \geq 0} \Lambda(b_i, k(b_i)) = F_i(b_i), \quad (45)
$$

where

$$
F_i(b_i) = (1 - \tau)(1 - \nu)\Delta_i b_i + Bk(b_i) - 0.5\lambda(1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s^2)b_i^2 \quad (46)
$$

and $B = \frac{(1 - \alpha)(\gamma - \beta)}{\alpha \gamma - \beta}$.

In sum, the manager’s optimal contracting problem for period $[T - 1, T]$, conditional on the firm’s solvency in the beginning of this period, is completely specified by $b_i^*$. The optimal capital investment of shareholders $k_i^*$ is $k(b_i^*)$, the manager’s optimal effort $\eta_i^*$ is $\eta(b_i^*, k_i^*)$, and her optimal cash compensation $a_i^*$ is $a(b_i^*, k_i^*)$. Note that the contract exists in period $[T - 1, T]$ if and only if the firm is solvent at date $i = T - 1$. If shareholders’ continuation value based on their beliefs at date $i$ is non-positive, bankruptcy is declared and the relationship is terminated at this date, that is, $T_b = T - 1$.

**The Inductive Step**

We now set $i = T - 2$ and consider the optimal contracting problem for period $[i, i + 1]$. Suppose that the firm is still solvent at date $i$. By the law of iterated expectations, the manager’s conditional expected utility is

$$
CEU_i = E_i^M \left[ - \exp \left( - \lambda \left( \sum_{j=i}^{T_b-1} (c_j^m - \kappa \eta_j^i) \right) \right) \right]
= - \exp \left( - \lambda \Lambda(a_i, b_i, k_i, \eta_i) \right) \left( - CEU_{i+1}^* \right), \quad (47)
$$
where \( \Lambda(a_i, b_i, k_i, \eta_i) \) is given by (38). The last term \(-CEU^*_{i+1}\) equals one if bankruptcy occurs at date \( i + 1 \) because the manager’s payoff is zero. If no bankruptcy is declared, it is the conditional expected utility at date \( i + 1 \), which we determined earlier by the first step in the process of backward induction. In either case, the term is unaffected by the contractual parameters in the current period. Consequently, as in the analysis of period \([T - 1, T]\), the contractual parameters maximize the objective function \( \Lambda(a_i, b_i, k_i, \eta_i) \). Moreover, the participation constraint for shareholders is now given by

\[
CV_i = E_i^S \left[ c_i^S - k_i + \max \left\{ CV_{i+1}, 0 \right\} \right] \geq E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} (1 - \tau) \left( (1 - \nu)(\Theta + N_{j+1}) - d \right) \right]. \tag{48}
\]

If \( CV_{i+1} > 0 \) so that \( T_b > i + 1 \), then it follows from the analysis of the period \([T - 1, T]\) that

\[
CV_{i+1} = E_{i+1}^S \left[ \sum_{j=i+1}^{T_b-1} e^{-r(j-i)} (1 - \tau) \left( (1 - \nu)(\Theta + N_{j+1}) - d \right) \right]
\]

It immediately follows from the above that, regardless of whether bankruptcy is declared at date \( i + 1 \), shareholders’ participation constraint (48) is equivalent to the following:

\[
E_i^S \left[ c_i^S - k_i \right] \geq E_i^S \left[ (1 - \tau) \left( (1 - \nu)(\Theta + N_{i+1}) - d \right) \right],
\]

which is identical in form to (40). Consequently, the optimization problem that determines the optimal contractual parameters is identical to the one for the period \([T - 1, T]\). Hence, the optimal contractual parameters for period \([T - 2, T - 1]\) can be determined in the same manner as in the contracting problem for period \([T - 1, T]\). The above arguments can clearly be extended by backward induction to any period \([i, i + 1]\) for \( i < T - 2 \). Q.E.D.

**Proof of Theorem 2**

As mentioned earlier, the bankruptcy time is endogenously determined by the optimal stopping problem (27). The equity value at date \( i \leq T - 1 \) is given by (48). We denote the continuation value as \( CV_i(\mu_i^S) \) to make explicit its dependence on shareholders’ mean posterior assessment of project quality at each date. Let \( Z \) denote a standard normal random variable. By (4), (5), and (6), the continuation value in (48) can be expressed as

\[
CV_i(\mu_i^S) = (1 - \tau) \left( (1 - \nu)\mu_i^S - d \right) + E_i^S \left[ \max \left\{ CV_{i+1}(\mu_i^S + \sigma_i^S Z), 0 \right\} \right], \tag{49}
\]

where we replace \( \mu_{i+1}^S \) with \( \mu_i^S + \sigma_i^S Z \), and \( \sigma_i^S \) is defined in (28). We first need to show that the continuation value, \( CV_i(\cdot) \), is a continuous, non-decreasing function of \( \mu_i^S \) for \( 0 \leq i \leq T - 1 \). To do so, we use backward induction. For the proof of its continuity, we show that there exist positive
constants $\kappa_i^1$, $\kappa_i^2$ such that
\[ CV_i(\mu_i^S) \leq \kappa_i^1 + \kappa_i^2 \max\{\mu_i^S, 0\}. \tag{50} \]

The assertions of monotonicity and continuity for date $T - 1$ immediately follow because $CV_{T-1}(\mu_{T-1}^S) = (1 - \tau)(1 - \nu)\mu_{T-1}^S - d$. Suppose the assertion is true for dates $i + 1, \ldots, T - 2$. Then we only need to demonstrate based on the assumption that the assertions also hold for date $i$.

The monotonicity of $CV_i(\cdot)$ is a direct consequence of the fact that the expectation on the right-hand side of (49) is taken with respect to the standard normal density and the monotonicity of $CV_{i+1}(\cdot)$ by the inductive assumption.

The proof of continuity of $CV_i(\cdot)$ will follow from (49) if the limit and expectation operators may be interchanged, since $CV_{i+1}(\cdot)$ is continuous in $\mu_{i+1}^S$ and, therefore, in $\mu_i^S$ by the inductive assumption. Due to the assumed continuity for date $i + 1$, the function $CV_{i+1}(\cdot)$ is bounded by a positive function whose expectation,

\[ E\left[\kappa_{i+1}^1 + \kappa_{i+1}^2 \max\{\mu_{i+1}^S, 0\}\right] = \kappa_{i+1}^1 + \kappa_{i+1}^2 \left[\frac{\sigma_i^\mu}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\mu_i^S}{\sigma_i^\mu}\right)^2} + \mu_i^S P\left(Z > -\frac{\mu_i^S}{\sigma_i^\mu}\right)\right], \tag{51} \]

is finite, and thus the interchange is justified by the dominated convergence theorem. Further, using (51), we can show that the expectation on the right-hand side of (49) is bounded above by

\[ \left(\kappa_{i+1}^1 + \kappa_{i+1}^2 \frac{\sigma_i^\mu}{\sqrt{2\pi}}\right) + \kappa_{i+1}^2 \max\{\mu_i^S, 0\}. \tag{52} \]

Hence it is possible to define positive constants $\kappa_i^1$ and $\kappa_i^2$ for which (50) holds for date $i$, as required.

So far, we have seen that each function $CV_i(\cdot)$ is continuous and non-decreasing. By (49), each $CV_i(\cdot)$ is negative for sufficiently small $\mu_i^S$. Since each $CV_i(\cdot)$ is obviously positive for sufficiently high $\mu_i^S$, there exists a unique value $\mu_i^*$ for which $CV_i(\mu_i^*) = 0$. By the dynamic programming principle of optimality, bankruptcy is declared at date $i$ if and only if $\mu_i^S \leq \mu_i^*$. Q.E.D.

**Proof of Theorem 3**

First, from the proof of Theorem 2, we know that $CV_i(\cdot)$ is a continuous, non-decreasing function of $\mu_i^S$ and, therefore, of $\mu_0^S$ because of its linearity on $\mu_i^S$, as shown in (6).

We now demonstrate that $CV_i(\cdot)$ increases with the initial transient risk $\sigma_0^2$, but decreases with the intrinsic risk $s^2$. Using backward induction, as in the proof of its monotonicity and continuity, we first show that each $CV_i(\cdot)$ is convex in $\mu_i^S$. It obviously holds for date $T - 1$ because it is a linear function of $\mu_{T-1}^S$. Suppose the assertion is true for dates $i + 1, \ldots, T - 2$. Based on the assumption, we need to establish the assertion for date $i$. By the definition of a convex function, we need to show that, for any two points $x$ and $y$ in the domain of $CV_i(\cdot)$ and any $t \in [0, 1]$, $CV_i(tx + (1 - t)y) \leq tCV_i(x) + (1 - t)CV_i(y)$. Using (49) and the inductive assumption that $CV_{i+1}(\cdot)$ is convex in $\mu_{i+1}^S$, we see that the left hand side of this inequality is less than or equal to
(1 − τ)(1 − ν)(tx + (1 − t)y) − d + E_i^S \left[ \max \left\{ tCV_{i+1}(x + \sigma_i^sZ) + (1 − t)CV_{i+1}(y + \sigma_i^sZ), 0 \right\} \right],
which is also less than or equal to its right hand side. Hence the equality is satisfied for date i, which completes the proof of the convex property of CV_i(.).

By (49), we notice that the risk parameters, \sigma_i^2 and s^2, affect each CV_i(\cdot) only through \sigma_i^s. It is straightforward to show that \sigma_i^s, defined in (28), increases with \sigma_i^2, but declines with s^2. Let \bar{\mu}_i^{S} denote a normal random variable with the same mean \mu_i^S and greater variance (\bar{\sigma}_i^s)^2 attributed to an increase in \sigma_i^2 or a decrease in s^2. Then the distribution of \mu_i^{S} second-order stochastically dominates that of \bar{\mu}_i^{S}. By the convexity of max\{CV_i^{S}(\cdot), 0\}, it follows that
\begin{equation}
E_i^S \left[ \max \left\{ CV_i^{S}(\bar{\mu}_i^{S}), 0 \right\} \right] \geq E_i^S \left[ \max \left\{ CV_i^{S}(\mu_i^{S}), 0 \right\} \right],
\end{equation}
which implies that each CV_i(\cdot) increases with \sigma_i^2, but declines with s^2.

As shown by Theorem 2, the optimal bankruptcy trigger, \mu_i^*, is uniquely determined by the condition, CV_i(\mu_i^*) = 0. Since CV_i(\cdot) increases with \mu_i^S, increases with \sigma_i^2, but decreases with s^2, the bankruptcy time increases with \mu_i^S, increases with \sigma_i^2, and decreases with s^2. Q.E.D.

**Proof of Theorem 4**

The manager’s objective function specified in (21) for the optimal choice of long-term debt coupon payment can be written as
\begin{equation}
G(d^*(\pi), \pi) := \max_d \left[ g_{\text{initial}}(LD(0) + S(0) − I) + \sum_{i=0}^{T_b-1} F_i(b_i^*(\pi), \pi) \right],
\end{equation}
where \( F_i(\cdot) \) is given in (46). We denote the indirect objective function by \( G \) and indicate the dependence of \( G \) on the parameter \( \pi \in \{\Delta_0, \lambda, \kappa\} \). Since
\begin{equation}
\frac{\partial d^*}{\partial \pi} = -\frac{\partial^2 G/\partial d \partial \pi}{\partial^2 G/\partial d^2} \bigg|_{d=d^*},
\end{equation}
due to the implicit function theorem and \( \partial^2 G/\partial d^2|_{d=d^*} < 0 \) by the second order condition for a maximum, the sign of (55) is identical to that of \( \partial^2 G/\partial d \partial \pi \). Moreover, the manager’s initial payoff—her share of the firm’s net proceeds from external financing—does not depend on parameter \( \pi \), which in turn implies that
\begin{equation}
\frac{\partial^2 G/\partial d \partial \pi}{\partial d^2} = \frac{\partial^2 \sum_{i=0}^{T_b-1} F_i(b_i^*(\pi), \pi)}{\partial d \partial \pi}.
\end{equation}

By (46) and the definition of \( b_i^* \), we notice that the function \( Q \equiv \sum_{i=0}^{T_b-1} F_i(b_i^*(\pi), \pi) \) only depends on \( d \) through the time of bankruptcy \( T_b \). As shown in (49), each CV_i(\cdot) decreases with \( d \) so that the bankruptcy time, \( T_b \), decreases monotonically with \( d \). It can be easily shown by (43), (44), and (46) that \( F_i(\cdot) \) increases with \( \Delta_0 \) and declines with \( \lambda \) and/or \( \kappa \), which implies that the sign of
(56) is non-positive (non-negative) when \( \pi = \Delta_0 \) (respectively, \( \pi \in \{ \lambda, \kappa \} \)). Therefore, long-term debt decreases monotonically with the initial degree of managerial optimism, but increases with the manager’s risk aversion and/or disutility of effort. Q.E.D.

**Proof of Proposition 2**

In what follows, we solve the manager’s optimal contracting problem when shareholders have bargaining power \( \phi \). The procedure is similar to the proof of Theorem 1, and we only focus on the contracting problem for period \([i, i+1]\) with \( i = T - 1 \), because the same arguments apply to the earlier dates.

Suppose that the firm is solvent at date \( i \) and has a long-term debt structure \( d \). The manager’s objective function from her conditional expected utility is given by \( \Lambda(a_i, b_i, k_i, \eta_i) \) in (38), and, therefore, her optimal effort function is also determined by \( \eta(b_i, k_i) \) in (39). As mentioned in the text, however, we have the modified participation constraint for shareholders that additionally requires a portion of the equilibrium discretionary earnings at the level rationally anticipated by shareholders (recall that the discretionary earnings are observable but non-verifiable), which is denoted by \( A k_i^\alpha \tilde{\eta}_i^\beta \):

\[
E_i^S \left[ e_i^s - k_i \right] \geq E_i^S \left[ (1 - \tau)(1 - \nu)(\Theta + N_{i+1} + \phi A k_i^\alpha \tilde{\eta}_i^\beta) - d \right].
\]

This constraint must be satisfied with equality, which derives the manager’s cash compensation function as follows:

\[
a_i = a(b_i, k_i) = (1 - \tau)\left( d - (1 - \nu)\mu_i^S \right) b_i + (1 - \tau)(1 - \nu) A k_i^\alpha \eta(b_i, k_i)^\beta (1 - b_i) - k_i - (1 - \tau)(1 - \nu) \phi A k_i^\alpha \tilde{\eta}_i^\beta.
\]

After some algebra, we can rewrite the objective function as

\[
\Lambda(b_i, k_i) = (1 - \tau)(1 - \nu) \Delta_i b_i + \psi(b_i) k_i^{\alpha - \beta} - k_i - 0.5 \lambda(1 - \tau)^2 (1 - \nu)^2 (\sigma_i^2 + s^2) b_i^2 - (1 - \tau)(1 - \nu) \phi A k_i^\alpha \tilde{\eta}_i^\beta,
\]

where \( \psi(b_i) \) is given by (43). As in the proof of Theorem 1, the equilibrium pay-performance sensitivity and the capital investment maximize \( \Lambda(b_i, k_i) \). Compared with (42), there is an additional term regarding the rationally anticipated equilibrium discretionary surplus \( A k_i^\alpha \tilde{\eta}_i^\beta \), but this term does not influence the manager’s optimization problem. Consequently, the optimal pay-performance sensitivity, effort and investment are given by their values \((b_i^*, \eta_i^*, k_i^*)\) in Theorem 1, that is, they are unaffected by the bargaining power parameter \( \phi \). In a rational expectations equilibrium, all agents correctly anticipate the optimal contractual parameters. Consequently, we must have

\[
\tilde{b}_i = b_i^*, \quad \tilde{\eta}_i = \eta_i^*, \quad \tilde{k}_i = k_i^*.
\]
The allocation of bargaining power, however, affects the manager’s cash compensation as follows.

\[ a_i(\phi) = (1 - \tau)(d - (1 - \nu)\mu_i^S)b_i^* + (1 - \tau)(1 - \nu)Ak_i^{*\alpha}\eta_i^{*\beta}(1 - b_i^* - \phi) - k_i^*. \]  

Finally, notice that the market value of equity when shareholders have nonzero bargaining power is

\[ S(i) = E_i^S \left[ \sum_{j=1}^{T_0(\phi) - 1} e^{-r(j-i)(1-\tau)}((1-\nu)(\Theta + N_{j+1} + \phi Ak_j^{*\alpha}\eta_j^{*\beta}) - d) \right]. \]  

Appendix B: Numerical Implementation and Calibration

As noted in the text, we estimate the baseline parameter values that minimize the distance between model-predicted and observed values of the statistics. To this end, we use the Nelder-Mead optimization method—a simplex search method available in MATLAB. Since the numerical method may end up with a local minimum solution, we repeat the optimization routine multiple times by randomizing the choice of the initial seed to increase the likelihood of obtaining a global minimum.

We now describe the computation of the model-predicted statistics for a candidate parameter vector \( \pi \). First, conditional on the firm being solvent, the equilibrium value of the discretionary earnings is deterministic and, therefore, can be pre-computed for each date \( i; i = 0, 1, \ldots, T - 1 \).

Next, we simulate the evolution of the stochastic base earnings process using a discretization approach. Instead of discretizing the values of \( \Theta \) and \( N_{i+1} \) directly, we model the evolution of \( \mu_i^S \) because it evolves based on the realizations of the random parts in a Bayesian manner. At date 0, shareholders’ mean assessment of the core output is \( \mu_0^S \). Let \( n(i) \) denote the number of states at date \( i > 0 \), which linearly increases from period to period, that is, \( n(i) = Mi \) where \( M \) is set to 20, and let \( \mu_{i,j}^S \) denote shareholders’ mean assessment of the core output at the \( j^{th} \) state at date \( i, j = 1, \ldots, n(i) \). The lattice is designed so that the minimal and maximal states at date \( i, \mu_{i,1}^S \) and \( \mu_{i,n(i)}^S \), respectively, are \( \kappa = 2.5 \) standard deviations below and above the minimal and maximal states at date \( i - 1 \). The values for the remaining \( n(i) - 2 \) states are equally spaced between the minimum and maximum states. Based on the values of \( \mu_{i,j}^S \) in the lattice and the normality of earnings process, we also compute the transition probabilities, \( p_{i,j+1,k}^+ \) —the probability that shareholders’ mean assessment will transition from state \( \mu_{i,j}^S \) at date \( i \) to state \( \mu_{i+1,k}^S \) at date \( i + 1 \).

Fix a long-term debt coupon payment, \( d \). We now describe the determination of the bankruptcy time for a given \( d \). Let \( CV_{i,j} \) denote the continuation value of shareholders at state \( \mu_{i,j}^S \). To get the final condition for backward induction, we assume that the firm’s shareholders, whether existing shareholders or existing bondholders due to bankruptcy, continue to operate the firm as long as the state values of \( \mu_{T,j}^S \) are positive, and thus \( CV_{T,j} = (1 - \tau)(1 - \nu)\mu_{T,j}^S/(1 - e^{-r}) \) if \( \mu_{T,j}^S > 0 \), and
$CV_{T,j} = 0$ otherwise. At earlier dates the continuation values are

$$CV_{i,j} = (1 - \tau)(1 - \nu)\mu_{i,j}^S - d + e^{-r}\sum_{k=1}^{n(i+1)} p_{i,j}^{i+1,k} \max\{CV_{i+1,k}, 0\}.$$ 

Starting from period $[T-1,T]$ and working backwards through time, we use dynamic programming to compute the continuation values for all states and dates. Since the true continuation value function is continuous and increasing, we complete the approximation to $CV_i(\cdot)$ by linear interpolation. We then determine the optimal trigger $\mu_i^*$ that solves $CV_i(\mu_i^*) = 0$ for each date $i$.

It remains to determine the optimal long-term coupon payment at date zero. We also exploit the Nelder-Mead optimization method to solve the problem specified in (54). For a given $d$, $LD(0) + S(0)$, $I(= AV(0))$, and $\sum_{i=0}^{T_b-1} F_i$ in the objective function are computed by working backwards through time with the consideration of bankruptcy states. The optimization method evaluates the objective function at differing $d$ values and derives the best solution. Given the optimal solution $d^*$, we finally compute the model-predicted statistics at date zero by backward induction.

References


Table 1: **Observed and Predicted Statistics for General Firms.** The table displays the observed values of the statistics that are used to calibrate the model and their predicted values from the model. We scale all variables, except for the inside equity stake, by the asset value. The observed inside equity stake is consistent with the estimate of Holderness et al (1999), but the other observed values are based on a sample of general firms from the annual Compustat industrial files over the period of 1993-2003. The predicted statistics are estimated from the model, which characterizes the manager’s optimal financing and contracting choices and, therefore, the firm’s dynamic capital structure, using a discretization approach.

<table>
<thead>
<tr>
<th>Observed</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-term Debt Ratio</td>
<td>0.0943</td>
</tr>
<tr>
<td>Short-term Debt Ratio</td>
<td>-0.0754</td>
</tr>
<tr>
<td>Firm Value Ratio</td>
<td>1.1306</td>
</tr>
<tr>
<td>Inside Equity Stake</td>
<td>0.1440</td>
</tr>
<tr>
<td>Investment Ratio</td>
<td>0.0441</td>
</tr>
<tr>
<td>EBITM Ratio</td>
<td>0.0966</td>
</tr>
<tr>
<td>Net Income Ratio</td>
<td>0.0189</td>
</tr>
</tbody>
</table>

Table 2: **Baseline Parameter Values for General Firms.** The table reports the baseline parameter values obtained by the model calibration for general firms. The parameters are classified into three categories: *Technology* parameters including the total-factor productivity, $A$, the intrinsic risk, $s$, and the sensitivity of working capital, $\nu$. *Belief* parameters consisting of the mean assessment of project quality by shareholders, $\mu_0^S$, the degree of heterogeneous beliefs, $\Delta_0$, and the initial transient risk, $\sigma_0$. *Preference* parameters incorporating the manager’s risk aversion parameter, $\lambda$, and disutility of effort parameters, $\kappa$ and $\gamma/\beta$.

<table>
<thead>
<tr>
<th>Technology Parameters</th>
<th>Belief Parameters</th>
<th>Preference Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$s$</td>
<td>$\nu$</td>
</tr>
<tr>
<td>20.48</td>
<td>39.36</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Table 3: **Observed and Predicted Statistics for Firms that Go Public.** The table shows the observed values and model-predicted values of the statistics for IPO firms. While the observed inside equity stake is consistent with the estimate of Mikkelson et al (1997), the other observed values are based on a sample of the firms that go public over the period of 1993-2003 from the annual Compustat industrial files.

<table>
<thead>
<tr>
<th>Observed</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-term Debt Ratio</td>
<td>0.0161</td>
</tr>
<tr>
<td>Short-term Debt Ratio</td>
<td>-0.3348</td>
</tr>
<tr>
<td>Firm Value Ratio</td>
<td>1.9926</td>
</tr>
<tr>
<td>Inside Equity Stake</td>
<td>0.1590</td>
</tr>
<tr>
<td>Investment Ratio</td>
<td>0.0471</td>
</tr>
<tr>
<td>EBITM Ratio</td>
<td>0.0966</td>
</tr>
<tr>
<td>Net Income Ratio</td>
<td>0.0189</td>
</tr>
</tbody>
</table>

Table 4: **Baseline Parameter Values for Firms that Go Public.** The table displays the baseline parameter values obtained by the model calibration for IPO firms.

<table>
<thead>
<tr>
<th>Technology Parameters</th>
<th>Belief Parameters</th>
<th>Preference Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$s$</td>
<td>$\nu$</td>
</tr>
<tr>
<td>75.66</td>
<td>42.83</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Figure 1: The Effects of Managerial Optimism

Figure 2: The Effects of Transient Risk

Figure 3: The Effects of Intrinsic Risk
Figure 4: The Effects of Expected Profitability

Figure 5: The Effects of Managerial Optimism for Varying Levels of Shareholders’ Bargaining Power

Figure 6: The Effects of Transient Risk for Varying Levels of Shareholders’ Bargaining Power
Figure 7: The Effects of Intrinsic Risk for Varying Levels of Shareholders’ Bargaining Power

Figure 8: The Effects of Expected Profitability for Varying Levels of Shareholders’ Bargaining Power